Evaluating Private Equity performance using Stochastic Discount Factors*

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Abstract

We examine the performance of 2,790 private equity (PE) funds incepted during 1979-2008 using Stochastic Discount Factors (SDFs) implied by the two leading consumption-based asset pricing models (CBAPMs)—external habit and long-run risks—as their assumptions appear consistent with investment objectives of avid PE investors. In contrast to CAPM-based inference, venture funds did not destroy value under these CBAPMs in post-2000 vintages and may even have outperformed buyouts and generalists in the full sample. We find that 2007-08 venture vintages provide a better hedge for post-crises consumption shocks than other types of PE, and that the temporal variation in PE excess returns is significantly smaller under CBAPMs. Our contribution is also methodological. We extend the realized risk premia matching insight of Korteweg and Nagel (2016) to a more general class of SDFs, namely portfolio-specific discount factors that reflect non-tradeable assets unspanned by standard benchmarks. To this end, we propose a more efficient estimation of SDF parameters in this context and develop a finite sample bias correction for NPV-based inference with long-duration assets.

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Historically, pension funds and endowments have been the cornerstone investors in private equity (PE) funds, yet the standard performance metrics used to evaluate these funds may not always account for the specific investment objectives of these investors. Beginning with Ljungqvist and Richardson (2003) and Cochrane (2005), a growing literature studies the risk and return characteristics of private capital investments within the context of CAPM or public market-based factor models.[1] While these models are natural benchmarks that span relevant risks, few endowment or pension funds specify maximizing the excess return over a public market benchmark as their policy objective. Rather, their mandates focus on hedging consumption (and/or production) risks for various beneficiaries. Furthermore, these beneficiaries effectively represent non-tradeable assets and liabilities that are not necessarily spanned by publicly traded assets (Gârleanu and Panageas, 2017). Recognizing this mismatch, Cejnek, Franz, and Stoughton (2015) argue that recursive preferences, such as those in Epstein and Zin (1989), are more appropriate for modeling endowment funds where smoothing expenditures and/or time variation in wealth are paramount. In this paper, we apply time-inseparable preferences and consumption-based asset pricing models to evaluate the performance of a large sample of PE funds relative to these non-tradeable discount factors.

Our goal is not to test the asset pricing models or argue which model is more applicable to all endowments or pensions funds, but to examine if conclusions about PE fund performance depend on the model choice while also providing evidence that the selected discount factors are plausibly relevant for those investor types. We posit that CBAPMs may convey the variation in the marginal utility (for investment returns) of pension plans and university endowments complementary to that derived from the publicly traded factors. We examine the stated investment policies and performance reports of these institutions to assess whether the preferences of endowments and pension plans map closely to those of investors with *risk-sensitive preferences* and near infinite *investment horizons*.[2] In addition to supporting the preference for early resolution of uncertainty regarding the amount of funding available in the future, most documents explicitly acknowledge the limitations in forecasting current economic and market trends. These facts map closely to settings studies in Bidder and Dew-Becker (2016) and Jagannathan and Liu (2019) who show that when investors are learning about the dynamics of the economy, assets are priced as though long-run risks are present.

Specifically, we consider the long-run risk (LRR) model of Bansal and Yaron (2004) and

[1] See Kaplan and Sensoy (2015) for a survey, Sorensen and Jagannathan (2015) and Korteweg and Nagel (2016) or a power-uliity investor perspective, and Gupta and Van Nieuwerburgh (2019) for the mimicking portfolio and multi-asset discount factor.

[2] For example, Dew-Becker and Giglio (2016) show that investors with Epstein-Zin preferences heavily weight low frequency trends lasting a century or longer.

the external habit model of Campbell and Cochrane (1999). To estimate the time series of the stochastic discount factors (SDFs) implied by these models, we follow Colacito and Croce (2011) and Ghosh, Julliard, and Taylor (2016) by extracting the series of consumption growth innovations from a panel of macroeconomic and financial variables. We compare the temporal variation of the model-implied SDF series to the real growth in gifts to U.S. university endowments. We find a robust time series correlation between the two series even after controlling for public market returns. A similar statistical relation is present between the CBAPM SDFs and average contributions growth to U.S. public pension plans. Therefore, assets that deliver positive NPV under CBAPM SDFs indeed help hedge the risks of non-tradeable assets held by the endowments and pension plans.

We then apply these SDFs to evaluate the cash flows of PE funds incepted between 1979 and 2008 and compare the resulting NPVs with those obtained under CAPM SDFs. This sample of cash flows is net of all fees, extends through December 2018, and includes 1,281 venture and 1,510 other private equity funds from the Burgiss database. The natural interpretation for an SDF is a ratio of marginal utilities of consumption across different scenarios (e.g., see Cochrane, 2009). As such, the SDF is inversely proportional to the market return realization in CAPM, and it is decreasing in consumption growth in CBAPMs. Hence, we assume that the aggregate U.S. consumption shocks are a reasonable proxy of marginal utility of those PE investors. While we acknowledge that it is not the best proxy for a given university endowment or pension plan, we provide statistical evidence that it is at least as good as the market return for an average U.S. university endowment or public pension plan.

Our key findings are as follows. First, we show that the growth in capital committed to PE funds in general (and to buyout funds especially) took place during the period when financial asset returns have provided an extraordinary good hedge to the consumption shocks. This result obtains from the canonical versions of these SDFs taken from the literature as is ("off-the-shelf") . These estimates suggest very large and positive NPVs not only with the PE fund cash flows but also with the cash flows of PE fund replicating portfolios that invest in small growth and value stocks, as well the with quarterly returns on these public portfolios if instrumented with PE activity levels as measured by invested asset value relative to public market capitalization.

Second, unlike under a power utility CAPM, post-2000 venture funds did not destroy value according to both CBAPMs, and performed better than buyout and generalists in the full sample (1979–2008 vintages). More specifically, venture funds incepted during 2007–2008 have produced significantly positive NPVs of 15 to 30 cents per dollar of committed capital, which stands in sharp contrast with NPV losses of 5 to 60 cents delivered by other PE funds. Third, we document a notably lower variation of PE fund NPVs across vintage years under CBAPMs, and the LRR model especially, in comparison to that under CAPMs. This result is consistent with CBAPMs better capturing the time variation in risk premia in private investment markets.

