Size and Focus of a Venture Capitalist's Portfolio

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We take a portfolio approach to analyze the investment strategy of a venture capitalist (VC) and show that portfolio size and scope affect both the entrepreneurs' and the VC's incentives to exert effort. A small portfolio improves entrepreneurial incentives because it allows the VC to concentrate the limited human capital on a smaller number of startups, adding more value. A large and focused portfolio is beneficial because it allows the VC to reallocate the limited resources and human capital in the case of startup failure and allows the VC to extract greater rents from the entrepreneurs. We show that the VC finds it optimal to limit portfolio size when startups have higher payoff potential—that is, when providing strong entrepreneurial incentives is most valuable. The VC expands portfolio size only when startup fundamentals are more moderate and when he can form a sufficiently focused portfolio. Finally, we show that the VC may find it optimal to engage in portfolio management by divesting some of the startups early since this strategy allows him to extract a greater surplus. (*JEL* G24)

The existing theoretical work on venture capital has so far concentrated on a venture capitalist's (VC) investment in a *single* entrepreneurial startup in which the VC provides monetary and non-monetary resources to turn the entrepreneur's project idea into a viable business.[1] However, VC funds typically invest in more than one startup at any given time, and engage in active portfolio management to maximize the return from their investment. In recent research, Kaplan and Schoar (2005) find substantial heterogeneity in performance across private equity funds of different size. They also find that better-performing VC funds grow proportionally slower, and they argue that better VCs may choose to stay small (by deliberately limiting the amount of capital raised) to avoid

[1] Inderst, Mueller, and Muennich (2007); Kanniainen and Keuschnigg (2003); and Bernile, Cumming, and Lyandres (2005) provide notable exceptions.

dilution from allocating their limited amount of human capital over a large number of startups.[2]

In this paper, we take a portfolio approach to analyze the investment strategy of a venture capitalist and investigate the optimal size of a VC's portfolio. More specifically, we address the following questions: What determines the size of a VC's portfolio? What are the benefits and costs of having a small versus a large portfolio? What are the strategic aspects of managing a portfolio of startups? Do VCs prefer having a diversified portfolio as opposed to a focused one? How do size and focus of a VC's portfolio affect performance?

We show that holding a small portfolio can be an optimal strategy for a VC even if the VC has access to a large number of potentially profitable startups in the economy. Our analysis starts with the notion that both entrepreneurial effort and VC human capital are essential inputs for a given startup's success.[3] In addition, a key feature of our analysis is to recognize that VC human capital is a fixed resource in limited supply, and cannot be expanded easily. The basic trade-off we investigate is whether a VC should concentrate all the human capital and resources on a small number of startups, or spread them over a larger number of startups. Even if the VC has the ability to raise capital sufficient for a large number of startups, doing so may not be the optimal strategy if the VC cannot back up the monetary investment by the human capital. We show that spreading the human capital and diluting the value-adding capability over a large number of startups may affect performance so adversely that the VC may find it optimal to limit the size of the portfolio.

Our analysis shows that the size and scope of the VC's portfolio affect both entrepreneurial incentives to exert effort and the VC's incentives to make startup-specific investment. A small portfolio benefits the VC for two different reasons. The first is that a smaller portfolio allows the VC to add more value to each startup and, as a response, each entrepreneur finds it optimal to exert higher effort, improving the success potential of the startup. The second benefit of a small portfolio is that it limits the VC's ability to extract rents from startups by inducing competition for the limited human capital and resources. In other words, by holding a small portfolio, the VC commits not to exploit the entrepreneurs by threatening to take resources away from one startup and transferring them to another one. This commitment proves beneficial for *ex ante* entrepreneurial incentives.

There are benefits associated with holding a large portfolio as well. The first is that having a larger number of startups increases the VC's *ex post* bargaining advantage when the startups compete for the limited human capital at a future project's stage. Thus, increasing the number of startups in the portfolio allows

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[2] For anecdotal evidence on the relevance of fund size, see *The Economist*, "Aftershock-Venture Capital" (April 2, 2005): "Some venture capital funds say they have turned away money from investors in order to keep fund sizes down to an amount that can be managed responsibly."

[3] For example, Kortum and Lerner (2000) document that VC-backed firms produce a greater number of and more valuable patents. Furthermore, Bottazzi, DaRin, and Hellmann (2008) find that more active VCs are associated with better startup performance.

the VC to extract a higher surplus from the entrepreneurs. The second benefit of a large portfolio is that it allows the VC to reallocate resources from one startup to another in case one startup fails. We show that the magnitude of both benefits associated with a large portfolio becomes greater as the startups in the VC's portfolio have more related technologies, that is, as the VC's ability to form a focused portfolio increases. This follows from the fact that a more focused portfolio increases both the VC's rent extraction ability and resource reallocation efficiency.

Our main results hinge on the balance between the benefits and the costs of a small versus large portfolio and the VC's ability to form a focused portfolio. We find that a small portfolio is more desirable when startups have a higher potential payoff, lower risk, and less related technologies. These are exactly the conditions under which promoting strong entrepreneurial incentives outweighs the cost of a reduction in the VC's rent extraction ability and resource reallocation efficiency. In contrast, when startups have a lower expected payoff, higher risk, and more related technologies, it becomes more desirable for the VC to form a larger portfolio. Note that a larger portfolio weakens entrepreneurial incentives, but this proves to be less costly for startups with lower expected payoffs and higher risks, since entrepreneurial effort will be lower in such startups even in a small portfolio.

Our model also highlights the value of active portfolio management, a common VC practice observed in real life. We show that a VC with a large portfolio may find it optimal to divest one of the startups early, even if the company's early stage performance is positive. An active portfolio management strategy is desirable from the VC's point of view since it allows the VC to add more value to the remaining startups in the portfolio and to extract more surplus from them. We find that the strategy of early divestiture is optimal when the startups in the portfolio have a high degree of relatedness, and hence, when the VC has a focused portfolio.

Our paper makes several novel contributions. This is the first paper, to our knowledge, that studies the interaction of size and scope of a VC's portfolio. We analyze the costs and benefits of a large versus a small portfolio as well as a focused versus a diversified portfolio. Our paper shows that a VC may prefer to limit portfolio size even if he has access to a large number of potentially profitable startups. Note that the VC's desire to limit portfolio size is not the outcome of the assumption that the supply of good projects is limited, but it derives from the benefit of providing entrepreneurs with stronger incentives. In our model, the VC may prefer to limit the portfolio size precisely because expanding portfolio size will have a negative spillover effect on the existing startups.[4] Furthermore, we show that the VC finds it desirable to have larger

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[4] Thus our paper provides a theoretical explanation for the observation in Kaplan and Schoar (2005) that "passing up less profitable (but potentially still positive NPV projects) could only be an optimal choice for the GP if there are negative spillover effects on the inframarginal deals from engaging in these investments" (p. 1822).

portfolios only when he can form a portfolio of sufficiently related startups, that is, when he can efficiently reallocate resources from one startup to another. Since the ability to reallocate resources proves to be most valuable for startups with high risk and failure rates, this implies that VCs investing in high-tech and risky industries are more likely to form larger portfolios.

Note that the contribution of our paper is not limited to VC investment only. Our paper also addresses the more general topic of the theory of the firm by studying both project size and scope together, and adds to this literature in terms of the optimal number of divisions as well as their relatedness in a given firm. The existing research on the theory of the firm and internal capital markets considers the advantages and disadvantages of firms with a large number of divisions (see, for example, Gertner, Scharfstein, and Stein 1994), but is largely silent about the relatedness of divisions within a firm and its impact on optimal firm size. Stein (1997) shows that firms benefit from being focused because it increases headquarters' gains from "winner-picking." Our model implies that firms benefit from pursuing multiple related activities because doing so allows them to reduce divisional managers' rent extraction ability and to redeploy assets more efficiently within the firm.

In an extension of our model, we analyze how changes in the supply of VCs relative to the supply of entrepreneurial ideas affect optimal portfolio size. We show that an increase in the availability of VCs, keeping all else being constant, leads to a reduction in the optimal portfolio size and an improvement in startup success. Similarly, an increase in the supply of entrepreneurial projects in the economy results in an increase in the optimal portfolio size.

Our work is related to several papers in the recent literature. In a recent paper, Inderst, Mueller, and Muennich (2007) show that VCs may benefit from limiting the amount of capital they raise by having "shallow pockets," since competition for a VC's limited funds provides entrepreneurs with stronger incentives. In our paper, in contrast, competition among startups affects entrepreneurs' incentives negatively. More importantly, the main objective of our paper is to investigate the size and focus of a VC's portfolio. Inderst, Mueller, and Muennich (2007) abstract from determining the optimal size of the VC's portfolio by assuming a fixed number of startups and do not consider the benefits and costs of having a focused portfolio, a key novel feature of our model. Our work is also related to the benefits of having a "narrow business" discussed in Rotemberg and Saloner (1994). They find that firms benefit from pursuing a narrow-business strategy because being "narrow" reduces the negative impact of competition on managerial incentives. In our model, a small portfolio reduces the VC's rent extraction ability, resulting in stronger entrepreneurial incentives.

In Kanniainen and Keuschnigg (2003), further extended by Bernile, Cumming, and Lyandres (2005), adding an additional startup to the portfolio always weakens both the VC's and the entrepreneurs' incentives. In contrast, in our paper, due to the complementarity between entrepreneurial effort and the VC's investment incentives, a large portfolio may result in stronger incentives

for the VC and prove beneficial for both the entrepreneurs and the VC. In addition, in our paper, the VC's ability to extract surplus depends on the degree of relatedness of the startups, and thus portfolio focus. Differently from Kanniainen and Keuschnigg (2003) and Bernile, Cumming, and Lyandres (2005), we derive the optimal size of the VC's portfolio by analyzing the combined impact of portfolio size and focus on incentives.

Our work also contributes to the literature emphasizing the active role of VCs in adding value to their startups, such as Casamatta (2003), Michelacci and Suarez (2004), Repullo and Suarez (2004), and Inderst and Mueller (2004), among others. These papers consider the incentive problems between a single VC and a single entrepreneur, while in our paper we analyze the VC's optimal investment strategy at a portfolio level.

The paper is organized as follows. Section 1 describes our basic model. Section 2 analyzes the basic model and determines the optimal size of the VC's portfolio. Section 3 analyzes the VC's engagement in active portfolio management. Section 4 presents the extensions of our model and discusses the robustness of our results. Section 5 provides the empirical implications of our model. Section 6 concludes. All proofs are in the Appendix.

1. The Model

We consider an economy endowed with two types of risk-neutral agents: VCs and wealth-constrained entrepreneurs. Entrepreneurs are endowed with a project idea that can be turned, with the collaboration of a VC, into a final marketable product. The development of the entrepreneurs' project ideas into a final marketable product requires two stages. The outcome of the first stage is either a success or a failure. If the first stage is successful, then the project is further developed and commercialized during its second stage. If it is a failure, it has no value and is abandoned.

The development of each startup requires the active involvement of both the VC and the entrepreneur, which is described below. A critical feature of our model is that VCs provide capital as well as other value-adding activities for turning entrepreneurs' ideas into viable businesses. We assume initially that VC human capital is a scarce resource in the economy, and that VCs have access to a large supply of entrepreneurs. This assumption reflects the notion that it takes time and experience to accumulate skills and human capital to become a VC.[5] We relax this assumption in Section 4 and study the impact of VC competition for entrepreneurial startups on optimal portfolio size.

There are four dates in our economy, with no discounting. At $t = 0$, the VC chooses the number η of startups to invest in the portfolio. He may invest

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[5] For example, see *The Economist*, "Once Burnt, Still Hopeful" (November 27, 2004): "Perhaps there are simply just a few people in private equity who are very much better at it than their rivals" in explaining the substantial performance gap across private equity funds.

in either one or two startups, or he may decide to make no investment; thus, $\eta \in \{0, 1, 2\}$.

At $t = 1$, the VC makes a noncontractible startup-specific investment at a personal fixed cost of c, with $c > 0$. The VC's investment can be interpreted as the effort to acquire all the project-specific skills and human capital that add value to the startup, including, for example, learning about its technology and business opportunities, and developing all the skills useful in managing the startup. For short, we will refer to these efforts as the VC's startup-specific investment.[6]

We assume that the VC's initial human capital investment with one or two startups has the same cost c. This assumption captures the notion that the VC has only limited time and resources at his disposal, and that he cannot expand the investment proportionally when he has two startups in the portfolio rather than only one. In other words, given the limited resources, the VC can either concentrate all the resources and human capital on only one startup, or spread the resources and human capital over two startups, incurring the same cost c in each case.

The VC's startup-specific investment increases the value of the project, and each startup will have a higher value with the VC's investment than without it. Thus, a VC's investment and entrepreneurial effort are both necessary and complementary inputs for the success of each startup. For simplicity, we normalize the startup payoff to zero if the VC does not make the initial startup-specific investment. Note that this is not a critical assumption. All we need for our results to hold is that the potential payoff from a given startup is higher with the VC's investment than without it.

The project's payoff depends on the number of startups in the VC's portfolio. If a VC invests in only one startup, he concentrates all the initial human capital investment on one project only. In this case, the payoff from the startup, if successful in the first stage and continued during its second stage, is 2Δ.

If the VC invests in two startups, he allocates the initial human capital investment between the two startups. In this case, the projects' payoffs depend on whether only one or both startups are successful in the first stage. If both startups are successful in the first stage (state SS) and if they are both continued into the second stage, each startup generates a payoff of Δ. Note that this feature is an implication of the assumption that a VC who chooses one startup can specialize all the initial investment on one startup only and, as a result, the startup will have a higher payoff than in the case where the VC allocates the initial investment over two startups. Since the cost c of initial investment is the same regardless of whether the VC invests in one or two startups, this assumption implies that the VC can obtain the same total potential payoff from the portfolio, which is then divided among the number of startups in the

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[6] Thus, in our setting, the startup-specific investment represents all the noncontractible VC activities that add value to a venture. For analytical simplicity, we do not explicitly consider the VC's monetary investment in the startups.

portfolio. This "linear" payoff structure ensures that none of our results are driven by the presence of economies or diseconomies of scale in the VC's "production technology."