The second and third results obtain under the recalibrated SDFs of the respective type that minimize the pricing error on relevant public assets with importance weights determined by PE activity level and with the bias-corrected estimators developed in section V. While we show simulation-based evidence of good statistical properties of the NPV estimators we utilize and correct for cross sectional mispricing possibility (i.e., compare buyouts with small value, venture with small growth), we do not claim that our approach absolves criticism of these CBAPMs (see, e.g., Ghosh, Julliard, and Taylor, 2016). We do note, however, that largely same criticism applies to CAMP which nevertheless remains a workhorse model in evaluating PE performance. Rather we promote an agnostic view that, when none of the models perform very well, it is useful to examine an ensemble of estimates, and/or weight them according to investor-specific portfolio situation.

Our analysis contributes to the literature by shedding light on the performance of a comprehensive sample of PE funds through the lens of modern macro-finance models. Previous studies have considered the (conditional) CAPM and tradeable factor models for risk-adjusting PE returns (see, inter alia, Korteweg and Sorensen, 2010; Franzoni, Nowak, and Phalippou, 2012; Robinson and Sensoy, 2013; Ang, Chen, Goetzmann, and Phalippou, 2017; Korteweg and Nagel, 2016; Gupta and Van Nieuwerburgh, 2019) while prior empirical evidence on Habit and LRR models has been confined to publicly traded assets (Constantinides and Ghosh, 2011; Breeden, Litzenberger, and Jia, 2015, among others). Our results also contribute to the discussion on what, why, and how institutional investors invest in PE.[3] Our findings are consistent with preference for growth over value by investors with recursive utility under I-CAPM as shown in Campbell, Giglio, Polk, and Turley (2018).

Our contribution is also methodological. We show that cash flow NPV-based measures of performance for long-duration investment vehicles like PE funds are biased relative to per-period abnormal return estimates. This bias is related to both the compounding of idiosyncratic returns and the short effective time series of PE fund returns stemming from the high degree of overlap in fund lives. Adjusting for this bias is necessary for correct inference on PE fund performance and interpretation in the context of the asset pricing model under consideration. Our simulations suggest that this bias (i) can be quite large, (ii) is primarily of finite sample nature, but (iii) is present even asymptotically under certain realistic conditions.[4] Among these conditions are measurement errors and/or temporal dependencies

[3] See, e.g., Lerner, Schoar, and Wongsunwai 2007; Lucas and Zeldes 2009; Bernstein, Lerner, and Schoar 2013; Ang, Papanikolaou, and Westerfield 2014; Gilbert and Hrdlicka 2015; Robinson and Sensoy 2016.

[4] While acknowledging that these conditions may also indicate the model misspecifications (see, e.g.,

which are endemic to the type of SDFs considered in this study. We develop two complementary bootstrap-based methods to correct for the compounding error bias and propose a more efficient GMM procedure to estimate the SDF parameters relatively to the GPME method of Korteweg and Nagel (2016). Specifically, we use standard time series overidentified GMM on periodic quarterly returns of publicly traded benchmarks and account for differences in PE activity levels across time periods using the instrumented portfolio approach (Cochrane, 1996). This method adopts the realized risk premia matching insight of Korteweg and Nagel (2016) while avoiding some of its drawbacks which hinder the application of the GPME method in short samples or when the SDF is not a tradeable portfolio.

The paper proceeds by first outlining the CBAPMs and evaluating PE fund performance relative to the canonical versions of these models from the prior literature. We then discuss the limitations and refinements to these results. Section VII concludes and contemplates the application to SDFs featuring with different underlying shock series.

I. Models

In this section, we outline the models that we use to evaluate performance of PE funds. Specifically, we utilize a power-utility CAPM, the long-run risk model of Bansal and Yaron (2004), and the external habit model of Campbell and Cochrane (1999). We rely on the SDF representation of the extant models, which is generally equivalent to the *beta method* where the asset risk-factor exposure are explicitly estimated (Jagannathan and Wang, 2002).

A. CAPM

Sorensen and Jagannathan (2015) show an equivalence between the PME metric of Kaplan and Schoar (2005) and the SDF implied by the log-utility CAPM of Rubinstein (1976). In this model, the state of the world is summarized by the return on the public equity market portfolio. Specifically, the log SDF is equal to

$$m_{t+1} = -r_{m,t+1} \tag{1}$$

where $r_{m,t+1}$ is the log return on the public equity market portfolio.

Korteweg and Nagel (2016) consider a generalization of the PME, in which the state of the world is summarized by a vector of factor returns, f_{t+1}. The log SDF is then exponentially

Chernov, Lochstoer, and Lundeby, 2018), we emphasize that models' evaluation is outside the scope of our study while some non-tradeable assets can imply SDFs that violate these standard conditions.

affine in these factors such that

$$m_{t+1} = a - bf_{t+1}. \tag{2}$$

We focus on the case where the market return is the only factor and the SDF mirrors the one implied by a power-utility CAPM. Specifically, the log SDF is given by

$$m_{t+1} = a - \gamma r_{m,t+1} \tag{3}$$

where $r_{m,t+1}$ is the log return on the public equity market portfolio, a governs the unconditional mean of the log SDF, and γ is the coefficient of relative risk aversion. PME is then a special case of GPME where $a = 0$ and $\gamma = 1$. Pricing assets in this framework requires us to specify a series of public equity market returns, $\{r_m\}_{t \in T}$, and a parameter vector, $\theta = (a, \gamma)$.

B. Long-run Risks model

The long-run risk model of Bansal and Yaron (2004) couples a small, but persistent, shock in the conditional expectation of consumption growth with the non-time separable utility in the form of Epstein and Zin (1989) preferences. These features form an SDF more volatile than the observed consumption growth. Specifically, consumption growth is modeled as

$$\begin{aligned} \Delta c_{t+1} &= \mu + x_t + \sigma \eta_{t+1} \\ x_{t+1} &= \rho x_t + \psi_e \sigma e_{t+1}, \end{aligned} \tag{4}$$

where e_{t+1}, η_{t+1} are independent mean-zero shocks with unit unit variance, Δc_t is real log growth in consumption, and x_t is the persistent long-run risk factor. In this model, the log SDF is equal to

$$\begin{aligned} m_{t+1} &= \bar{m} - 1/\psi \cdot x_t - \gamma \sigma \eta_{t+1} - \big(\gamma - 1/\psi\big)/\big(1 - \rho \kappa_c\big) \kappa_c \psi_e \sigma e_{t+1} \\ &= \mathbb{E}_t[m_{t+1}] - \gamma \cdot \sigma \eta_{t+1} - f(\gamma) \cdot \psi_e \sigma e_{t+1} \quad , \end{aligned} \tag{5}$$

where $\bar{m}$ is the unconditional mean of the log SDF, ψ is the intertemporal elasticity of substitution, γ is the coefficient of relative risk aversion, and κ_c is the coefficient in the Campbell and Shiller (1988) approximation detailed in Bansal and Yaron (2004). In order to price assets with the SDF implied by this model, we need to specify a series of innovations to the consumption growth process, $\{\eta, e\}_{t \in T}$ estimated from the data, and a parameter vector, $\theta = (\bar{m}, \sigma, \psi_e, \rho, \psi, \gamma, \kappa_c)$.

One of the characteristic features of investors in Bansal and Yaron model is aversion to uncertainty about long-term future consumption path (as captured by $\psi > 1$) and a preference to smooth the variation in wealth appear consistent with the mandates and preferences that some avid PE fund investors exhibit. For example, Harvard University intends its endowment "to ensure that it has the financial resources to *confidently* maintain and expand its preeminence in teaching, learning and research for future generations." While CalPERS Investment Beliefs state "*ensuring the ability* to pay promised benefits by maintaining an adequate funding status... consider[ing] *the impact of its actions on future generations* of members and taxpayers... [taking] advantage of *factors that materialize slowly* such as demographic trends."

C. External Habit model

The external habit model of Campbell and Cochrane (1999) relies on the multiplicative effect of a surplus consumption ratio, S_t, to lever consumption volatility:

$$\begin{aligned} \Delta c_{t+1} &= g + u_{t+1} \\ s_{t+1} &= (1-\phi)\bar{s} + \phi s_t + \lambda(s_t)u_{t+1} \quad , \end{aligned} \tag{6}$$

where Δc_t is real log growth in consumption, subject to temporarily independent socks u_t; s_t is the log surplus consumption ratio; $\lambda(s_t) = \frac{1}{\bar{s}}\sqrt{1-2(s_t-\bar{s})}-1$ when $s \leq s_{max}$, 0 elsewhere, $\bar{S} = \sigma\sqrt{\frac{\gamma}{1-\phi-b/\gamma}}$ is the steady-state surplus-consumption ratio, and $s_{max} = \bar{s} + \frac{1-\bar{S}^2}{2}$ is the upper bound of the log surplus-consumption ratio. In this model, the log SDF is equal to

$$\begin{aligned} m_{t+1} &= \bar{m} - \gamma(\phi-1)s_t - \gamma\big(1+\lambda(s_t)\big)u_{t+1} \\ &= \mathbb{E}_t[m_{t+1}] - \gamma\big(1+\lambda(s_t)\big)u_{t+1} \quad , \end{aligned} \tag{7}$$

where $\bar{m}$ is the unconditional mean of the log SDF and γ is the coefficient of relative risk aversion. As before in order to price assets with the SDF implied by this model, we require a series of innovations to consumption growth, $\{u\}_{t\in T}$ estimated from the data, and a parameter vector, $\theta = (\bar{m}, \gamma, \phi, b, \sigma)$.

One of the characteristic features of investors in Campbell and Cochrane model is a preference for consumption smoothing. This also appears consistent with the mandates and preferences that some avid PE fund investors exhibit. For example, the Spending Policy of Yale University endowment "balances the competing objectives of providing a stable flow of income to the operating budget and protecting the real value of the Endowment over time."

II. Data

In this section, we describe our sample of PE fund cash flows and the construction of the SDF series we use to evaluate PE fund performance. We then give suggestive evidence on empirical relevance of CBAPM SDFs for some core PE fund investors such as university endowments and pension plans.

A. *PE funds*

Our PE dataset is from Burgiss, which obtains cash flow data from investors (limited partners) in PE funds and cross-validates these data across several investors. Harris, Jenkinson, and Kaplan (2014) and Brown, Harris, Jenkinson, Kaplan, and Robinson (2015) find that this dataset is representative of the universe of PE funds.

Our sample contains cash flows and valuation histories for 1,281 venture, 1,105 buyout, and 405 generalist PE funds.[5] The cash flows span the period from Q3 1979 to Q4 2018 and represent a complete transaction history between each PE fund and its investors. We define contributions as cash flows from investors to PE funds and distributions as cash flows from the PE funds back to investors. Contributions include management fees, while distributions are net of fees.

Panel A of Table I reports fund counts and summary statistics for our sample. In this table, the money multiple (also called Total Value to Paid-In capital or TVPI) is computed as a sum of the last net asset value reported (NAV) and the distributions repaid to investors up to that date divided by the sum of contributions received from investors over the life of each fund. The table shows significant variation in PE fund returns, both over time and across funds. The peaks in performance (unadjusted for risk) correspond to the mid 1990s vintages for venture funds and the late 1980s and early 2000s for buyouts and generalists.

The final column, labeled $lNAV^r/\sum D^r$, presents a measure of fund resolution rates. Specifically, this column presents the fund size-weighted average ratio between the last reported NAV and the sum of distributions preceding it. Superscript r denotes that both series are adjusted for inflation—i.e., "real". For the majority of our analysis, we use real cash flows since the consumption-based SDF series are defined in real terms. Hence, this measure of fund resolution rate will tend to be somewhat more conservative than those based

[5] Following the classification scheme of the 2018 Burgiss Manager Universe dataset, we exclude funds that invest primarily in debt securities, *'Real Assets'* (including Real Estate), as well as funds that are *'Not elsewhere classified'* and *'Unknown'* according to $asset\_class1$-field. In addition to the funds classified as *'Generalist'* as per $asset\_class1$-field, we reclassify as generalists those funds that have the following $asset\_class1$- and $asset\_class2$-field values: *'Equity'-'Expansion Capital'*, *'Equity'-'Unknown'*, and *'Equity'-'Generalist'*. We only include funds that by 2019 made at least one distribution to their investors.

on nominal cash flows by reducing the (fund-specific) period-T value of distributions that precede the latest NAVs. Nonetheless, we see that funds incepted before 2002 are either fully resolved or report NAVs that are less than 10% of total distributions on average.[6] However after the 2003 vintage, the resolution rate drops dramatically, especially for venture funds.