An important feature of the model is that if one of the startups fails in the first stage (state SF), the VC can concentrate all the resources and human capital exclusively on the successful startup, obtaining a payoff equal to $(1 + \phi)\Delta$, with $0 \le \phi \le 1$. The value of the parameter ϕ depends on the ability of the VC to transfer the startup-specific investment from one startup to the other. Thus, ϕ depends on the degree of relatedness of the two startups, and we interpret it as representing the degree of "focus," or scope, of the VC's portfolio.

Entrepreneurs play a key role during both the first and the second stages of the project. At $t = 1$, after observing the number of startups the VC invests in the portfolio, each entrepreneur exerts effort p, at a cost of $\frac{k}{2}p^2$.[7] The parameter k measures the cost of exerting effort, with $k > 1$. Entrepreneurial effort determines the success probability of the first stage of the project, which becomes known at $t = 2$.

If the first stage of a project is a success, the second stage needs the active participation of both the VC and the entrepreneur. We assume that contracts are incomplete in the sense that it is not feasible to contract *ex ante* on the participation of either the entrepreneur or the VC to the second stage of the project.[8] Note that this assumption is plausible particularly in the context of VC investment. VCs very often finance entrepreneurial projects surrounded by great *ex ante* uncertainty. Not only is it very difficult to describe the final outcome of the project *ex ante*, but it is also often impossible to contract *ex ante* on the level of the VC's and the entrepreneurs' involvement, the amount of human capital and resources to be allocated to the project, and the contingencies (such as the state and progress of the project) under which resources will be available to the startups in the future. If either the entrepreneur or the VC does not participate in the second stage, the project payoff is normalized to zero. We relax this assumption in Section 4, where we allow the entrepreneur to switch to a new VC and to continue with the startup without the original VC.

The inability to contract *ex ante* on the continuation of the project implies that the division of the total surplus between the VC and the entrepreneur(s) is determined through bargaining at the interim stage at $t = 2$. The outcome of this bargaining process is important because it determines the VC's and the

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[7] Note also that, while entrepreneurial effort p is modeled as a continuous variable, with $p \in [0, 1]$, the VC's input is a binary choice between making the initial investment, and thus paying the cost c, or not. We make these assumptions for simplicity, since modeling the VC's investment as a continuous variable as well considerably reduces the analytical tractability of the model.

[8] Thus, contracts are incomplete in the sense of Grossman and Hart (1986) and Hart and Moore (1990, 1994). We recognize that contracts are a very important aspect of venture capital financing in real life. Kaplan and Stromberg (2003) document that VCs indeed use complex contracts designed to mitigate adverse selection, moral hazard, and holdup problems. The main assumption of our paper is that after all these contractual features are accounted for, VC contracts still contain a significant degree of residual incompleteness.

entrepreneurs' payoffs and, thus, their incentives to exert effort and the VC's portfolio-formation decision.

We characterize the payoffs that result from bargaining between the VC and the entrepreneur(s) by using the notion of Shapley value (see, for example, Myerson 1991 and Winter 2002). According to this solution concept, each player obtains the expected value of their marginal contribution to all coalitions that can be formed with all other players engaged in bargaining. While the notion of Shapley value can be justified solely on an axiomatic ground, we will argue later that players' payoffs can be obtained in our model as the outcome of a suitable extensive form bargaining game between the VC and the entrepreneur(s).

To obtain the Shapley value, we first need to specify the set of players engaged in the bargaining process, which we denote by N. The Shapley value is then defined as follows. Let C be a possible (sub)coalition of players from the set of all players engaged in bargaining N, that is, $C \subseteq N$. Let $\Pi_T(C)$ be the total payoff that can be obtained by the players in C if they cooperate, that is, by the (sub)coalition $C \subseteq N$, with $\Pi_T(\varnothing) = 0$. The Shapley value for player $i \in N$, denoted by v_i, is then given by

$$v_i = \sum_{C \subseteq N-i} \frac{|C|!(|N| - |C| - 1)!}{|N|!} (\Pi_T(C \cup i) - \Pi_T(C)). \qquad (1)$$

Intuitively, the Shapley value reflects the notion that each player's payoff from bargaining depends on the player's marginal contribution to the total payoff, given what the other players can obtain by themselves or by forming subcoalitions.

At $t = 3$, the payoff from the project is realized and distributed between the VC and the entrepreneur(s).

2. Model Analysis

2.1 The VC invests in one startup

If the VC invests in only one startup, $\eta = 1$, bargaining at $t = 2$ will take place between the VC and the entrepreneur if the project is successful. Define the Shapley values for the VC and the entrepreneur in this case as v_V^1 and v_E^1, respectively. The VC's expected profit, π_V^1, is then given by

$$\pi_V^1 \equiv p v_V^1 - c, \qquad (2)$$

and the entrepreneur's expected profit, π_E^1, is given by

$$\pi_E^1 \equiv p v_E^1 - \frac{k}{2} p^2. \qquad (3)$$

The surplus allocation between the VC and the entrepreneur(s) (that is, their Shapley values) is then determined as follows. The set of bargaining players is given by the VC and the successful entrepreneur, yielding $N = \{V, E\}$. The total payoff that the VC and the entrepreneur obtain, if they continue with the startup, is 2Δ. This implies that the coalition formed by the VC and the entrepreneur generates a total payoff equal to $\Pi_T^1(V, E) = 2\Delta$. If the VC does not continue with the startup, or the entrepreneur does not participate in the second stage, the project is terminated and both the VC and the entrepreneur obtain a zero payoff. Thus, $\Pi_T^1(V) = \Pi_T^1(E) = 0$. This implies that the Shapley values for the VC and the entrepreneur, v_V^1 and v_E^1, are

$$v_V^1 = \frac{\Pi_T^1(V, E) - \Pi_T^1(E)}{2} = \Delta, \tag{4}$$

$$v_E^1 = \frac{\Pi_T^1(V, E) - \Pi_T^1(V)}{2} = \Delta. \tag{5}$$

From Equations (4) and (5), it is easy to see that the VC and the entrepreneur share equally the total payoff 2Δ that they jointly generate from the project.[9]

In anticipation of the outcome of the bargaining with the VC, the entrepreneur determines her level of effort, p, by maximizing her expected profit π_E^1. By substituting the Shapley value from Equation (5) into Equation (3), we obtain that the effort level p is determined by maximizing the entrepreneur's expected profit, π_E^1, where

$$\pi_E^1 = p\Delta - \frac{k}{2}p^2. \tag{6}$$

Similarly, by substituting the Shapley value (4) into the VC's expected profit (2), we obtain that

$$\pi_V^1 = p\Delta - c. \tag{7}$$

Proposition 1. *If the VC makes the startup-specific investment at $t = 1$, the optimal level of effort exerted by the entrepreneur, p^{1*}, is*

$$p^{1*} \equiv \frac{\Delta}{k}. \tag{8}$$

The corresponding levels of expected profits for the VC and the entrepreneur are

$$\pi_V^{1*} = \frac{\Delta^2}{k} - c, \quad \pi_E^{1*} = \frac{\Delta^2}{2k}. \tag{9}$$

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[9] Note that the payoffs (4) and (5) are also equal to the equilibrium payoffs that the entrepreneur and the VC obtain in a traditional (extensive form) bargaining game with alternating offers, in which the entrepreneur and the VC have the same bargaining power and no outside option (see, for example, Binmore, Rubinstein, and Wolinski 1986).

The VC has an incentive to make the startup-specific investment only if he has a positive expected profit, as presented in the following lemma.

Lemma 1. *The VC makes the startup-specific investment if and only if* $\Delta \geq \Delta_m \equiv \sqrt{ck}$.

If the VC does not make the investment at $t = 1$ (that is, if $\Delta < \Delta_m$), the payoff from the project is zero, the entrepreneur does not exert any effort, and both parties obtain zero profits.

2.2 The VC invests in two startups

When the VC invests in two startups, $\eta = 2$, bargaining over payoffs between the VC and the entrepreneurs depends on whether only one project has, or both projects have, a successful outcome at their first stage, $t = 2$. There are three different possible cases (states of the world): (i) both projects are successful in their first stage—state SS, (ii) one project is successful while the other one is a failure—state SF,[10] (iii) both projects are a failure—state FF.

In the simplest case, where both entrepreneurs fail in the first stage, state FF, both startups are terminated and all agents obtain zero payoffs. Define $i, j = 1, 2$, as then the Shapley value in states SS and SF for the VC and entrepreneur with i, $i = 1, 2$, as $\{v_V^2(SS), v_V^2(SF)\}$ and $\{v_{E_i}^2(SS), v_{E_i}^2(SF)\}$, respectively. The VC's expected profit, π_V^2, is then given by

$$\pi_V^2 \equiv p_i p_j v_V^2(SS) + p_i(1 - p_j)v_V^2(SF) + p_j(1 - p_i)v_V^2(SF) - c,$$
$$i, j = 1, 2, \ i \neq j, \tag{10}$$

and entrepreneur i's expected profit, $\pi_{E_i}^2$, is given by

$$\pi_{E_i}^2 \equiv p_i p_j v_{E_i}^2(SS) + p_i(1 - p_j)v_{E_i}^2(SF) - \frac{k}{2}p_i^2, \quad i, j = 1, 2, i \neq j. \tag{11}$$

The surplus allocation between the VC and the entrepreneur(s) (that is, their Shapley value) is then determined as follows. If the VC has only one successful startup in the portfolio, state SF, the set of bargaining players is given by the VC and the successful entrepreneur, say entrepreneur i, yielding $N = \{V, E_i\}$. In this case, the VC can reallocate the human capital to, and concentrate exclusively on, the startup i, increasing its total payoff from Δ to $(1 + \phi)\Delta$. Thus, the total payoff of the coalition formed by the VC and entrepreneur i is $\Pi_T^{2,SF}(V, E_i) = (1 + \phi)\Delta$. If the startup is not continued, both the VC and the entrepreneur obtain zero payoff, giving that $\Pi_T^{2,SF}(V) = \Pi_T^{2,SF}(E_i) = 0$. This

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[10] Note that given that the two entrepreneurs are identical, it is irrelevant which of the two projects is successful. Therefore, we will treat these two separate but symmetric cases effectively as a single case.

implies that the Shapley values for the VC and the entrepreneur, $v_V^2(SF)$ and $v_{E_i}^2(SF)$, are

$$v_V^2(SF) = \frac{\Pi_T^{2,SF}(V, E_i) - \Pi_T^{2,SF}(E_i)}{2} = \frac{(1 + \phi)\Delta}{2}, \tag{12}$$

$$v_{E_i}^2(SF) = \frac{\Pi_T^{2,SF}(V, E_i) - \Pi_T^{2,SF}(V)}{2} = \frac{(1 + \phi)\Delta}{2}. \tag{13}$$

Note that both the VC's and the entrepreneur's payoffs are increasing in ϕ. This is because when the VC has a portfolio with greater focus, he can transfer the resources more efficiently to the successful startup, which benefits both the VC and the entrepreneur of the successful startup.

If both startups are successful at $t = 2$, state SS, the VC engages in a process of (multilateral) bargaining with both entrepreneurs. In this case, the set of bargaining players is given by the VC and the two successful entrepreneurs, yielding $N = \{VC, E_1, E_2\}$. If the VC continues with both startups, the coalition of the VC and the two entrepreneurs generates the full value 2Δ of the projects. Thus, the coalition's payoff is $\Pi_T^{2,SS}(VC, E_1, E_2) = 2\Delta$. If the VC forms a coalition with one startup only, he will be able to reallocate the human capital, increasing the startup's payoff from Δ to $(1 + \phi)\Delta$. Thus, the payoff of the coalition formed by the VC with only one entrepreneur is $\Pi_T^{2,SS}(V, E_i) = (1 + \phi)\Delta$, for $i = 1, 2$. This implies that the Shapley values for the VC and the entrepreneurs, $v_V^2(SS)$ and $v_{E_i}^2(SS)$, are

$$v_V^2(SS) = \frac{2\Pi_T^{2,SS}(V, E_1, E_2) + 3\Pi_T^{2,SS}(V, E_i)}{6} = \frac{3 + \phi}{6}2\Delta, \tag{14}$$

$$v_{E_i}^2(SS) = \frac{2\left(\Pi_T^{2,SS}(V, E_1, E_2) - \Pi_T^{2,SS}(V, E_j)\right) + \Pi_T^{2,SS}(V, E_i)}{6}$$

$$= \frac{3 - \phi}{6}\Delta. \tag{15}$$

Note that the payoffs (14) and (15) can be obtained again as the outcome of an appropriate extensive form alternating-offers bargaining game.[11] By comparing the VC's payoff in the SS state, $v_V^2(SS)$, with v_V^1 and noting that $\frac{3+\phi}{6} \geq \frac{1}{2}$ for all $0 \leq \phi \leq 1$, it is easy to see that the VC obtains a (weakly) greater fraction of the total surplus, 2Δ, when he has two successful startups in the portfolio than when

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[11] This can be seen as follows (see Stole and Zwiebel 1996a, 1996b; Inderst and Wey 2003 obtain a similar result, although in a different context). The VC leads individual bargaining sessions with one startup at a time, starting, say, with entrepreneur i. If the VC and entrepreneur i reach an agreement, the VC starts a round of bargaining with entrepreneur j. If bargaining between the VC and entrepreneur i breaks down without an agreement, entrepreneur i drops from the bargaining process and the VC engages in bargaining with entrepreneur j, where both players have zero outside options. Equilibrium payoffs are subject to the condition that if the VC reaches an agreement with entrepreneur i and bargaining with entrepreneur j breaks down, then entrepreneur j drops from the bargaining process and the VC and entrepreneur i renegotiate their original agreement through bargaining, where now both players have zero outside option.

he has only one. This happens because having two successful startups decreases each entrepreneur's marginal contribution to the grand coalition $\{V, E_1, E_2\}$ and, thus, his Shapley value. In other words, the presence of a second startup, and the ability to transfer resources from one startup to the other, creates "competition" between entrepreneurs, allowing the VC to extract more surplus from each startup.

The VC's surplus, $v_V^2(SS)$, is increasing in the degree of portfolio focus, ϕ, whereas each entrepreneur's surplus, $v_{E_i}^2(SS)$, is decreasing in the level of portfolio focus, ϕ. Thus, in this state, differently from the SF state, a greater level of portfolio focus ϕ benefits the VC but hurts the entrepreneurs. This is because a greater level of portfolio focus allows the VC to reallocate the resources more efficiently across startups and reduces each entrepreneur's marginal contribution to the grand coalition. As a result, the VC extracts a greater surplus. Note that the VC's ability to transfer resources from one startup to another is critical, since, when $\phi = 0$, we obtain that $v_V^2(SS) = v_V^1$, implying that the VC extracts the same surplus with both one and two successful startups.

We now characterize the entrepreneurs' choice of effort. If the VC makes the specific investment for each startup, entrepreneur i determines her effort level p_i by maximizing her expected profit $\pi_{E_i}^2$. By substituting the Shapley values (13) and (15) into (11), we obtain that the effort level p_i is determined by maximizing the entrepreneur's expected profit $\pi_{E_i}^2$, where

$$\pi_{E_i}^2 = p_i p_j \frac{(3-\phi)\Delta}{6} + p_i(1-p_j)\frac{(1+\phi)\Delta}{2} - \frac{k}{2}p_i^2, \quad i, j = 1, 2, \ i \neq j. \tag{16}$$

Similarly, by substituting the Shapley values (12) and (14) into the VC's expected profit (10), we obtain that

$$\pi_V^2 = p_i p_j \frac{(3+\phi)2\Delta}{6} + p_i(1-p_j)\frac{(1+\phi)\Delta}{2} + p_j(1-p_i)\frac{(1+\phi)\Delta}{2} - c, \\ i, j = 1, 2, \ i \neq j. \tag{17}$$

The first-order condition of Equation (16) is

$$p_i(p_j) = \frac{(3(1+\phi) - 4\phi p_j)\Delta}{6k}. \tag{18}$$

Note that the effort exerted by entrepreneur i decreases in the effort exerted by entrepreneur j, and hence the effort levels are strategic substitutes. This is because when the VC has two startups in the portfolio, in state SS he extracts a higher surplus from each entrepreneur, reducing their expected profits and their incentives to exert effort. The following proposition characterizes the Nash equilibrium.

Proposition 2. *If the VC makes the startup-specific investment for each startup at $t = 1$, the Nash equilibrium level of effort, denoted by p^{2*}, is*

$$p^{2*} \equiv \frac{3(1 + \phi)\Delta}{2(2\phi\Delta + 3k)}. \tag{19}$$

The corresponding levels of expected profits for the VC and the entrepreneurs are

$$\pi_V^{2*} = \frac{3(\phi\Delta + 3k)(1 + \phi)^2\Delta^2}{2(2\phi\Delta + 3k)^2} - c, \tag{20}$$

$$\pi_{E_1}^{2*} = \pi_{E_2}^{2*} = \left[\frac{3((1 + \phi)\Delta)}{2(2\phi\Delta + 3k)}\right]^2 \frac{k}{2}. \tag{21}$$

It is easy to verify that the equilibrium level of entrepreneurial effort, p^{2*}, is increasing in the degree of portfolio focus, ϕ. Portfolio focus has two opposing effects on the level of effort chosen by the entrepreneur. On the one hand, a higher degree of focus allows the VC to extract more surplus from each entrepreneur in the SS state, with a negative effect on entrepreneurial effort. On the other hand, a more focused portfolio allows the VC to reallocate the resources more efficiently to the successful startup in the SF state, where only one of the startups is successful in its first stage, with a positive effect on entrepreneurial effort. As it turns out, the second effect dominates the first effect, and the overall impact of an increase in focus on the level of effort and the expected profits is always positive.[12]

2.3 Portfolio size and incentives

The impact of portfolio size on effort incentives is a key driver of the optimal size of the VC's portfolio. Hence, an important question is whether the entrepreneurs and the VC have stronger incentives to exert effort in a large or a small portfolio.

Portfolio size affects the entrepreneurs' and the VC's incentives as follows. First, a large portfolio allows the VC to extract greater rents, affecting entrepreneurial incentives negatively and the VC's investment incentives positively. Second, a large portfolio benefits the VC because it allows him to reallocate the human capital from one startup to another in case one of the startups fails. This affects the VC's incentives positively, and is stronger when the level of portfolio focus is higher. Third, in a large portfolio, dilution from spreading the VC's resources and human capital over two startups lowers the payoff from exerting effort, and thus reduces both the VC's and the entrepreneurs' incentives to exert effort. This implies that, if the VC makes the startup-specific investment, entrepreneurial incentives to exert effort are always stronger when

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[12] This implies that the VC would always prefer, all else being equal, to invest in portfolios with greater focus. Remember, however, that the parameter ϕ is exogenous in our model, and it characterizes the population of investment projects that the VC can invest in.

the VC has only one startup in the portfolio rather than two.[13] The following lemma presents this result formally.

Lemma 2. *If the VC makes the startup-specific investment with both one and two startups in the portfolio, each entrepreneur always has greater incentives to exert effort when her startup is the only startup in the VC's portfolio:*

$$p^{1*} > p^{2*}.$$

Furthermore, the difference between the levels of effort, $p^{1*} - p^{2*}$, increases in project payoff, Δ, and decreases in the degree of focus, ϕ, and in the entrepreneur's cost of exerting effort, k.

The property that $p^{1*} - p^{2*}$ is increasing in the project payoff Δ is due to the fact that when the VC has two startups, each entrepreneur benefits relatively less from an increase in the project payoff since the VC extracts a greater fraction of the incremental surplus from the entrepreneurs. The difference $p^{1*} - p^{2*}$ is decreasing in the level of focus ϕ. This can be seen by noting that an increase in the degree of focus ϕ increases p^{2*} while it has no effect on p^{1*}, reducing the difference between the two effort levels. Finally, an increase in the cost of effort k always reduces entrepreneurial effort, but relatively more when the VC has only one startup, that is, when the entrepreneur exerts more effort.

Note that, in our paper, the effect of competition for the VC's scarce resources on entrepreneurial effort is different from that in Inderst, Mueller, and Muennich (2007). They found that the VC benefits from limiting the amount of capital he raises since competition for limited funds provides entrepreneurs with stronger incentives. This happens because, by limiting in advance the resources available for the startup's second stage, the VC is committed to continue only the most valuable project. In this way, competition for the VC's limited funds leads each entrepreneur to exert more effort to increase the likelihood that the VC continues her project. In our model, on the contrary, competition in a large portfolio for the VC's limited human capital reduces the startup's payoff and allows the VC to extract more rents, reducing both the expected profits of the entrepreneurs and their incentives. By holding a small portfolio, the VC limits the extent of competition between startups, adds more value to each startup, and commits to extract lower rents from entrepreneurs, with a positive impact on entrepreneurial incentives.

Having examined the net impact of portfolio size on entrepreneurial incentives, it is also important to analyze the net effect of portfolio size on the VC's incentives. Intuitively, for startups with a large payoff and high degree of

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[13] Note that if the VC does not have the incentive to make the initial investment in a portfolio with one startup, the entrepreneurs will exert zero effort.

focus, the VC always has incentives to make the startup-specific investment in both a small and a large portfolio. Similarly, for startups with low payoff and low degree of focus, the VC will not have the incentives to make the startup-specific investment in either a small or a large portfolio. However, for startups with moderate payoff and moderate focus, it is possible that the VC will have the investment incentives in a small portfolio but not in a large portfolio, or vice versa.

For startups with a lower payoff and higher focus, the VC has the incentives to make the initial investment only if he holds a large portfolio. The intuition for this result is that, from Lemma 2, we know that the benefit of a small portfolio in terms of stronger entrepreneurial incentives, measured by $p^{1*} - p^{2*}$, is smaller at lower values of project payoff Δ and higher values of portfolio focus ϕ. In addition, from Equation (14), we have that the VC's rent extraction benefit from a large portfolio is greater at higher levels of ϕ. Hence, startups with a small payoff and a moderate degree of focus will become feasible only if the VC forms a large portfolio.

On the contrary, for startups with a moderate payoff but lower focus, the VC has the incentives to make the initial investment only in a small portfolio, not in a large portfolio. This is because, for startups with a moderate payoff, holding a large portfolio results in weak entrepreneurial incentives and, thus, in low success probability. Furthermore, since the startups are only moderately related, the VC cannot benefit sufficiently from the rent extraction and the resource reallocation advantages of a large portfolio. Hence, startups become feasible only if the VC forms a small portfolio. The following lemma establishes these results formally.

Lemma 3. *There are critical values* $\{\phi_c, \Delta(\phi)\}$ *(defined in the Appendix) such that, for startups with a lower payoff (that is, $\Delta(\phi) \leq \Delta < \Delta_m$) and a high focus (that is, $\phi_c \leq \phi \leq 1$), the VC has the incentive to make the initial startup-specific investment in a large portfolio but not in a small portfolio. Conversely, for startups with a moderate payoff (that is, $\Delta(\phi) \leq \Delta < \Delta_m$) and a lower focus (that is, $\phi < \phi_c$) the VC has the incentives to make the initial startup-specific investment in a small portfolio but not in a large portfolio.*

The results in Lemma 3 differ from those in Kanniainen and Keuschnigg (2003), who found that the VC's incentives are always worse in a large portfolio than in a small one. This happens in their model because in a large portfolio, the VC adds less value to each startup, similar to our model, thus reducing entrepreneurial and the VC's incentives to exert effort. In contrast, in our model, with a large portfolio, the VC can extract greater rents from the entrepreneurs and reallocate the human capital among startups in case of a failure, increasing the expected profits. Thus, the resource reallocation and the rent extraction effects of our model may induce the VC to exert greater effort in a large portfolio than in a small one.

Finally, it is interesting to compare the above results with those in the recent literature on internal capital markets. For example, Stein (1997) shows that internal capital markets improve the internal allocation of resources by allowing headquarters to "pick winners" among competing projects, but does not examine its effect on effort. Closer to our paper, Stein (2002) argues that large hierarchical organizations may reduce divisional managers' incentives to exert effort. In Inderst and Laux (2005), competition for funds between two divisions has a positive effect on divisional incentives when the two divisions are sufficiently similar in terms of their growth potential, and a negative effect when they are very different. In our model, we show that the impact of competition, for limited resources, on incentives depends also on the degree of relatedness among divisions. Relatedness is important when divisions compete not only for capital allocations, as in the traditional internal capital markets models, but also for the (re)deployment of the firm's assets among divisions. Our model suggests that the degree of relatedness among divisions and, thus, firm focus, reduces the negative impact of competition on divisional incentives. It also implies that the ability to redeploy assets from one division to another allows headquarters to reduce the rents that divisional managers can extract from the firm's headquarters. In this way, firm focus limits the extent of divisional managers' rent-seeking behavior, as described in Meyer, Milgrom, and Roberts (1992) and Rajan, Servaes, and Zingales (2000), among others.

2.4 Optimal portfolio size

The VC chooses portfolio size as a result of the interaction of three distinct effects and their impact on incentives. The first one is the "rent extraction effect": the VC can extract greater rents with a larger portfolio. This effect always induces the VC to prefer (all else being equal) a larger portfolio. The second effect is the "resource allocation effect": by investing in two startups, rather than only one, the VC can reallocate *ex post* the resources and human capital from one startup to the other. The strength of this effect depends on the degree of focus of the portfolio, ϕ. If the success probability of each startup is fixed and the same regardless of whether the VC has one or two startups, this effect always leads the VC to prefer a large portfolio to a small one. The third effect is the "value dilution effect": a larger portfolio requires the VC to spread the fixed amount of resources and human capital over a larger number of startups. The result is that the VC adds lower value to each startup. Thus, the value dilution effect favors a small portfolio.

We can now establish the paper's first main results by comparing the VC's expected profit from holding a small portfolio, given by Equation (9), to that from holding a large portfolio, given by Equation (20). The VC's optimal portfolio size depends on the value of the project payoff, Δ, and portfolio focus, ϕ, which may fall in one of three possible regions (see Figure 1): the VC can invest in no startup at all (Region 0), in one startup (Region 1), or in two startups (Region 2), as summarized in the following proposition.
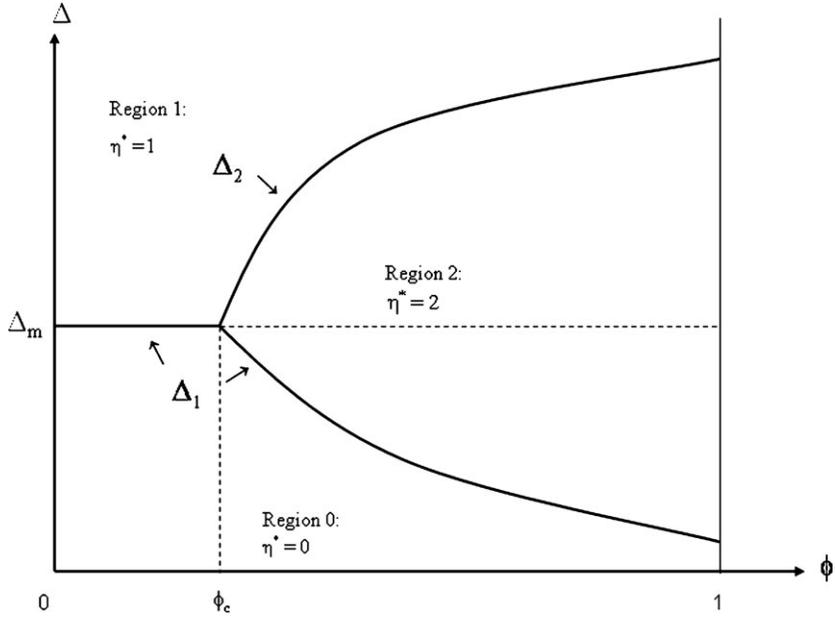
Figure 1
Optimal size of the VC's portfolio
The VC's optimal portfolio size depends on the value of the project's payoff, Δ, and portfolio focus, φ. In Region 0 the VC's expected profits are negative and he does not invest in any startup: $\eta^* = 0$. In Region 1 the VC invests in one startup: $\eta^* = 1$. In Region 2 the VC invests in two startups: $\eta^* = 2$. Δ_m is the value of Δ at which the VC earns zero expected profits when he invests in one startup only; Δ_1 is the value of Δ at which the VC earns zero expected profits when he invests in either one or two startups; Δ_2 is the value of Δ at which the VC is indifferent between investing in one or two startups.