Figure 1 plots the history of aggregate net asset values reported by the funds in our sample. NAVs peaked around 2012 at over $800bln as the investment period for 2007–08 vintages lapsed and the net cash flow back to investors became positive. As of the end of 2018, venture funds NAVs are approximately one-third of the total at $59bln, while the inflation-adjusted aggregate commitment size of venture funds is less than 11% of the PE total (untabulated). We exclude funds with a vintage after 2008 from our sample due to the low resolution rates of funds even in the 2009 vintage. We show later that the high weight of the last-reported NAV in the distributions series may confound the interpretation of inference about PE funds performance using the SDF method—as it effectively assumes that Q4 2018 distributions were a factor of 10 larger than an average quarter.

B. *Constructing the SDFs*

As outlined in section I, each of our SDFs require specifying the state variables that summarize the economy at each point in time. For the SDF implied by the power-utility CAPM, we follow Korteweg and Nagel and use the continuously compounded return on the CRSP value-weighted index as a proxy for $\{r_m\}_{t\in T}$.

We follow Colacito and Croce (2011) to obtain estimates of the latent long-run risk process x_t, as well as the innovations to it, $\psi_e \sigma e_{t+1} \equiv \epsilon_{x,t}$, and the consumption growth process, $\sigma\eta_{t+1} \equiv \epsilon_{c,t}$. We obtain estimates, $\hat{x}_t$, as the projection of quarterly growth in consumption (i.e., Δc_t) on the one lag of consumption growth, the consumption to output ratio, the price-dividend ratio, the risk free rate, and default spread. This regression is estimated over the period 1951 to 2018. Data on U.S. consumption of nondurables and services, gross domestic product, and population are from the National Income and Product Accounts of the Bureau of Economic Analysis. Consumption growth is real per capita personal consumption expenditures in non-durable goods and services. Where applicable, the data are adjusted for seasonality. Yields on 3-month Treasury bills, dividends, and dividend yields for the United States are from the Center for Research in Security Prices. Consumer price index inflation and the spread between BAA and AAA corporate bonds were obtained from the website of the Federal Reserve Bank of St. Louis. e_{t+1} is the fitted residual of an AR1-model estimated

[6] In unreported analysis, we verify that levels are similar on an equal-weighted basis and, in most cases, reflective of the 15-year life limit we impose (rather than actually having non-zero NAVs as of 2018).

on the $\hat{x}_t$-series, and $\eta_{t+1} = c_t - \hat{x}_{t-1}$.[7]

For the habits model, we require a series of innovations to consumption growth. Following Ghosh, Julliard, and Taylor (2016), we obtain consumption growth, the real per capita personal consumption expenditures in non-durable goods, from the National Income and Product Accounts of the Bureau of Economic Analysis. u_{t+1} is then the deviation in log consumption growth from its sample mean. The summary statistics for these macro economic and financial variables used to construct the LRR and the habit SDF shocks are reported in the first five rows of Panel B of Table I. While we construct the SDF series at quarterly frequency, the table reports data at quarterly frequency to ensure comparability with other series discussed below.

In the first part of the paper, we keep the parameters, θ, governing each SDF, fixed to those from the prior literature. We refer to these as "off-the-shelf" SDFs. For the CAPM SDF, we consider two sets of parameters. First, we denote the K-S CAPM as the SDF given in Equation 3 where $a = 0$ and $\gamma = 1$. Second, we denote the K-N CAPM as the SDF corresponding to the GPME estimates of venture funds in Korteweg and Nagel.[8] Specifically, we set $a = 0.012$, on a per quarter basis, and $\gamma = 2.65$.

For the SDF implied by the CBAPMs, we use the following calibrations to construct these "off-the-shelf" SDF series. For the long-run risk model, we follow Bansal and Yaron (2004) and set $\sigma = 0.0135$, $\psi_e = 0.1085$, $\rho = 0.979^3$, $\psi = 1.5$, $\gamma = 10$, and $\kappa_c = 0.9649$. For the external habits model, we follow Wachter (2005) and set $b = 0.011$, $\phi = \sqrt[4]{0.894}$, and $\gamma = 2$. σ is set to the sample standard deviation of log consumption growth as in Ghosh, Julliard, and Taylor (2016). We set the unconditional mean of CBAPMs so that the average log change is equal equal to that of the log-utility CAPM SDF.[9]

The next three rows panel Panel B of Table I for the 1979–2018 sample, at annual frequency December-to-December. Meanwhile, last three columns of the panel report correlations of each of the series with the respective SDF. It follows that all three SDFs positively co-move, albeit, at 0.691, the correlation is not particularly high even between the CBAPM SDFs and is predictably lower—at 0.347— between the Habit and the CAPM SDFs.

[7] We use in-sample estimates of AR(1) model on $\hat{x}_t$-series to obtain those residuals (rather than canonical parameters discussed below) to minimize autocorrelation in filtered shocks. The difference between the ρ and $\hat{\rho}$ is about 0.2 at quaterly frequency and is attributable to Error-in-Variable attenuation bias.

[8] The drift is corrected to reflect that series are inflation-adjusted in this case. However, it makes virtually no difference for inference about fund NPV since cash flows are also deflated accordingly.

[9] We do so to enhance the comparability of the results. Arguably, all three SDFs should have the same unconditional mean and, since measured with less error, the mean from the CAPM SDFs should be more informative than those of CBAPMs. The results are qualitatively similar if we use a different common mean. This issue is addressed more systematically in section VI.A and does affect the bias-corrected NPV estimates reported in section VI.B.

C. *PE fund investors and consumption-based SDFs*

We next examine the information content of these series relative to the cash flow risk of the nontradeable assets effectively held by backbone PE fund investors, such as university endowments and public pension plans. These non-investment cash flows—namely, claims on alumni base and pension plan contributors—are quite large relative to the investment income of these investors. For example, alumni gifts are on average 17% of the operating budget of a typical U.S. university with an annualized variance of 35% (Gilbert and Hrdlicka, 2015) compared to typical investment returns are 5-to-8% per year.[10]

To this end, we explore the following series. Δ*UEd Gifts* is the growth rate of gifts to US institutional endowment funds obtained from the Council for Financial Aid to Education. ΔSPP is the growth rate of contributions to state pension plans, equally weighted by assets, obtained from the Center for Retirement Research at Boston College.[11] Both series are adjusted for inflation and available at the annual frequency. A negative correlation between these series and an SDF indicates that the asset payout is low in a high utility state and vice verse. Moreover if CBAPM SDFs explain variation in, say alumni gifts growth, beyond that of CAPM then it is more likely that endowment overall portfolio risk is not "spanned" by standard tradeable risk factors.