Proposition 3.   *There are critical values $\{\Delta_1(\phi, k), \Delta_2(\phi, k)\}$ (defined in the Appendix) such that the VC's optimal portfolio size is as follows:*

(i)   *for low project payoff $(0 \leq \Delta < \Delta_1)$, the VC invests in no startup, $\eta^* = 0$ (Region 0);*

(ii) *for high project payoff $(\Delta \geq \Delta_2)$, the VC invests in one startup only, $\eta^* = 1$ (Region 1);*

(iii) *for moderate project payoff and high focus $(\Delta_1 \leq \Delta < \Delta_2$ and $\phi_c \leq \phi \leq 1)$, the VC invests in two startups, $\eta^* = 2$ (Region 2).*

Furthermore, $\frac{\partial \Delta_1(\phi,k)}{\partial \phi} \leq 0$ and $\frac{\partial \Delta_2(\phi,k)}{\partial \phi} \geq 0$.

Two key insights emerge from Proposition 3. The first is that the VC finds it optimal to hold a small portfolio when the project payoff, Δ, is relatively high, that is, when $\Delta \geq \Delta_2$ (Region 1). In this region, the benefit of a small portfolio in terms of better entrepreneurial incentives dominates the advantages of a large portfolio in terms of rent extraction and resource reallocation. The intuition for this result is as follows. From Lemma 2, we know that the difference in the

entrepreneurs' effort levels in a small and a large portfolio (that is, $p^{1*} - p^{2*}$) is greater when project payoff Δ is larger. This implies that the negative impact on entrepreneurial incentives of holding a large portfolio is greater at higher values of Δ. Hence, for startups with a large Δ, the VC prefers to give up the greater rent extraction ability and the resource allocation advantages of a large portfolio for the benefits of stronger entrepreneurial incentives of a small portfolio.

Note that the parameter Δ can be interpreted as representing the project's residual expected value, after the first stage is completed. Thus, a greater value of Δ characterizes startups with either larger ultimate payoff or greater ultimate success probability. Proposition 3, therefore, implies that the VC finds it optimal to have a smaller portfolio when he has access to startups with higher payoff potential. By holding a small portfolio, the VC boosts entrepreneurial incentives and increases the success probability of the startups. A small-size portfolio, therefore, is desirable precisely because it allows the VC to obtain superior *ex post* performance from the investment.

The second insight from Proposition 3 is that the VC finds it optimal to hold a large portfolio when the project payoff Δ becomes moderate and when he can form a portfolio with sufficient focus, that is, when $\Delta_1 \leq \Delta < \Delta_2$ and $\phi_c \leq \phi \leq 1$ (Region 2). The intuition for this result is as follows. In this region, a large value of ϕ implies that the rent extraction and resource reallocation benefits of a large portfolio are significant. In addition, the difference between the levels of entrepreneurial effort in a small and a large portfolio is smaller at moderate values of Δ, as established in Lemma 2. Thus, the benefits of a large portfolio dominate its cost in terms of weaker entrepreneurial incentives, and the VC selects a portfolio with two startups. Note also that $\frac{\partial \Delta_2(\phi, k)}{\partial \phi} \geq 0$ implies that, for a given project payoff Δ, the VC finds it optimal to expand the portfolio only at sufficiently high values of ϕ (see again Figure 1). Thus, the VC is willing to increase the size of the portfolio only if he can form a portfolio with sufficient focus by investing in highly related startups.

It is interesting to note that for some parameter values in this region (that is, when $\Delta_1 \leq \Delta < \Delta_m$) both the VC and the entrepreneurs are better off with a large portfolio than a small one. This happens because, as discussed in Lemma 3, with a small portfolio, the VC cannot extract sufficient rents from the entrepreneur to compensate him for the cost of making the initial investment, c. In this case, anticipating that the VC is not willing to make the initial startup investment, the entrepreneur does not exert effort either, and the project is not undertaken even if it is potentially profitable. Investing in a large portfolio, however, provides the VC with the rent extraction and resource reallocation benefits and induces him to make the required initial investment. Anticipating the improved incentives of the VC, the entrepreneurs exert effort and the projects become viable. Thus, both the entrepreneurs and the VC turn out to be better off with a large portfolio.

The above result has an interesting implication that entrepreneurial ideas with a moderate value may be economically viable only if the VC can form a large portfolio with a sufficient degree of focus. If we interpret Δ as measuring the size of a startup, this implies that VCs would be willing to invest in small businesses only if they are able to combine such startups in a portfolio of sufficient size and focus. It also implies that entrepreneurs with smaller businesses may have an incentive to cluster in similar or related industries so that they can be financed by a common VC. A policy implication from this result is that encouraging small business creation in the same or related industries may increase available VC financing, with a potential positive effect on social welfare, because VCs will be willing to provide monetary and human capital to such businesses only if they have a common industry focus.

Finally, when the level of Δ is very low (that is, when $0 \leq \Delta < \Delta_1$; Region 0), the VC does not invest in any startup. In this region, the project payoff is so low that the VC cannot recover the initial investment cost c. As a result, the VC does not make any startup-specific investment and the entrepreneurs do not exert any effort. Hence the project opportunities cannot be exploited.

An increase in the entrepreneur's cost of exerting effort k makes larger portfolios more desirable, as established in the following corollary.

Corollary 1. $\quad \frac{\partial \Delta_2(\phi, k)}{\partial k} \geq 0$

The intuition for this result is as follows. An increase in k reduces entrepreneurial effort, which leads to a lower success probability for each startup and, therefore, to a riskier portfolio. As a result, the VC's willingness to hold a larger portfolio increases, since a lower success rate for each startup increases the importance of the resource reallocation benefit of large portfolios. This implies that the VC is more likely to invest in risky projects with low success rates only if he can form large portfolios of startups with sufficiently related technologies.

3. Portfolio Management

In this section we show that the VC can increase the expected profits by committing to divest one of the successful startups early, that is, by engaging in active portfolio management. This strategy can be optimal since the possibility of divesting one of the startups, even when it is successful, allows the VC to extract more surplus from the remaining one. Hence, this section helps shed some light on why VCs may make seemingly socially inefficient decisions by divesting some of their startups prematurely in order to maximize their own welfare.[14]

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[14] See, for example, *The Economist*, "Once Burnt, Still Hopeful" (November 27, 2004), which reports that "Google's founders would have preferred to wait longer to do their IPO, but had to rush it because venture capitalists, including Kleiner Perkins, wanted to cash in."

We modify our basic model as follows. At the beginning of the game, the VC can invest in one or two startups. Furthermore, if the VC chooses to invest in two startups, we assume that at this time he can also commit to continue only one of the startups in the event that they are both successful (that is, in the SS state). The VC can make this commitment, for example, by limiting the total amount of resources he raises at the beginning of the game. As discussed in Inderst, Mueller, and Muennich (2007), one possibility is that the VC raises an amount of capital that is sufficient to fund only one startup at the development stage. This choice, together with the fact that VC funds are typically close-end, effectively commits the VC to continue at $t = 2$ only one startup, even when both startups are successful. We assume that the proceeds from divesting the startup are lower than the proceeds from continuing with the original VC and, for simplicity, we normalize them to zero.[15] The rest of the model unfolds as before.

The commitment to continue only one successful startup changes the payoff that the VC and the entrepreneurs obtain in state SS. We assume that the VC selects, with equal probability, one of the two successful startups, say startup i, and starts a process of bargaining with entrepreneur i to determine the payoff for the continuation of startup i only. A critical feature here is that while bargaining with entrepreneur i, the VC still has the option to continue with the other startup. As we will show below, under certain conditions, this strategy allows the VC to extract more surplus from startup i.

We characterize the outcome of the (bilateral) bargaining process between the VC and entrepreneur i by using again the Shapley value. The set of bargaining players is given by the VC and entrepreneur i, yielding $N = \{V, E_i\}$. The total payoff from the coalition of the VC with entrepreneur i is $\Pi_T^{2B,SS}(V, E_i) = (1 + \phi)\Delta$. The difference with the basic model is that when bargaining with entrepreneur i, the VC also has the possibility of bargaining with the other entrepreneur for the continuation of her startup only. This implies that the VC's payoff without entrepreneur i is not zero, but rather is the payoff obtained from bargaining with the other entrepreneur only, given by Equation (12). Thus, $\Pi_T^{2B,SS}(V) = \frac{(1+\phi)}{2}\Delta$. Note that we can interpret this payoff as the value of the VC's outside option while bargaining with entrepreneur i. Entrepreneur i still obtains zero payoff if she does not continue with her startup with the VC, that is, $\Pi_T^{2B,SS}(E_i) = 0$. Thus, the Shapley value for the VC and entrepreneur i are now given, respectively, by

$$v_V^{2B}(SS) = \frac{\Pi_T^{2B,SS}(V, E_i) + \Pi_T^{2B,SS}(V)}{2} = \frac{3(1 + \phi)\Delta}{4}, \tag{22}$$

$$v_{E_i}^{2B}(SS) = \frac{\Pi_T^{2B,SS}(V, E_i) - \Pi_T^{2B,SS}(V)}{6} = \frac{(1 + \phi)\Delta}{4}. \tag{23}$$

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[15] Normalizing divestiture payoff to zero is only a simplification. Our results go through as long as divestiture payoff is lower than the payoff with the incumbent VC. This assumption reflects that the incumbent VC, because of his initial startup-specific investment, can generate a greater payoff.

By contrasting Equations (14) and (22), it is easy to see that $\frac{3+\phi}{6} < \frac{3}{4}$ for all $0 \leq \phi \leq 1$. This implies that, by committing at $t = 0$ to divest one of the successful startups, the VC can extract a greater fraction of the surplus, but at the cost of reducing the overall surplus from 2Δ to $(1 + \phi)\Delta$. By direct calculation, it is easy to see that $v_V^{2B}(SS) \geq v_V^2(SS)$ for $\phi \geq \frac{3}{5}$. Thus, at high levels of portfolio focus, committing to active portfolio management results in an increase in the VC's rent extraction ability.

In state SF, the VC has only one successful startup and, therefore, the payoffs are the same as in the basic case. Anticipating her payoffs in different states of the world, entrepreneur i determines her level of effort p_i by maximizing her expected profit, $\pi_{E_i}^{2B}$, where

$$\pi_{E_i}^{2B} \equiv p_i p_j \frac{1}{2} \frac{(1 + \phi)\Delta}{4} + p_i(1 - p_j)\frac{(1 + \phi)\Delta}{2} - \frac{k}{2}p_i^2,$$
$$i, j = 1, 2, \ i \neq j. \tag{24}$$

Correspondingly, the VC's expected profit, π_V^{2B}, is

$$\pi_V^{2B} \equiv p_i p_j \frac{3(1 + \phi)\Delta}{4} + p_i(1 - p_j)\frac{(1 + \phi)\Delta}{2} + p_j(1 - p_i)\frac{(1 + \phi)\Delta}{2} - c,$$
$$i, j = 1, 2, \ i \neq j. \tag{25}$$

The following proposition characterizes the Nash equilibrium of the effort level subgame.

Proposition 4. *The Nash equilibrium level of effort, denoted by* p^{2B*}, *is*

$$p^{2B*} \equiv \frac{4(1 + \phi)\Delta}{3(1 + \phi)\Delta + 8k}. \tag{26}$$

The corresponding levels of expected profits for the VC and the entrepreneurs are

$$\pi_V^{2B*} = \frac{8((1 + \phi)\Delta + 4k)(1 + \phi)^2\Delta^2}{(3(1 + \phi)\Delta + 8k)^2} - c, \tag{27}$$

$$\pi_{E_1}^{2B*} = \pi_{E_2}^{2B*} = \frac{8k(1 + \phi)^2\Delta^2}{(3(1 + \phi)\Delta + 8k)^2}. \tag{28}$$

Note that divesting a successful startup reduces the entrepreneurs' expected profits and, thus, their effort. This happens because each entrepreneur anticipates that their startups will be divested in state SS with probability $\frac{1}{2}$, generating zero payoff. This loss is not fully compensated by the expectation that the startup's payoff will increase, if continued, from Δ to $(1 + \phi)\Delta$, which generates a greater payoff to the entrepreneur, given by $v_{E_i}^{2B}(SS) > v_{E_i}^2(SS)$. This

implies that portfolio management lowers each entrepreneur's expected profits and results in a lower level of effort compared to before, giving $p^{2B*} < p^{2*}$.[16]

At the beginning of the game, the VC must choose the portfolio size and, in case he chooses two startups, whether or not to raise sufficient resources for both startups if they are successful during their first stage. The VC makes the latter choice by trading off the benefit of portfolio management, in terms of greater rent extraction ability, against the costs of reduced entrepreneurial effort and the efficiency loss due to the premature divestiture of a successful startup. As it turns out, both costs of portfolio management decrease in ϕ and, thus, the VC's willingness to engage in portfolio management increases in the level of portfolio focus, as established in the following lemma.

Lemma 4. *There is a threshold value $\phi_P(\Delta)$ (defined in the Appendix), with $\phi_P(\Delta) \in (\frac{27}{37}, 1]$, such that the VC's expected profit with portfolio management, π_V^{2B*}, is greater than that without portfolio management, π_V^{2*}, if and only if $\phi \geq \phi_P(\Delta)$.*

Based on Lemma 3, we characterize the VC's *ex ante* choice of portfolio size in the following proposition (for expositional simplicity, we set $c = 0$).

Proposition 5. *Let $c = 0$. If $\phi \leq \phi_P$, the optimal size of the VC's portfolio is the same as characterized in Proposition 3, and the VC raises resources sufficient to continue with both startups, if they are successful.*

If $\phi > \phi_P$, the VC raises resources sufficient to continue only with one startup. Furthermore, there is a critical value $\Delta^B(\phi, k)$ (defined in the Appendix) such that the VC's optimal portfolio size is as follows:

 (i) *for high project payoff ($\Delta > \Delta^B$), the VC invests in one startup only, $\eta^* = 1$;*
 (ii) *for moderate project payoff ($\Delta \leq \Delta^B$), the VC invests in two startups, $\eta^* = 2$, and the VC divests one successful startup in state SS (that is, he engages in active portfolio management).*

Furthermore, $\frac{\partial \Delta^B(\phi, k)}{\partial \phi} > 0$.

Proposition 5 shows that the focus of the VC's portfolio is a key determinant of the VC's overall portfolio strategy. When portfolio focus is low (that is, when $\phi < \phi_P$), the VC cannot reallocate resources efficiently across startups. In this case, at $t = 0$ the VC raises resources that are sufficient to continue with both startups, and the VC's portfolio choice is the same as in Proposition 3. When portfolio focus is sufficiently large (that is, when $\phi \geq \phi_P$), the VC can reallocate resources across startups with little efficiency loss. As a result, the VC

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finds it optimal to engage in active portfolio management. This means that the VC raises resources that are sufficient to continue only one successful startup at the interim stage, committing himself to divest one of the successful startups in state SS. Note that the main results of our basic model hold in this case as well. When project payoff is sufficiently large (that is, when $\Delta \geq \Delta^B$), the VC still finds it optimal to hold a small portfolio, while for moderate project payoff and high portfolio focus (that is, for $\Delta < \Delta_2^B$ and $\phi \geq \phi_P$), the VC expands portfolio size to two startups. Furthermore, the threshold level Δ^B is an increasing function of ϕ, confirming the earlier result that large portfolios are more likely to be optimal when the VC can form a more focused portfolio.

4. Competition for Entrepreneurs

In our basic model, we assume that VC human capital is in fixed supply and study the portfolio problem of a single VC facing a large supply of entrepreneurs. In this section, we relax this assumption and introduce competition by VCs for entrepreneurs, and examine its effect on optimal portfolio size.[17] In Section 4.1, we consider competition among VCs for startups *ex ante*, at the time of portfolio formation, and in Section 4.2, we consider competition in the interim, when the VC and the entrepreneur(s) bargain for the continuation of a successful startup. For analytical tractability, we analyze these cases separately, and for notational simplicity, we set $k = 1$.

4.1 *Ex ante* competition for entrepreneurs
We model *ex ante* competition for startups by assuming that the entrepreneurs bargain with the VC at the time of portfolio formation for their inclusion in the VC's portfolio. The effect of this bargaining is to reallocate the expected surplus among the VC and the entrepreneur(s).[18] While bargaining with the VC, entrepreneurs have an outside option whose value increases with the level of competition for entrepreneurial projects or the scarcity of entrepreneurial startups relative to the supply of VC human capital. We modify the basic model as follows.

At the beginning of the game at $t = 0$, the VC moves first and selects one or two startups. By choosing whether to form a small or a large portfolio, the VC determines whether to bargain with one or two entrepreneurs. If the VC selects one startup, he bargains with only one entrepreneur and, therefore, the set of bargaining parties is $N = \{V, E\}$. We characterize the outcome of the bargaining process between the VC and the entrepreneur by using again their

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[17] The effect of competition among VCs on effort and surplus allocation is examined also in Inderst and Mueller (2004). In that paper, however, VCs invest in only one startup at a time.

[18] This surplus reallocation may happen, for example, through side transfers between the VC and the entrepreneur(s). For expositional simplicity, we assume that the nonnegativity constraint on the entrepreneurs' wealth is always satisfied.

Shapley values, denoted now by $\tilde{v}_V^1$ and $\tilde{v}_E^1$, respectively. From Equation (9), the payoff for the coalition of the VC with the entrepreneur is given by the total expected profits from the project, $\pi_T^{1*} \equiv \pi_V^{1*} + \pi_E^{1*}$; thus, $\tilde{\Pi}_T^1(V, E) = \pi_T^{1*}$.

The difference from the basic model is that if the VC does not invest in the startup, the entrepreneur can turn to another VC. For simplicity, we model this option for the entrepreneur by assuming that the entrepreneur will find another VC with probability θ_0. The probability θ_0 can be interpreted as a measure of the scarcity of entrepreneurs relative to VCs, and therefore the tightness of the VC market.[19] In other words, an increase in θ_0 represents an increase in the supply of VCs relative to entrepreneurial projects and therefore greater competition for startups. This implies that the entrepreneur's payoff without the original VC is not zero, but is equal to the expected payoff that she would get from bargaining with another VC, that is, $\tilde{\Pi}_T^1(E) = \theta_0 \tilde{v}_E^1$. Hence this payoff can be interpreted as representing the value of the entrepreneur's outside option while bargaining with the initial VC. We continue to assume that the VC alone obtains zero payoff, $\tilde{\Pi}_T^1(V) = 0$, that is, the VC still has no outside option. Under this specification, the Shapley value for the entrepreneur is

$$\tilde{v}_E^1 = \frac{1}{2}\left(\tilde{\Pi}_T^1(E) + \tilde{\Pi}_T^1(V, E)\right) = \frac{\theta_0 \tilde{v}_E^1 + \pi_T^{1*}}{2}, \tag{29}$$

yielding

$$\tilde{v}_E^1 = \frac{\pi_T^{1*}}{2 - \theta_0}. \tag{30}$$

Correspondingly, the Shapley value for the VC is

$$\tilde{v}_V^1 = \frac{1}{2}\left(\tilde{\Pi}_T^1(V) + \tilde{\Pi}_T^1(V, E) - \tilde{\Pi}_T^1(E)\right) = \frac{(1 - \theta_0)\pi_T^{1*}}{2 - \theta_0}. \tag{31}$$

If the VC selects two startups, the set of bargaining parties is now $N = \{V, E_1, E_2\}$. From Equations (20) and (21), the total payoff from the coalition of the VC with both entrepreneurs is equal to the total expected profits from both startups: $\pi_T^{2*} \equiv \pi_V^{2*} + 2\pi_{E_i}^{2*}$. Thus, $\tilde{\Pi}_T^2(V, E_1, E_2) = \pi_T^{2*}$. The payoff of the coalition formed by the VC with only one entrepreneur, say entrepreneur i, is the expected profit the VC can obtain with only one startup, giving $\tilde{\Pi}_T^2(V, E_i) = \pi_T^{1*}$. The entrepreneur will be able to find another VC with probability θ_0, implying that her payoff without the original VC is equal to the expected payoff that she would obtain from bargaining with the new VC. Thus, $\tilde{\Pi}_T^2(E_i) = \theta_0 \tilde{v}_E^1$. Note that this payoff represents the value of the entrepreneur's outside option while bargaining with the initial VC. Thus, the Shapley value

for each entrepreneur, denoted by $\tilde{v}_{E_i}^2$, is now equal to

$$\tilde{v}_{E_i}^2 = \frac{\tilde{\Pi}_T^2(V, E_1, E_2) - \tilde{\Pi}_T^2(V, E_i) + \tilde{\Pi}_T^2(E_i)}{3}$$

$$= \frac{2\pi_T^{2*} - \pi_T^{1*} + \left(2\pi_T^{1*} - \pi_T^{2*}\right)\theta_0}{3(2 - \theta_0)}, \tag{32}$$

and for the VC, denoted by $\tilde{v}_V^2$, is

$$\tilde{v}_V^2 = \frac{\tilde{\Pi}_T^2(V, E_1, E_2) - 3\tilde{\Pi}_T^2(E_i) + \tilde{\Pi}_T^2(V, E_i)}{3}$$

$$= \frac{2(\pi_T^{2*} + \pi_T^{1*}) - \left(\pi_T^{2*} + 4\pi_T^{1*}\right)\theta_0}{3(2 - \theta_0)}. \tag{33}$$

The VC chooses the size of the portfolio by comparing the *ex ante* expected profits from one startup, given by Equation (31), and from two startups, given by Equation (33), as presented in the following proposition.

Proposition 6. *Let* $\theta_0 \in [\tilde{\theta}_{c,1}; \tilde{\theta}_{c,2}]$ *(defined in the Appendix). There are critical values* $\{\phi_{\theta_0}, \tilde{\Delta}_1(\phi, \theta_0), \tilde{\Delta}_2(\phi, \theta_0)\}$ *(defined in the Appendix) such that the VC's optimal portfolio size is as follows:*

(i) *for low project payoff* $(0 \leq \Delta < \tilde{\Delta}_1)$, *the VC invests in no startup,* $\eta^* = 0$;

(ii) *for high project payoff* $(\Delta \geq \tilde{\Delta}_2)$, *the VC invests in one startup only,* $\eta^* = 1$;

(iii) *for moderate project payoff and high focus* $(\tilde{\Delta}_1 \leq \Delta < \tilde{\Delta}_2$ *and* $\phi_c \leq \phi \leq 1)$, *the VC invests in two startups,* $\eta^* = 2$.

Furthermore, $\frac{\partial \tilde{\Delta}_1(\phi, \theta_0)}{\partial \phi} \leq 0$ *and* $\frac{\partial \tilde{\Delta}_2(\phi, \theta_0)}{\partial \phi} \geq 0$.

Proposition 6 mirrors Proposition 3 and confirms that our earlier results are robust to relaxing the assumption that VC human capital is *ex ante* scarce. Furthermore, it generates the additional result that the VC finds it more desirable to hold smaller portfolios when the extent of competition for entrepreneurial startups is greater (that is, as θ_0 increases). This property is formally established in the following corollary.

Corollary 2. $\frac{\partial \tilde{\Delta}_2(\phi, \theta_0)}{\partial \theta_0} \leq 0$ *for* $\theta_0 > \theta_{c,1}$

The intuition for this result is as follows. An increase in θ_0 improves the bargaining position of the entrepreneurs and reduces the rent extraction ability of the VC in both a small and a large portfolio. However, the reduction in the rent extraction ability of the VC is greater (at the margin) in a large portfolio than

in a small one. This is because in a large portfolio, the VC must bargain with a greater number of entrepreneurs, each of whom possesses an outside option. Thus, an increase in the value of the entrepreneur's outside option has a greater negative impact on the VC's payoff when he forms a large portfolio than when he forms a small portfolio, reducing the desirability of large portfolios. This implies that the VC is more likely to choose a smaller portfolio as the intensity of competition for entrepreneurial projects increases or as the availability of VC human capital relative to the supply of entrepreneurs increases.

4.2 Interim competition for entrepreneurs

We model interim competition for entrepreneurial startups by assuming that if an entrepreneur with a successful startup does not continue the project with the original VC at $t = 2$, with probability θ_1 she will find a new VC to continue her startup in the second stage. The basic model is modified as follows.

If, at $t = 0$, the VC selects one startup only and the project is successful, the VC bargains with one entrepreneur only and the set of bargaining parties is $N = \{V, E\}$. The Shapley values for the VC and the entrepreneur, denoted respectively by $\check{v}_V^1$ and $\check{v}_E^1$, are determined as follows. The total payoff of the coalition formed by the VC with the entrepreneur is $\check{\Pi}_T^1(V, E) = 2\Delta$. If the VC does not continue with the startup, the entrepreneur will find, with probability θ_1, a new VC with whom to bargain for the continuation of her startup. This implies that the entrepreneur's payoff without the original VC is not zero, but is the expected payoff from bargaining with the new VC, that is, $\check{\Pi}_T^1(E) = \theta_1 \check{v}_E^1$. This payoff represents the value of the entrepreneur's outside option while bargaining with the original VC. The original VC alone still obtains a zero payoff. Under these conditions, the Shapley value for the entrepreneur is

$$\check{v}_E^1 = \frac{1}{2}\left(\check{\Pi}_T^1(E) + \check{\Pi}_T^1(V, E)\right) = \frac{\theta_1 \check{v}_E^1 + 2\Delta}{2}, \tag{34}$$

yielding

$$\check{v}_E^1 = \frac{2\Delta}{2 - \theta_1}. \tag{35}$$

Similarly, the Shapley value for the VC is

$$\check{v}_V^1 = \frac{1}{2}\left(\check{\Pi}_T^1(V) + \check{\Pi}_T^1(V, E_i) - \check{\Pi}_T^1(E_i)\right) = \frac{(1 - \theta_1)2\Delta}{2 - \theta_1}. \tag{36}$$

The entrepreneur chooses her level of effort p to maximize her expected profit, resulting in $\check{p}^{1*} \equiv \frac{2\Delta}{2 - \theta_1}$. The VC's expected profit from a small portfolio, denoted by $\check{\pi}_V^{1*}$, is

$$\check{\pi}_V^{1*} = \frac{2\Delta^2}{(2 - \theta_1)^2}. \tag{37}$$

If the VC selects two startups, $\eta = 2$, the outcome of the bargaining process will depend on the number of successful startups in the VC's portfolio. If the VC has only one successful startup, say startup i, he will bargain with the successful entrepreneur, and the set of bargaining parties is $N = \{V, E_i\}$. The total payoff from the coalition between the VC and the successful entrepreneur is $\check{\Pi}_T^{2,SF}(V, E_i) = (1 + \phi)\Delta$. If the VC does not continue with the successful startup, with probability θ_1 the entrepreneur will find another VC with whom to bargain for the continuation of her project and the entrepreneur's payoff will be the expected payoff form bargaining with the new VC, yielding that $\check{\Pi}_T^{2,SF}(E_i) = \theta_1 \check{v}_{E_i}^1$. This payoff represents the value of the entrepreneur's outside option while bargaining with the initial VC. The VC alone obtains a zero payoff. The Shapley value for the entrepreneur, $\check{v}_{E_i}^2(SF)$, is

$$\check{v}_{E_i}^2(SF) = \frac{\check{\Pi}_T^{2,SF}(V, E_i) - \check{\Pi}_T^{2,SF}(V)}{2} = \frac{(2 - \theta_1)\phi + 2 + \theta_1}{2(2 - \theta_1)}\Delta, \qquad (38)$$

and for the VC, $\check{v}_V^2(SF)$, is

$$\check{v}_V^2(SF) = \frac{\check{\Pi}_T^{2,SF}(V, E_i) - \check{\Pi}_T^{2,SF}(E_i)}{2} = \frac{2(1 + \phi) - (3 + \phi)\theta_1}{2(2 - \theta_1)}\Delta. \qquad (39)$$