However, from the correlation analysis in panel B it appears that Δ*UEd Gifts* does not meaningfully correlate with either of the SDFs while for CAPM the correlation is even weakly positive. One potential explanation for this fact is endogeneity in endowment gift growth whereby the universities are able to receive more donations during times the returns on their financial investments are low. In other words, the supply constraint on donations is not binding. Another potential explanation is that the dates in the academic fiscal year are simply misaligned with calendar year.

Panel C of table I shows that shifting the SDF series to measure changes from June-to-June indeed has a significant effect on measuring these correlations. The panel reports regression results of Δ*UEd Gifts* on our three SDF series, where the dependent and explanatory variables are standardized to have zero mean and unit variance. In the first column, Δ*UEd Gifts* is regressed on the negative of the cumulative log market return over previous 12 months ending in June. The correlation increases from roughly zero to 0.508 and is statistically different from zero using standard errors robust to serial autocorrelation. This result is consistent with the binding supply constraint on donations, as well as with results in Gilbert and Hrdlicka (2015) who use a different dataset that starts in 1993. Therefore it is informative to compare the extent CBAPM SDFs explain the residual variation in the

[10] 2018 NACUBO-TIAA Study of Endowments.

[11] The SPP series are only available from 2002.

Δ*UEd Gifts* time series.

Columns 2 and 3 of panel C show that the correlations between Δ*UEd Gifts* and the CBAPM SDFs also increase notably. These increases suggest that accounting for the intra-year variation in CBAPM SDFs is empirically important. Moreover, the correlations are of similar magnitudes as those with the CAPM SDF. The next two columns show that a significant portion of that co-movement is actually orthogonal to that of the CAPM. The coefficient for the standardized Habit and LRR SDFs remain statistically significant even if the market return is added as an additional explanatory variable. The coefficient on the market return becomes insignificant at conventional levels in this multivariate setting. Finally, the last column of the panel suggests that each of the three SDFs exhibits about same amount of independent covariation with the growth in gifts to university endowments.

Panel D of Table I shows the realized correlations over the past 10 years. As in panel C, the changes are measured from June to June. We see that, if anything, the statistical link between endowment gifts and CBAPMs appears to have strengthened recently. For example, the correlation between Δ*UEd Gifts* and the Habit SDF rose from -0.51 to -0.70. This correlation is also larger in magnitude than the correlation between Δ*UEd Gifts* and consumption growth. Contributions to state pension plans are more strongly correlated with CBAPM SDFs than with our CAPM-like SDFs—especially in the LRR case—even though the span of ΔSPP data is too short for formal tests.

We note that probably fewer endogeneity concerns apply to the variation in ΔSPP, which is a function of labor force size and real earnings growth, whereas the intensity of outreach to alumni is the universities' discretion. Yet both are imperfect proxies and subject to potentially large measurement errors. However, these measurement errors should be attenuating the correlation coefficient towards zero across all three SDFs. The possibility that some gifts are in the form of marketable securities may be amplifying the correlations with CAPM SDFs rather than the CBAPM SDFs.

We note that these correlation results also extend to alternatively calibrated models of each respective SDF type (CAPM, Habits, LRR). The correlations are exactly the same for factors that are affine transformations of those we consider—e.g., calibrations that assume different risk aversion levels (slope) or different unconditional mean of the SDF (drift).

D. When do SDFs disagree?

We now examine SDF correlations at horizons that commensurate with the duration of PE funds. Panel A of Figure 2 depicts 24-quarter rolling log returns at semi-annual intervals for our four SDF series: the K-S CAPM, the K-N CAPM, the LRR SDF and the Habit

SDF. This panel suggests that over longer horizons the SDF series may depart notably. For example, the CAPM SDFs "disagreed" with CBAPM SDFs over the utility of payouts from 6-year old investments harvested during the 2002-07 period and, even more so, during the post-GFS period. In the latter episode, the Habit and LRR SDFs reflected subdued consumption growth rates, despite markets rallying strongly (and vice verse in the former episode).

There are also instances of notable disagreement within model types. For example, the Habit model regards 2017-18 as a low utility state for payouts after the consumption surplus ratio rallied. In contrast, the cumulative innovation in the persistent component of the LRR model has been net-zero since 2012. The disagreement between CAPM calibrations is limited to magnitude, by construction. These differences become meaningful after strong trends in market returns, such as during late 90s and post-GFS.[12]

III. PE performance with "off-the-shelf" SDFs

We now apply these SDFs to evaluate the performance of PE funds. Specifically, we compare the NPVs of PE funds using the SDFs implied by CBAPMs with the NPVs calculated using the CAPM SDF. As indicated in the previous section, we construct each SDF series using parameters from prior literature.

Our goal is a sample counterpart of the following expectation:

$$NPV = \sum_{\tau=0}^{T} \sum_{s \in S(\tau)} C_{s,\tau} \cdot M_{s,\tau} \cdot p\{s(\tau)\} \quad , \tag{8}$$

where $p\{s(\tau)\}$ denotes the probability of realization of state s specific to horizon τ, such that $\sum_{s \in S(\tau)} p\{s(\tau)\} = 1$, and T is the last period of the typical fund operations, while $C_{s,\tau}$ and $M_{s,\tau}$ are the cash flow and discount factor realizations in that state. For example, $M_{s,24}$ is the gross return on the SDF over a 24-quarter interval, log of which is plotted in figure 2.

A natural estimator of the expectation in (8) is:

$$\overline{NPV} = \frac{1}{N} \sum_{i=1}^{N} \sum_{t=b(i)}^{T(i)} C_{it} M_{b(i):t} \quad , \tag{9}$$

[12] In untabulated analysis, we verify that correlations between endowment gift growth and our SDF series remains tight at longer horizons. For example, the correlations of 5-to-7-year changes are -0.56 to -0.66 for CAPM calibrations and -0.72 to -0.74 [-0.76 to -0.79] for LRR [Habit] SDF. The correlations between endowment gift growth and the innovations in the two CBAPM SDFs are consistently stronger than the correlation between endowment gift growth and cumulative consumption growth.

where C_{it} is the cash flow on calendar quarter t made by fund $i = 1, ..., N$ and t denotes the time between calendar quarters $b(i)$ and $T(i)$ that the fund was operating while $M_{b(i):t}$ is the gross return on the SDF between quarters $b(i)$ to t. Note that if the same number of funds started each quarter and each fund invested in one deal held for 24-quarters (i.e., had one negative cash flow at $b(i)$ and one positive at $T(i)$, s.t. $\tau = T(i) - b(i) = 24$), estimates of $p\{s(24)\}$ would be equal across the state realizations plotted in panel A of 2. In practice however, estimates of $p\{s(\tau)\}$ given in (9) will reflect large differences in the number of funds with non-zero cash flows in τth quarter since inception. This number is generally proportional to but not fully explained by either the number of funds of a given age or the portfolio values of those funds.