If both startups are successful, the VC will engage in multilateral bargaining with both entrepreneurs, and the set of bargaining parties is $N = \{V, E_1, E_2\}$. The total payoff from the coalition of the VC with both entrepreneurs is 2Δ, while the payoff of the coalition formed by the VC with only one entrepreneur is $(1 + \phi)\Delta$. If the VC does not continue one of the successful startups (say, startup i), with probability θ_1 the entrepreneur will find another VC with whom to bargain for the continuation of her project, and, thus, her expected payoff will be equal to $\check{\Pi}_T^{2,SS}(E_i) = \theta_1 \check{v}_{E_i}^1$. This payoff represents the value of the entrepreneur's outside option while bargaining with the initial VC. The VC alone obtains zero payoff. This implies that the Shapley value for each entrepreneur is $\check{v}_{E_i}^2(SS) = \frac{1}{3}(\check{\Pi}_T^2(V, E_1, E_2) - \check{\Pi}_T^2(V, E_j) + \check{\Pi}_T^2(E_i))$, yielding

$$\check{v}_{E_i}^2(SS) = \frac{(6 + 3\theta_1 - \phi(2 - \theta_1))\Delta}{6(2 - \theta_1)}, \qquad (40)$$

and for the VC, it is $\check{v}_V^2(SS) = \frac{1}{3}(\check{\Pi}_T^2(V, E_1, E_2) - 3\check{\Pi}_T^2(E_i) + \check{\Pi}_T^2(V, E_i))$, yielding

$$\check{v}_V^2(SS) = \frac{(6 - 9\theta_1 + \phi(2 - \theta_1))\Delta}{3(2 - \theta_1)}. \qquad (41)$$

Anticipating their payoffs in different states of the world, the entrepreneurs choose the level of effort they exert by maximizing their expected profit, giving

$$\check{p}^{2*}(\phi, \theta_1) \equiv \frac{3\Delta((1-\phi)\,\theta_1 + 2(\phi+1))}{2(2-\theta_1)\,(3+2\Delta\phi)}. \tag{42}$$

Note that the ability to switch, at the interim stage, to a new VC introduces an outside option while the entrepreneur bargains with the original VC. This outside option allows the entrepreneur(s) to extract more surplus from the original VC, with a positive effect on their effort incentives and, thus, on the success probability of the startups, as summarized in the following lemma.

Lemma 5. *Entrepreneurial effort $\check{p}^{2*}(\phi, \theta_1)$ is strictly increasing in θ_1.*

The VC's expected profit from holding a large portfolio, denoted by $\check{\pi}_V^{2*}$, is

$$\check{\pi}_V^{2*} = \frac{3((\Delta\phi(7+\phi) + 3(\phi+3))\,\theta_1 - 2(1+\phi)\,(\Delta\phi+3))((\phi-1)\theta_1 - 2(1+\phi))\Delta^2}{2\,(2\Delta\phi+3)^2\,(2-\theta_1)^2}. \tag{43}$$

The VC chooses the size of the portfolio by comparing the expected profit from one startup in Equation (37) with that from two startups in Equation (43). The following proposition characterizes the optimal size of the VC's portfolio.

Proposition 7. *Let $\theta_1 \leq \check{\theta}_{c,1}$ (defined in the Appendix). There are critical values $\{\phi_{\theta_1}, \check{\Delta}_1(\phi, \theta_1), \check{\Delta}_2(\phi, \theta_1)\}$ (defined in the Appendix) such that the VC's optimal portfolio size is as follows:*

- (i) *for low project payoff ($0 \leq \Delta < \check{\Delta}_1$), the VC invests in no startup, $\eta^* = 0$;*
- (ii) *for high project payoff ($\Delta \geq \check{\Delta}_2$), the VC invests in one startup only, $\eta^* = 1$;*
- (iii) *for moderate project payoff and high focus ($\check{\Delta}_1 \leq \Delta < \check{\Delta}_2$ and $\phi_{\theta_1} \leq \phi \leq 1$), the VC invests in two startups, $\eta^* = 2$.*

Furthermore, $\frac{\partial \check{\Delta}_1(\phi,\theta_1)}{\partial \phi} \leq 0$ and $\frac{\partial \check{\Delta}_2(\phi,\theta_1)}{\partial \phi} < 0$.

Proposition 7 mirrors Proposition 3 and confirms that our earlier results are robust when the entrepreneurs can switch to a new VC at the interim stage. Furthermore, introducing competition for entrepreneurial startups generates the additional result that the VC finds it more desirable to hold smaller portfolios when the level of *ex post* competition is greater. The following corollary establishes this result formally.

Corollary 3. $\frac{\partial \check{\Delta}_2(\phi,\theta_1)}{\partial \theta_1} < 0$

The intuition for this result is as follows. The entrepreneurs' ability to switch to a new VC reduces the VC's rent extraction ability. The reduction in the VC's rent extraction ability is greater in a large portfolio, where the VC bargains with a greater number of entrepreneurs. Thus, an increase in the level of *ex post* competition (or the ability of the entrepreneur to switch to a new VC) reduces the benefit of large portfolios relative to smaller portfolios, and leads the VC to prefer smaller portfolios.

5. Empirical Implications

The empirical predictions of our model hinge on the factors that affect the value of the critical parameters, especially the project payoff, Δ, and portfolio focus, ϕ.

(i) VCs investing in startups with higher ex ante quality hold smaller portfolios. High-quality startups are expected to generate greater value if successful, and therefore are characterized by a greater value of Δ. Our model shows that the VC finds it more desirable to limit portfolio size for such startups because doing so leads to strong entrepreneurial incentives, which proves to be most desirable for high-quality startups. Stronger entrepreneurial incentives improve startup success probability and, therefore, generate a superior portfolio performance and profits. Thus, this prediction helps explain the finding of Kaplan and Schoar (2005) that better-performing VC funds may choose to stay small.

(ii) VCs investing in high-risk technologies manage larger and more focused portfolios. In our analysis, the VC benefits from investing in related startups (high ϕ) because focus allows a more efficient reallocation of human capital from one startup to another. Reallocation of human capital is more likely when the (endogenous) failure probability for startups is relatively high, which happens when Δ is small or k is large. A greater level of focus reduces the *ex post* inefficiency associated with spreading the VC's resources across several startups, and increases the benefits of *ex post* resource reallocation. This implies that focused portfolios are more desirable (all else being equal) for startups that invest in technologies with high uncertainty and failure rates. This prediction is consistent with the findings of Cumming (2006). This paper documents that large portfolios are more likely to be observed for VCs investing in life sciences (rather than other high-tech industries), and argues that this strategy emerges because there are greater complementarities (i.e., higher focus) across entrepreneurial firms in the life sciences industry.

(iii) VCs expand their portfolios only if they have access to investment opportunities in the sector of their specialization. In our model, forming a portfolio with a high degree of focus benefits the VC in two ways. First, focus allows the VC to extract more surplus from the startups and, second, it facilitates the reallocation of human capital and resources among startups. In both cases, managing a focused portfolio increases the VC's investment incentives and rewards

the human capital acquisition. This implies that VCs refrain from investing in startups that are not related to their area of specialization, and that they are more likely to increase their investments when their industry of specialization experiences a positive technological shock (an increase in investment opportunities). This prediction is consistent with the findings of Gompers et al. (2004), which document that venture capital firms with the most industry-specific human capital and experience react most to an increase in investment opportunities in the sectors of their specialization. The explanation they offer for this evidence is that it is more difficult for diversified and less specialized VCs to redeploy their human capital from the sectors of their current investment to the sector experiencing an increase in investment opportunities.

(iv) VCs with the ability and the experience to create synergies across startups hold larger portfolios. Our model establishes that VCs with a better ability of reallocating resources from one startup to another are more willing to hold larger portfolios. One interpretation of this result is that experienced VCs will be better at reshuffling resources and generating synergies across startups and hence will have a greater willingness to hold larger portfolios.

(v) An increase in the availability of VC financing increases entrepreneurial effort and leads to a better success rate for startups. In Section 4.2, we show that availability of *ex post* VC financing increases the entrepreneurs' ability to extract rents from the VC and, hence, their effort. A greater level of entrepreneurial effort leads to a higher success probability for startups. Thus, greater availability of VC financing is positively correlated with better success rates for startups. This is, to our knowledge, a new and testable implication.

(vi) Industry clustering of smaller entrepreneurial businesses increases VCs willingness to provide financing to such businesses, with a positive impact on the creation and development of new businesses. Our analysis shows that smaller-size entrepreneurial projects characterized by moderate Δ will become financially viable only if the VC can combine them in his portfolio and create a positive externality between them through the reallocation effect. Hence, for a small-size startup, existence of competing but related businesses in the same industry does not necessarily represent a threat, but rather it may contribute to the development and commercialization of the business by increasing VCs' willingness to provide financing for it. This is a novel prediction of our model with the potential policy implication that encouraging small business development in similar industries or geographical locations may increase prospects of such businesses to receive VC funding.

Our model can also be applied to other domains such as leveraged buyout funds. In this context, managers of a private equity fund specializing in leveraged buyouts will face trade-offs very similar to the ones we analyzed in this paper. On the one hand, by investing in a larger number of target companies, the fund manager will add less value to each company (the "value-dilution" effect). On the other hand, a larger portfolio will allow the portfolio manager to extract more surplus from the target companies (the "rent extraction effect")

and to reallocate his human capital more effectively among portfolio companies as needed (the "resource reallocation" effect). Thus, the optimal size of the fund will emerge by trade-offs very similar to the ones discussed in our paper.

6. Conclusions

This paper studies the size and focus of a VC's portfolio. We have identified three main effects of portfolio size on the VC's and entrepreneurs' incentives. The first one is the rent extraction effect: The VC can extract higher rents in a larger portfolio because of the ability to reallocate his limited resources from one startup to another. This effect, everything else being constant, leads to stronger incentives for the VC and weaker incentives for the entrepreneurs. The second effect is the resource allocation effect: The VC benefits from investing in a large number of startups because this allows *ex post* reallocation of resources from one startup to another, after observing whether they have been successful or not. This effect has a positive impact both on the VC's investment incentives and on the entrepreneurial incentives. The third effect is the value dilution effect: A larger portfolio requires the VC to spread his limited resources across a large number of startups, diluting the VC's value-adding role, with a negative impact on both the VC's and the entrepreneur's incentives.

Our paper has several implications for VC portfolio management. One key message is that limiting portfolio size may prove to be beneficial for a VC despite the ability to add a large number of startups to his portfolio. This result originates from the fact that VC human capital is a scarce resource, and committing it to a fewer number of startups results in stronger entrepreneurial incentives. In addition, limiting portfolio size is most desirable for startups with a high payoff potential since improving entrepreneurial incentives is most valuable for such startups. A larger portfolio becomes optimal when startup payoffs become moderate and when the VC's ability to form a focused portfolio increases. A high level of focus increases the benefits of a large portfolio in terms of the VC's rent extraction and resource reallocation ability. We also show that entrepreneurs with smaller businesses may benefit from belonging to a large portfolio, rather than a small one, even if this means that the VC can extract more surplus from them. This happens when a large portfolio is the only way to enable the VC to make the startup-specific investments necessary for the success of the startups in the portfolio. In addition, clustering of smaller-size businesses with a common industry focus may increase their prospects of obtaining VC funding and may prove to be beneficial for both entrepreneurs and VCs. Finally, we show that VCs can create value by engaging in portfolio management, a real-life practice employed by many VCs. Portfolio management refers to early divestitures of some startups to extract higher surplus from the remaining startups. We find that the VC benefits from portfolio management when the relatedness of the startups in the portfolio is high.

Appendix

Proof of Proposition 1. The first-order condition of Equation (6) with respect to p is $\Delta = kp$, which, if solved for p, gives Equation (8). Substituting Equation (8) into the entrepreneur's and the VC's objective functions, given by Equations (6) and (7) respectively, gives Equation (9).

Proof of Lemma 1. The VC will make the investment if and only if the expected profits are nonnegative. It is straightforward to see that the VC's profits, given in Equation (9), are nonnegative if and only if $\Delta \geq \Delta_m = \sqrt{ck}$.

Proof of Proposition 2. Since the reaction functions of the two entrepreneurs are symmetric, the Nash equilibrium of the effort choice subgame is obtained by setting $p_j \equiv p_i$ in the first-order condition (18), and then solving for p_i, giving Equation (19). Substituting the Nash equilibrium level of effort (19) into the entrepreneurs' objective function, Equation (16), and in the VC's objective function, Equation (17), gives the VC's (20) and the entrepreneurs' expected profits (21).

Proof of Lemma 2. Direct comparison of p^{1*} with p^{2*} reveals that $p^{1*} > p^{2*}$ if and only if

$$\frac{2}{k} > \frac{1+\phi}{k + \frac{2\Delta\phi}{3}},$$

which is always true for $\phi \leq 1$. From Equations (8) and (19), we have that

$$p^{1*} - p^{2*} = \Delta \frac{3k(1-\phi) + 4\Delta\phi}{2k\,(3k + 2\Delta\phi)}. \tag{44}$$

Differentiating Equation (44) with respect to Δ gives

$$\frac{\partial(p^{1*} - p^{2*})}{\partial\Delta} = \frac{9k^2(1-\phi) + 8\Delta\phi\,(3k + \Delta\phi)}{2k\,(3k + 2\Delta\phi)^2} > 0.$$

Differentiating Equation (44) with respect to ϕ gives

$$\frac{\partial(p^{1*} - p^{2*})}{\partial\phi} = -\Delta \frac{3\,(3k - 2\Delta)}{2\,(3k + 2\Delta\phi)^2} < 0,$$

since $p^{1*} < 1$ implies that $\Delta < k$. Differentiating Equation (44) with respect to k yields

$$\frac{\partial(p^{1*} - p^{2*})}{\partial k} = -\Delta \frac{9k^2\,(1-\phi) + 8\Delta\phi\,(3k + \Delta\phi)}{2k^2\,(3k + 2\Delta\phi)^2} < 0.$$

Proof of Lemma 3. If the VC selects only one startup, $\eta = 1$, from Lemma 1 we have that the VC earns positive expected profits if and only if $\Delta \geq \Delta_m \equiv \sqrt{ck}$. If the VC selects a portfolio with two startups, $\eta = 2$, define $\Delta_0(\phi, k)$ implicitly by setting $\pi_V^{2*} = 0$ in Equation (20). Define now ϕ_c as the unique solution to $\pi_{VC}^{2*} = 0$ at $\Delta = \Delta_m$, and note that at $\phi = \phi_c$ we have that $\pi_V^{2*} = \pi_V^{1*} = 0$. It is straightforward to show that $\frac{\partial \pi_V^{2*}}{\partial\Delta} > 0$ and $\frac{\partial \pi_V^{2*}}{\partial\phi} > 0$. Thus, by the implicit function theorem, we have that $\frac{\partial \Delta_0(\phi,k)}{\partial\phi} < 0$. In addition, from $\frac{\partial \pi_V^{2*}}{\partial\Delta} > 0$ it follows that the VC earns positive expected profits if and only if $\Delta > \Delta_0(\phi, k)$. This implies that $\Delta_0 < \Delta_m$ for $\phi_c < \phi \leq 1$ and that $\Delta_0 > \Delta_m$ for $0 \leq \phi < \phi_c$. Thus, if $\phi_c \leq \phi < 1$ and $\Delta_0 \leq \Delta < \Delta_m$, the VC earns positive expected profits by investing in two startups (since $\Delta \geq \Delta_1$), but he earns negative expected profits by investing in one startup only (since $\Delta < \Delta_m$). If $\phi < \phi_c$ and $\Delta_m \leq \Delta < \Delta_0$, the VC earns positive expected profits by investing in one startup (since $\Delta \geq \Delta_m$), but he earns negative expected profits by investing in one startup only (since $\Delta < \Delta_1$).

Proof of Proposition 3. Consider first the case in which the VC selects only one startup, $\eta = 1$. In this case, from Lemma 1, we have that the VC earns positive expected profits if and only if $\Delta \geq \Delta_m$. Consider now the case in which the VC selects a portfolio with two startups, $\eta = 2$. From Lemma 3, we know that the VC earns positive expected profits if and only if $\Delta > \Delta_0(\phi, k)$. Part (i) of the proposition is obtained by setting $\Delta_1(\phi, k) \equiv min\{\Delta_m; \Delta_0(\phi, k)\}$. Note that $\frac{\partial \Delta_1(\phi,k)}{\partial \phi} \leq 0$ since $\frac{\partial \Delta_0(\phi,k)}{\partial \phi} < 0$ and Δ_m is independent of ϕ. Let now $\phi_c \leq \phi < 1$. In this case, if $\Delta_1 \leq \Delta < \Delta_m$, the VC earns positive expected profits by investing in two startups (since $\Delta \geq \Delta_1$), but earns negative expected profits by investing in one startup only (since $\Delta < \Delta_m$). Thus, the VC optimally selects two startups. Let now $\Delta \geq \Delta_m$. In this case, the VC earns positive expected profits with either one or two startups, and both choices are feasible. Therefore, the VC compares the expected profits with one startup only, π_V^{1*}, with the expected profits with two startups, π_V^{2*}, to decide whether to invest in one or two startups. Using Equations (9) and (20), we have that the VC invests in one startup if and only if

$$\frac{\Delta^2}{k} > \frac{6(3k + \Delta\phi)(1 + \phi)^2 \Delta^2}{4(3k + 2\Delta\phi)^2}. \tag{45}$$

Rearranging the inequality, we obtain that $\pi_V^{1*} > \pi_V^{2*}$ if and only if

$$P_1 \equiv 64\phi^2 \Delta^2 + 24k\phi(7 - \phi(2 - \phi))\Delta - 72k^2(\phi(2 + \phi) - 1) > 0.$$

Note that P_1 is a convex parabola in Δ with two roots, $\mathring{\Delta}_1$ and $\mathring{\Delta}_2$, given by

$$\mathring{\Delta}_1 \equiv \frac{3k}{16\phi}(\phi(2 + \phi) - 7 - (1 + \phi)\sqrt{\phi^2 + 2\phi + 17}),$$

$$\mathring{\Delta}_2 \equiv \frac{3k}{16\phi}(\phi(2 + \phi) - 7 + (1 + \phi)\sqrt{\phi^2 + 2\phi + 17}).$$

It is straightforward to show that $\mathring{\Delta}_1 < 0$ for all $0 < \phi < 1$, $k > 0$. We also have that $\mathring{\Delta}_2 > 0$ for $\phi > \phi_c$. This can be seen by noting that at $\phi = \phi_c$, we have that $\mathring{\Delta}_2(\phi^M, k) = \mathring{\Delta}_1(\phi_c, k) = \Delta_m > 0$ and

$$\frac{\partial \mathring{\Delta}_2}{\partial \phi} = \frac{3kH(\phi)}{16\phi^2\sqrt{\phi^2 + 2\phi + 17}} > \frac{3k(7\sqrt{17} - 17)}{16\phi^2\sqrt{\phi^2 + 2\phi + 17}} > 0,$$

since $H(\phi) \equiv \phi(\phi^2 + \phi - 1) - 17 + (\phi^2 + 7)\sqrt{\phi^2 + 2\phi + 17}$ is increasing in ϕ. Therefore, $\pi_V^{1*} > \pi_V^{2*}$ for $\Delta > \mathring{\Delta}_2$ and $\pi_V^{1*} \leq \pi_V^{2*}$ for $\Delta_m \leq \Delta < \mathring{\Delta}_2$. Define then $\Delta_2(\phi, k) \equiv \mathring{\Delta}_2(\phi, k)$ for $\phi_c \leq \phi < 1$. Finally, note that $\mathring{\Delta}_2(\phi^M, k) = \mathring{\Delta}_1(\phi^M, k) = \Delta_m > 0$ at $\phi = \phi_c$. Thus, for $0 \leq \phi < \phi_c$, $\Delta \geq \Delta_m$ implies $\Delta > \mathring{\Delta}_2$ since $\frac{\partial \mathring{\Delta}_2}{\partial \phi} > 0$ and, hence, that $\pi_V^{1*} > \pi_V^{2*}$. Define then $\Delta_2(\phi, k) \equiv \Delta_m$ for $0 \leq \phi < \phi_c$, obtaining (ii) and (iii). Note that we also have shown that $\frac{\partial \mathring{\Delta}_2}{\partial \phi} > 0$, which implies that $\frac{\partial \Delta_2}{\partial \phi} \geq 0$, concluding the proof.

Proof of Corollary 1. From the proof of Proposition 3, it is straightforward to see that $\mathring{\Delta}_2$ is a linear function of k and positive for $\phi > \phi_c$. This implies that $\frac{\partial \Delta_2}{\partial k} > 0$ for all $\phi > \phi_c$.

Proof of Proposition 4. The first-order condition of Equation (24) is

$$p_i^B(p_j) = (1 + \phi)\frac{4 - 3p_j}{8k}\Delta. \tag{46}$$

Since the reaction functions of the two entrepreneurs are symmetric, the Nash equilibrium of the effort choice subgame is obtained by setting $p_j = p_i^B$ in the first-order condition (46) and solving

for p_i^B, giving (26). Substituting the Nash equilibrium level of effort (26) into the entrepreneurs' objective function (24) and the VC's objective function (25) gives Equations (27) and (28).

Proof of Lemma 4. The VC's expected profit with portfolio management, given by Equation (27), is greater than that without portfolio management given by Equation (20) if and only if

$$\frac{8((1+\phi)\Delta + 4k)(1+\phi)^2\Delta^2}{(3(1+\phi)\Delta + 8k)^2} > \frac{3(\phi\Delta + 3k)(1+\phi)^2\Delta^2}{2(2\phi\Delta + 3k)^2}.$$

Rearranging the inequality, we obtain that $\pi_V^{2B*} > \pi_V^{2*}$ if and only if

$$P_2 \equiv \phi(37\phi - 27)(1+\phi)^3\Delta^2 + (223\phi^2 - 114\phi - 81)(1+\phi)^2k\Delta$$
$$+ 288(\phi - 1)(1+\phi)^2k^2) > 0.$$

Note that for $\phi > \frac{27}{37}$, P_2 is convex in Δ, and has two roots, Δ_1^B and Δ_2^B, given by

$$\Delta_1^B = \frac{(81 - \phi(223\phi - 114) - \sqrt{(145\phi^2 - 30\phi + 81)(7\phi - 9)^2})k}{2\phi(1+\phi)(37\phi - 27)}, \tag{47}$$

$$\Delta_2^B = \frac{(81 - \phi(223\phi - 114) + \sqrt{(145\phi^2 - 30\phi + 81)(7\phi - 9)^2})k}{2\phi(1+\phi)(37\phi - 27)}. \tag{48}$$

It is straightforward to show that $\Delta_1^B \leq 0$ for all values of ϕ and k. Hence, $P_2 > 0$ for $\Delta > \Delta_2^B(\phi)$. By direct calculation, it is easy to see that the numerator of the right-hand side of Equation (48) is an increasing function of ϕ and, when $\phi > \frac{27}{37}$, the denominator is a decreasing function of ϕ, implying that $\frac{\partial \Delta_2^B}{\partial \phi} > 0$. Hence, it follows that for $\phi > \frac{27}{37}$, we have that $\Delta_2^B(\phi)$ is invertible and that $\pi_V^{2B*} > \pi_V^{2*}$ for $\phi > \phi_P(\Delta)$, where $\phi_P(\Delta)$ is the inverse function of $\Delta_2^B(\phi)$. For $\phi \leq \frac{27}{37}$, it is straightforward to show that P_2 is concave in Δ and $\Delta_1^B \leq 0$ and $\Delta_2^B \leq 0$, implying that $P_2 \leq 0$ and $\pi_V^{2B*} \leq \pi_V^{2*}$, that is, portfolio management is never optimal.

Proof of Proposition 5. From the proof of Lemma 4, we have that for $\phi \leq \frac{27}{37}$, the VC's expected profit with portfolio management is lower than that without portfolio management, and hence the choice between a small and large portfolio is determined the same way as characterized in Proposition 3.

When $\phi > \frac{27}{37}$, using Equations (9) and (27), we have that the VC invests in one startup if and only if the expected profit from a small portfolio is greater than that from a large portfolio, that is,

$$\frac{\Delta^2}{k} > \frac{8(1+\phi)^2\Delta^2(4k + (1+\phi)\Delta)}{(8k + 3(1+\phi)\Delta)^2}.$$

Rearranging the inequality results in $\pi_V^{1*} > \pi_V^{2B*}$ if and only if

$$P_3 \equiv 36(1+\phi)^2\Delta^2 + 8k(1+\phi)(10 - 2\phi(1+\phi))\Delta + 128k^2(1 - \phi(2 - \phi)) > 0.$$

Note that P_3 is convex in Δ, and has two roots, Δ_1^B and Δ_2^B, given by

$$\Delta_1^B \equiv \frac{2k}{9(1+\phi)}(2\phi(2+\phi) - 10 - \sqrt{A}),$$

$$\Delta_2^B \equiv \frac{2k}{9(1+\phi)}(2\phi(2+\phi) - 10 + \sqrt{A}),$$

where $A \equiv (1+\phi)^2(4\phi(2+\phi) + 28)$. It is straightforward to show that Δ_1^B is always negative. Δ_2^B is positive for $\phi > \frac{27}{37}$. Set $\Delta^B \equiv \Delta_1^B$. It follows that $\pi_V^{1*} > \pi_V^{2B*}$ if $\Delta > \Delta^B$ and the VC

invests in one startup only. For $\Delta \leq \Delta^B$, we have that $\pi_V^{1*} \leq \pi_V^{2B*}$ and the VC invests in two startups and engages in portfolio management in the SS state by divesting one of the startups. Finally, taking the derivative of Δ^B with respect to ϕ yields that

$$\frac{\partial \Delta^B}{\partial \phi} = \frac{4k}{18} \frac{(2\phi(2+\phi)+14)\sqrt{A}+4(1+\phi)^4}{\sqrt{A}(1+\phi)^2} > 0$$

since $A \geq 0$.