Table II reports $\overline{NPV}$ for both venture and buyout funds for each the four SDFs plotted in Figure 2 across two weighting schemes. In columns 1 [3], we normalize C_{it} by the commitment size of each venture [buyout] fund (i.e., we equally weight each fund's cash flow per dollar of capital committed). In columns 2 [4], we rescale each fund's cash flows relative to the average commitment size within the funds vintage and type. Standard errors, reported in parentheses under each point estimate, are computed using the method of K-N that accounts for spatial distance in cash flow dates across funds.

The table reveals several interesting patterns. Under the K-S CAPM SDF, the NPV estimates of both venture and buyout funds are positive and have similar magnitudes, even though the standard errors are notably higher in the venture sample. The size-weighted NPVs of 17.7–19.4 cents per dollar of capital committed are about one standard deviation below the equally-weighted estimates (26.9–24.5 cents). In contrast under K-N CAPM SDF, the NPV estimates turn negative (albeit statistically insignificant) for venture funds under both weighting schemes. For buyout funds, they become closer to zero. The size-weighted strategy features NPVs that are 3 to 9 cents lower than the equally-weighted estimates, though this difference is small relative to the standard errors of 12- to 15-cents.

Using the CBAPM SDFs to evaluate PE fund performance, we find very large realized NPVs—on the order of several dollars per each dollar committed—for both venture and buyout funds. In contrast to the CAPM SDFs, CBAPMs suggest that a size-weighted strategy would have performed better than an equally-weighted one. The magnitudes are particularly large with the LRR SDF. Venture [buyout] NPVs are 3.9 to 5.8 [5.0 to 6.7] dollars and statistically significant (although they are not statistically different from each other). The Habit SDF suggests statistically higher NPVs for buyout funds than for venture funds—1.6 to 1.8 dollars versus 60 to 95 cents.

The results for CBAPMs are especially striking given that both models appear to price publicly-traded assets reasonably well during our sample period. Columns (5) through (7) of Table II report mean quarterly pricing errors for three publicly traded benchmarks—the CRSP value-weighted index (i.e., our proxy of the public market portfolio), and the Fama-French value-weighted Small Growth and Small Value portfolios. These mean quarterly pricing errors are defined as:

$$\overline{PxErr} = \frac{1}{T}\sum_{t=1}^{T}(R_t M_t - 1) \tag{10}$$

where t=1 [T] corresponds to Q1 1979 [Q4 2018] and R_t [M_t] denotes the gross return on the respective benchmarks [SDF] in that quarter. The average pricing error for the public market and Small Growth portfolios are 70 basis points, or less, per quarter for both CBAPMs. These pricing errors are much smaller economically than the average pricing errors of 1.5–1.8% for the Small Value portfolio. However given the 2% autocorrelation-robust standard error, the pricing errors for the Small Value portfolio are statistically insignificant and are of a similar magnitude as with CAMP models.

IV. Discussion

A straightforward interpretation of the NPV inference in Table II is that investments in PE funds were on average highly rewarding to investors whose marginal utility for returns is concordant with the variation of the habit and LRR SDFs. This could potentially explain the continued interest of endowments and public pension plans to PE fund investing.

Another interpretation however is that Table II indicates the coincidence of high marginal utility for payouts, as measured by CBAPM SDFs, and the peaks of the sample PE fund assets (and, hence, high magnitudes of subsequent payouts). In other words, these large NPVs may capture the timing of SDFs with commitments to PE funds, much less so the value that PE fund managers might have added through selection and nurturing specific investments. This appears a highly plausible explanation giving the time series of PE fund assets plotted in figure 1 and episodes of CBAPM SDF departures from the CAPM series as plotted in figure 2.

Yet another alternative explanation is that the “off-the-shelf” SDFs we use are misspecified for our context. For example, it could be that the measurement error on the consumption and the background risk shocks we extract from the macroeconomic data happened to be upward-biased during the times PE activity was high. Or it could be that the investors have been more (or less) risk-averse during the PE sample period, than on-average during the 20th century per the “canonical” calibrations.

To shed more light on this and related questions, we follow the well-established route in the performance evaluation literature (see, e.g., Farnsworth, Ferson, and Jackson, 2002) and construct replicating portfolios that do not imply any value-added by the manager. We then repeat the analysis in Table III using these hypothetical cash flows.

A. PE fund replication

We closely follow K-N to construct pseudo funds (also referred to as "benchmark funds") for each sample PE fund. The capital call amounts and the dates of all cash flows by each pseudo fund are assumed to exactly match those of the respective actual fund. The pseudo fund NAV on a given date is the date value of to-date capital calls invested in a public benchmark net of to-date distributions which magnitude is determined a fixed rule. Specifically, the pseudo fund distribution has two components. The first component is equal to the public benchmark return accumulated since the previous cash flow date. The second component pays out a fraction of the capital that was in the pseudo fund after the previous cash flow date, and the fraction is equal to $min\big((\tau - p)/(40 - p), 1\big)$ where τ [p] is the since-inception quarter of the current [previous] distribution by the actual fund. Thus, the distribution rule of pseudo funds captures the idea that the pace of distributions accelerate as the 10th anniversary approaches, yet depends also on the frequency of distributions by actual funds and returns on the public benchmark.

Panel A of figure 3 plots the aggregate asset value of pseudo funds investing in public market (i.e. CRSP index). The dashed line indicates the dollar values adjusted for inflation, whereas the solid red line plots same series but deflated by the CRSP index capitalization. Hence these series are naturally comparable with the as-reported aggregate NAVs of PE funds from figure 1. We note that the pseudo fund NAVs, while tracking the overall pattern and magnitude of actual NAVs rather closely, are less volatile during the episodes of high market volatility, such as around 2000-01 and 2008-09. The most striking difference from figure 1 however is a much lower residual NAV as of 4Q 2018. At \$95 bln, it is less than half of the \$215 bln reported by actual funds.

Panel B of figure 3 plot the aggregate distributions of pseudo funds in comparison to those of the actual funds as well as the aggregate capital calls (which are constructed to mimic the actual funds exactly). We see the two distributions series are highly correlated (at 81 [91]% in levels [logs], untabulated) but nevertheless depart markedly during some periods—e.g., during 1999–2001, before and after the GFS onset. Besides the difference in risk exposures and excess returns realized by actual funds, the discrepancies in distribution series also reflects the fact that actual fund distributions are not governed by the deterministic rule as are those of pseudo funds. Importantly, these differences reflect the discretion of fund fund managers—e.g., to slow down the distributions during 2014–2018, which CBAPM SDFs characterize a low marginal utility periods (as evident from figure 2).

Similarly, we construct pseudo funds investing in small growth and small value stocks.

B. Pseudo fund NPVs

Table III reports the NPV analysis similar to that Table II but on pseudo fund cash flows. In panel A, the pseudo fund cash flows are weighted equally, while in panel B – by the size of the actual funds. The rows in each panel correspond to the NPV point estimates and robust standard errors thereof by SDF type, exactly as in table II. In both panels, columns (1) and (2) report results for venture pseudo funds investing in, respectively, public market and small *growth*; while columns (3) and (4) do so for buyout pseudo funds investing in public market and small *value*.

Also as in table II, columns (5) through (7) of Table III report the average quarterly pricing errors from the broad public market, small growth, and small value portfolios. However, here we use weights proportional to the relative size of pseudo fund NAVs. Specifically, in Panel B we report

$$\overline{PxErr}^W = \sum_{t=1}^{T} w_{t-1}(R_t M_t - 1) \tag{11}$$

where $w_t = NAV_t^P / \sum_{\tau=1}^{T} NAV_\tau^P$ and NAV_τ^P is the end-of-the-quarter aggregate pseudo fund NAVs scaled by the public market cap. Whereas in Panel A we equally weight each pseudo fund NAVs that are non-zero of the respective calendar quarter.

We begin with an examination of the weighted quarterly pricing errors. First, the weighting does not make much difference for the K-S CAPM with the public market pricing error remaining zero for every period by construction. Small growth pricing errors move closer to zero but are still slightly negative at 30bps. Pricing errors on the small value portfolio are positive 90bps and statistically insignificant. For the K-N CAPM, the pricing error predictably improves from a negative 1.3% to 10bps since their estimation effectively imposes a similar weighting scheme.[13] The magnitude of pricing errors notably reduce on the small growth portfolio to -70bps but switch sign and increase in magnitude to 110 bps for small value. However the sensitivity of the CAPM pricing errors to the PE-activity weighting scheme are dwarfed by those of CBAPMs. CBAPM pricing errors remain positive but

[13] K-N use these pseudo fund cash flows to estimate a power-utility CAMP SDF so that the realized NPV of those cash flows is zero along with the NPV of the identically timed investments in risk free rate. K-N sample is different from ours—it includes only venture funds from Preqin with cash flows ending in 2012, albeit the span of vintages is the same—1979-2008.

increase by a factor of 2 to 5 and reach up to 3.7% per quarter.

Turning to the multi-period NPV analysis, we note that pseudo fund NPVs tend to be in the same direction as the respective $\overline{PxErr}^W$ but appear stronger statistically. The NPV magnitudes however are not particularly well explained by scaling the quarterly pricing errors by the pseudo fund durations. For example, the equally-weigthed NPV of pseudo funds investing in small value public equities funds is 3.9 dollars for each dollar invested (Panel A, column 4, fifth row). It is statistically significant at the 1% level and is 2.1 times greater than the NPV implied by compounding of the 3.7% quarterly pricing error for 17 quarters, the average duration of buyout pseudo funds (untabulated). Similar discrepancies—but of smaller relative magnitudes—are evident for the CAPM SDFs. For example, equally weighted venture pseudo funds show an 11 cent loss from investing in a broad equity index with K-N CAPM (panel A, column 1) even though the quarterly pricing error is positive (column 5).

Next, we note that weighting pseudo cash flows by the actual fund size (panel B) tends to increase the CBAPM-based NPV magnitudes by about the same amount as for the actual fund cash flows in table II while also increasing the magnitudes of the on the quarterly pricing errors (columns 5–7) by a factor 1.3 to 1.7. Finally, we note that the CBAPM-based NPVs of venture pseudo funds are lower than those of buyout pseudo funds by about the same amount as actual fund buyout NPVs exceed those of the venture. Moreover, this wedge appears greater for the size&style pseudo funds, especially for the habit SDF.

C. *Implications*

Overall, the results in Table III do suggest that the period of high PE investment activity took place when the performance of financial assets provided an extraordinarily good hedge against aggregate consumptions trends. On one hand, this correlation may have nothing to do with private fund investing. On the other hand, PE funds might have been particularly convenient “vessels” to gain these utility-improving long-duration exposures. In the end, PE fund managers are at least partially responsible for the decision of when to launch a fund and have discretion over the timing of its cash flows.[14]

This pseudo fund analysis also cautions against reading too much into the differences in NPVs of venture and buyout funds, despite it being statistically significant. Table I and figure 1 show that the venture funds are far less resolved than buyout funds as of Q4 2018. At this point in time, the habit SDF implies very low utility of payouts (figure 2). While following the literature, we “assume” that these NAVs are paid out then, we actually *know* that venture fund managers chose not to make distributions during this period.

[14] Gredil (2018) shows that these decisions reflect information not embedded in public market prices.

In the remainder of the paper, we provide refinements to this SDF-based methodology. Our goal is more efficient estimates of the value PE fund managers created with a particular focus on which portions of this performance is likely relevant on a forward-looking basis. While the efficiency gains are significant with CAPM-like SDFs too, their much larger with non-tradeable SDFs, such as those implied by LRR and Habit models, and in situations when the time span in the fund sample is short.