Proof of Proposition 6. The proof of this proposition is similar to that of Proposition 3. Consider first the case in which the VC selects only one startup, $\eta = 1$. In this case, from Equations (9) and (31), we have that the VC earns positive expected profits if and only if $\Delta \geq \Delta_{m,\theta_0} \equiv \sqrt{\frac{2c}{3}}$. Consider now the case in which the VC selects a portfolio with two startups, $\eta = 2$. Define $\Delta_0(\phi, \theta_0)$ implicitly by setting $\tilde{v}_V^2 = 0$ in Equation (33). By direct calculation, it is straightforward to show that $\frac{\partial \tilde{v}_V^2}{\partial \phi} > 0$ and that $\frac{\partial \tilde{v}_V^2}{\partial \Delta} > 0$ for $\theta_0 \leq \tilde{\theta}_{c,1} \equiv \frac{2(8\Delta^2\phi^2+2\phi(\phi^2+2\phi+13)\Delta+9\phi(2+\phi)+27)}{32\Delta^2\phi^2+2\phi(\phi^2+2\phi+49)\Delta+9\phi(2+\phi)+81} > 0$. Thus, by the implicit function theorem, for $\theta_0 \leq \tilde{\theta}_{c,1}$ we have that $\frac{\partial \Delta_0(\phi,\theta_0)}{\partial \phi} < 0$. In addition, from $\frac{\partial \tilde{v}_V^2}{\partial \Delta} > 0$ it follows that the VC earns positive expected profits if and only if $\Delta > \Delta_0(\phi, \theta_0)$. Part (i) of the proposition is obtained by setting $\widetilde{\Delta}_1(\theta_0, \phi) \equiv min\{\Delta_{m,\theta_0}; \Delta_0(\phi, \theta_0)\}$. Note that $\frac{\partial \widetilde{\Delta}_1(\theta_0,\phi)}{\partial \phi} \leq 0$ since $\frac{\partial \Delta_0(\phi,\theta_0)}{\partial \phi} < 0$ and Δ_{m,θ_0} is independent of ϕ. Define now $\tilde{\phi}_{\theta_0}$ as the unique solution to $\tilde{v}_V^2 = 0$ at $\Delta = \Delta_{m,\theta_0}$, and note that at $\phi = \tilde{\phi}_{\theta_0}$ we have $\tilde{v}_V^2 = \tilde{v}_V^1 = 0$. Let now $\tilde{\phi}_{\theta_0} \leq \phi < 1$. In this case, if $\Delta_0(\theta_0, \phi) \leq \Delta < \Delta_{m,\theta_0}$, the VC earns positive expected profits if he invests in two startups (since $\Delta \geq \Delta_0(\theta_0, \phi)$), but he earns negative expected profits if he invests in one startup only (since $\Delta < \Delta_{m,\theta_0}$). Thus, the VC optimally selects two startups. Let now $\Delta \geq \Delta_{m,\theta_0}$. In this case, the VC earns positive expected profits with both one or two startups, and both choices are feasible. Therefore, the VC compares the expected profits with one startup only, $\tilde{v}_V^1$, with the expected profits with two startups, $\tilde{v}_V^2$, to decide whether to invest in one or two startups.

Comparing Equations (31) with (33) yields that the expected profit from a small portfolio is larger than that from a large portfolio if and only if

$$G_0(\Delta, \phi) \equiv \frac{3}{2}\Delta^2 \left[1 + \theta_0 - (2-\theta_0)\frac{(1+\phi)^2(2\phi\Delta+9)}{2(2\phi\Delta+3)^2} \right] + (1-2\theta_0)c \geq 0.$$

Taking the partial derivative of G with respect to Δ yields that

$$\frac{\partial G_0}{\partial \Delta} = \frac{3\Delta(A\theta_0 + B)}{2(2\phi\Delta+3)^3},$$

where $A \equiv 16\theta_0^3\Delta^3 + 2\phi^2(2\phi + \phi^2 + 37)\Delta^2 + 9\phi(2\phi + 13 + \phi^2)\Delta + 27\phi(2+\phi) + 81 > 0$, and $B \equiv 16\phi^3\Delta^3 - 4\phi^2(2\phi + \phi^2 - 17)\Delta^2 - 18\phi(\phi^2 + 2\phi - 5)\Delta - 54\phi(2+\phi) < 0$. By direct differentiation, it can be verified that $\frac{\partial G_0}{\partial \phi} < 0$ and that $\frac{\partial G_0}{\partial \Delta} > 0$ for $\theta_0 > \tilde{\theta}_{c,2} \equiv \frac{2\phi(-8\Delta^3\phi^2+2\phi(\phi(2+\phi)-17)\Delta^2+(9\phi(2+\phi)-45)\Delta+27\phi+54)}{16\phi^3\Delta^3+2\phi^2(\phi(2+\phi)+37)\Delta^2+9\phi(\phi(2+\phi)+13)\Delta+27\phi(2+\phi)+81} > 0$. This implies that for $\theta_0 > \tilde{\theta}_{c,2}$, there is a $\widetilde{\Delta}_2(\phi, \theta_0)$ such that $\tilde{v}_V^1 \geq \tilde{v}_V^2$ if and only if $\Delta \geq \widetilde{\Delta}_2(\phi, \theta_0)$, with $\frac{\partial \widetilde{\Delta}_2(\phi,\theta_0)}{\partial \phi} > 0$ by implicit function differentiation. Hence, we have that for $\widetilde{\Delta}_1 \leq \Delta < \widetilde{\Delta}_2$ the VC sets $\eta^* = 2$, and for $\Delta \geq \widetilde{\Delta}_2(\phi, \theta_0)$ the VC sets $\eta^* = 1$, proving (iii) and (iv). Part (ii) of the proposition is easily verified by noting that $\widetilde{\Delta}_2(\phi_{\theta_0}) = \widetilde{\Delta}_1(\phi_{\theta_0}) = \Delta_{m,\theta_0} > 0$ at $\phi = \tilde{\phi}_{\theta_0}$. Thus, $\frac{\partial \widetilde{\Delta}_2(\phi,\theta_0)}{\partial \phi} > 0$ implies that for $0 \leq \phi < \tilde{\phi}_{\theta_0}$ and $\Delta \geq \Delta_{m,\theta_0}$, we have that $\Delta > \widetilde{\Delta}_2$ and, hence, that $\tilde{v}_V^1 \geq \tilde{v}_V^2$, concluding the proof.

Proof of Corollary 2. Taking the partial derivative of G_0 defined in the proof of Proposition 6 with respect to θ_0 yields that $\frac{\partial G_0}{\partial \theta_0} = \frac{3}{2}\Delta^2(1 + \frac{1}{2}(1+\phi)^2\frac{2\phi\Delta+9}{(2\phi\Delta+3)^2}) - 2c \geq 0$. It is straightforward to show that the positive NPV conditions for one-startup and two-startup portfolios imply that $\frac{\partial G_0}{\partial \theta_0} \geq 0$. Since we have that $\frac{\partial G_0}{\partial \Delta} \geq 0$ for $\theta_0 > \theta_{c,1}$, by the implicit function theorem we obtain that $\frac{\partial \widetilde{\Delta}_2(\phi,\theta_0)}{\partial \theta_0} \leq 0$.

Proof of Lemma 4. It is immediate to see from $\breve{p}^{1*} \equiv \frac{2\Delta}{2-\theta_1}$ and Equation (42) that $\breve{p}^{1*}$ and $\breve{p}^{2*}$ are increasing in θ_1.

Proof of Proposition 7. The proof of this proposition is similar to that of Proposition 3. Consider first the case in which the VC selects only one startup, $\eta = 1$. In this case, from Equation (36), we have that the VC earns positive expected profits if and only if $\Delta \geq \Delta_{m,\theta_1} \equiv \frac{2(2-\theta_1)\sqrt{((1-\theta_1)c)}}{4(1-\theta_1)}$. Consider now the case in which the VC selects a portfolio with two startups, $\eta = 2$. Define $\Delta_0(\phi,\theta_1)$ implicitly by setting $\breve{v}_V^2 = 0$ in Equation (43). It is straightforward to show that $\frac{\partial \breve{v}_V^2}{\partial \phi} > 0$ and that $\frac{\partial \breve{v}_V^2}{\partial \Delta} > 0$ for $\theta_1 \leq \breve{\theta}_{c,1} \equiv \frac{2(1+\phi)(2\phi^2\Delta^2+9\phi\Delta+18)}{2\phi^2(7+\phi)\Delta^2+9\phi(7+\phi)\Delta+18(3+\phi)} > 0$. Thus, by the implicit function theorem, for $\theta_1 \leq \breve{\theta}_{c,1}$, we have that $\frac{\partial \Delta_0(\phi,\theta_1)}{\partial \phi} < 0$. In addition, from $\frac{\partial \breve{v}_V^2}{\partial \Delta} > 0$ it follows that the VC earns positive expected profits if and only if $\Delta > \Delta_0(\phi,\theta_1)$. Part (i) of the proposition is obtained by setting $\breve{\Delta}_1(\phi,\theta_1) \equiv min\{\Delta_{m,\theta_1}; \Delta_0(\phi,\theta_1)\}$. Note that $\frac{\breve{\Delta}_1(\phi,\theta_1)}{\partial \phi} \leq 0$ since $\frac{\partial \Delta_0(\phi,\theta_1)}{\partial \phi} < 0$ and Δ_{m,θ_1} is independent of ϕ. Define now $\breve{\phi}_{\theta_1}$ as the unique solution to $\breve{v}_V^2 = 0$ at $\Delta = \Delta_{m,\theta_1}$, and note that at $\phi = \breve{\phi}_{\theta_1}$ we have that $\breve{v}_V^2 = \breve{v}_V^1 = 0$. Let now $\breve{\phi}_{\theta_1} \leq \phi < 1$. In this case, if $\Delta_1(\phi,\theta_1) \leq \Delta < \Delta_{m,\theta_1}$, the VC earns positive expected profits by investing in two startups, but he earns negative expected profits by investing in one startup only. Thus, the VC optimally selects two startups. Let now $\Delta \geq \Delta_{m,\theta_1}$. In this case, the VC earns positive expected profits with either one or two startups, and both choices are feasible. Therefore, the VC compares the expected profits with one startup only, $\breve{v}_V^1$, with the expected profits with two startups, $\breve{v}_V^2$, to decide whether to invest in one or two startups.

Comparing Equations (36) with (43) yields that the expected profit from a small portfolio is larger than that from a large portfolio for $\Delta \geq \breve{\Delta}_2$, which implies that a small portfolio is better than a large portfolio for $F \geq 0$ where

$$F = -32\phi^2(2-\theta_1)^2(1-\theta_1)\Delta^2 + (3(2-\theta_1)^2(\phi(\phi+7)\theta_1 - 2(1+\phi)\phi)((\phi-1)\theta_1 - 2 - 2\phi)$$
$$- 96\phi(2-\theta_1)^2(1-\theta_1))\Delta + (3(2-\theta_1)^2((3\phi+9)\theta_1 - 6 - 6\phi)((\phi-1)\theta_1 - 2 - 2\phi)$$
$$- 72(2-\theta_1)^2(1-\theta_1)).$$

F is a concave function with two roots $\Delta_1(\phi,\theta_1) \leq \Delta_2(\phi,\theta_1)$, where

$$\Delta_1(\phi,\theta_1) \equiv \frac{H - 3\sqrt{K}}{64(1-\theta_1)\phi}, \quad \Delta_2(\phi,\theta_1) \equiv \frac{H + 3\sqrt{K}}{64(1-\theta_1)\phi},$$

with $H \equiv 3(\phi+7)(\phi-1)\theta_1^2 - 12(\phi+5)(\phi-1)\theta_1 + 12\phi(2+\phi) - 84$ and $K \equiv ((\phi+7)^2\theta_1^2 - (4\phi(8+\phi)+92)\theta_1 + 4\phi(2+\phi) + 68)(-(1-\phi)\theta_1 - 2(1+\phi))^2$. Note that $\Delta_1(\phi,\theta_1)$ is always negative for all θ_1 and ϕ and that we have $0 \leq \Delta_2(\phi,\theta_1) \leq 1$ for $\phi \geq \sqrt{2} - 1$ and $\theta_1 \leq \frac{(4\phi^2+8\phi-4-4\sqrt{(2\phi^2+4\phi-2)})}{2(\phi^2+2\phi-3)}$. Define

$$G_1(\Delta,\phi) \equiv \frac{3((\Delta\phi(7+\phi) + 3(\phi+3))\theta_1 - 2(1+\phi)(\Delta\phi+3))((\phi-1)\theta_1 - 2(1+\phi))\Delta^2}{2(2\Delta\phi+3)^2(2-\theta_1)^2}$$
$$- \frac{4(1-\theta_1)\Delta^2}{(2-\theta_1)^2}.$$

It is straightforward to show that $\frac{\partial G_1}{\partial \phi} > 0$. In addition, since F is concave in θ_1 with two roots $\Delta_1(\phi, \theta_1) \leq \Delta_2(\phi, \theta_1)$, and $\frac{\partial F}{\partial \Delta} \leq 0$ for $\Delta \geq \Delta_2(\phi, \theta_1)$, we have that $\frac{\partial G_1}{\partial \Delta} \leq 0$ for $\Delta \geq \Delta_2(\phi, \theta_1)$. Hence, by the implicit function theorem, we obtain that $\frac{\partial \Delta_2(\phi, \theta_1)}{\partial \phi} > 0$. Define then $\check{\Delta}_2 \equiv \Delta_2(\phi, \theta_1)$ for $\check{\phi}_{\theta_1} \leq \phi < 1$. Finally, note that $\Delta_2(\phi, \theta_1) = \Delta_{m,\theta_1} > 0$ at $\phi = \check{\phi}_{\theta_1}$. Thus, for $0 \leq \phi < \check{\phi}_{\theta_1}$, $\Delta \geq \Delta_{m,\theta_1}$ implies $\Delta > \Delta_2(\phi, \theta_1)$ since $\frac{\partial \Delta_2(\phi, \theta_1)}{\partial \phi} > 0$ and, hence, that $\check{v}_V^2 > \check{v}_V^1$. Define then $\check{\Delta}_2(\phi, \theta_1) \equiv \Delta_{m,\theta_1}$ for $0 \leq \phi < \check{\phi}_{\theta_1}$, obtaining (ii) and (iii). Note that we also have that $\frac{\partial \Delta_2(\phi, \theta_1)}{\partial \phi} > 0$, which implies that $\frac{\partial \check{\Delta}_2}{\partial \phi} \geq 0$.

Proof of Corollary 3. We have that $\frac{\partial G_1}{\partial \Delta} < 0$ for $\Delta \geq \Delta_2(\phi, \theta_1)$. It is straightforward to show that $\frac{\partial G_1}{\partial \theta_1} < 0$, yielding $\frac{\partial \check{\Delta}_2(\phi, \theta_1)}{\partial \theta_1} < 0$ by the implicit function theorem.

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