There are some important dimension that we do not pursue in our analysis. These include enhancing the underlying asset pricing model (e.g., constructing a "better SDF" from many tradeable factors) and estimating the potentially time-varying exposures of PE funds to various tradeable factors (i.e., replicating PE fund exposures). These goals are accomplished in contemporaneous work by Gupta and Van Nieuwerburgh (2019) (henceforth, GvN). Instead, we focus on the structural parameters with clear economic interpretations of relatively simple SDFs. Furthermore, we assume that the alternative to investing in PE funds is the broad public equity market and limit our replication to the style and size portfolios standard to the asset pricing literature. We note, however, that even these straightforward public equity alternatives stretch the feasibility constraint for some investors. For example at its peak of over \$800 bln in 2012 (Figure 1), the pseudo fund NAVs actually *exceeded* the combined market cap of small growth and value stocks by \$100 bln.[15]

V. Refining the Methodology

The SDF-based inference is nested in the Generalize Method of Moments methodology[16] One important assumption we make is that quarterly returns on PE fund assets are simply poorly observed (rather than undefined).

A. NPV-based inference bias

The pricing error equations (10–11) amount to Euler restriction standard for asset pricing model tests. Accordingly, the evaluation of an investment strategy returning R_t involves two steps. In the first step, one estimates the SDF parameters given the relevant benchmarks:

$$\mathbb{E}\big[z_t\big(R^B_{t+1} \cdot M_{t+1}(\theta_B) - 1\big)\big] = 0 \quad . \tag{12}$$

[15] According to Fama-French definitions that correspond to the small growth and value return series utilized in this study and elsewhere. The average for 1999–2015 is \$680 bln [4.8% of the total market cap].

[16] See, e.g., Jagannathan and Wang (2002); Farnsworth, Ferson, and Jackson (2002); Sorensen and Jagannathan (2015); Korteweg and Nagel (2016) or Cochrane (2009); Campbell (2017) for textbook exposition.

where R_t^B is a vector of gross returns on the benchmark assets for period t, $M_t(\theta_B)$ is the series of SDF parametrized with $\theta = \theta_B$ that make equation (12) hold, while z_t are the instruments implementing the investment strategy (Hansen and Richard, 1987; Cochrane, 1996). If the case under evaluation involves a constant allocation to the strategy, then $z_t = 1$ for all t. Thus, z_t must be independent from the SDF pricing error:

$$e_{t+1}^B(\theta) := R_{t+1}^B \cdot M_{t+1}(\theta) - 1 \quad , \tag{13}$$

if the parameter vector, θ_B, is estimated, instruments need be lagged relative to the strategy returns and the SDF realization.

Ideally in the second step, we would like to perform the following test on the pricing errors implied by the strategy periodic returns:

$$\begin{gathered} e_t(\theta_B) = R_t \cdot M_t(\theta_B) - 1, \\ \mathrm{H_0}\colon\ \hat{\mathbb{E}}\big[z_{t-1}e_t(\theta_B)\big] = 0 \qquad \mathrm{H_A}\colon\ \hat{\mathbb{E}}\big[z_{t-1}e_t(\theta_B)\big] \neq 0 \quad . \end{gathered} \tag{14}$$

However, since the per period returns for PE funds cannot be observed reliably, it is natural to replace test (14) with a similar test based on the Net Present Value of PE fund cash flows:

$$\mathrm{H_0}\colon\ \hat{\mathbb{E}}\big[NPV_i(\theta^B)\big] = 0 \qquad \mathrm{H_A}\colon\ \hat{\mathbb{E}}\big[NPV_i(\theta^B)\big] \neq 0, \tag{15}$$

where $\hat{\mathbb{E}}\big[NPV_i(\theta^B)\big]$ is estimated by Equation (9).

The premise for equivalence between the two tests is the existence of a mapping between the sequence of fund per-period returns and its cash flows. Denote this mapping with $\delta_{it} \in [-\underline{C}_{it}, 1]$, such that:

$$C_{it} = -C_{i0} \cdot \delta_{it} R_{it} \cdot \prod_{\tau=s(i)}^{t-1} R_{i\tau} \cdot (1 - \delta_{i\tau}), \tag{16}$$

where R_{it} is the gross return on fund i assets in period t, $\underline{C}_{it}$ is fund i's uncalled capital as of period t, and the first cash flow for every fund is a capital call (i.e., $C_{i0} < 0$). It is important to note that this mapping is not invertible—for different sequences of per period returns one can have the same sequence of cash flows so long as the geometric average of returns between cash flows is the same.

While the two tests may indeed be equivalent in expectation, (15) will be biased relative to (14) for any feasible finite sample of PE fund cash flows—see PROPOSITION 1 in

Appendix.A1:

$$\overline{NPV_i}(\theta_B) = \left(1 + \overline{e_t}(\theta_B)\right)^{\text{FundDuration}} + \text{CompoundingError} \quad . \tag{17}$$

Besides the fund duration effect—which is (log) linear in the test statistic of Equation (14) and, therefore, inconsequential—the inference based on estimates of fund NPV is affected by compounding error bias. This bias relates to the sampling error which is slow to decay even if in expectation it is indeed zero and e_t's (per equation 14) are independently distributed.[17]

It is insightful to think about the origin of the compounding error bias as the difficulty in estimating the state probabilities $p\{s(\tau)\}$ per equation (8) for large τ—e.g., 4- to 10-year—reliably. That goal requires many non-overlapping periods of length τ and is virtually impossible with only 40-years of PE cash flow data. Thus, $p\{s(\tau)\}$-estimates for large τ are not only very noisy (so we cannot test for that component of GP skill), but also not very useful since the odds of the same returns path in following 10-20 years is zero.

Another way to think about the compounding error bias is as arising from the differences between the geometric and arithmetic averaging. The NPV estimates correspond to geometric averaging. While generally useful for portfolio construction (see, e.g., Hakansson, 1971), these geometric averages are measured over a sample of horizons which is too small to be representative. Neither is it independent from the per period pricing error realizations.

Appendix.A2 examines the properties of the compounding error bias via simulations of 44-quarter funds making uniform distributions from 20th quarter. The magnitude of the bias varies from 6 to 50 percent of the funds' annualized idiosyncratic return volatility depending on the SDF type and sample characteristics. The bias remains economically meaningful even if there are no measurement errors in the SDF, the fund sample spans a hundred of vintage years, and regardless of how big the number of funds per vintage is. The bias also increases as the span of vintage years in the fund sample contracts and/or as the fund sample becomes unbalanced across vintage years. Importantly, the sign and the magnitude of the bias depend on the level of abnormal returns and its path.

In other words, reliable estimates of $\mathbb{E}\left[NPV\right]$ are infeasible and are not as practically relevant as the expected per period return per test (14). Consequently, the weighted quarterly pricing errors per columns (5)–(7) of Table III are more informative of the SDF misspecification than the pseudo fund NPVs, since the latter reflect the compounding error bias.

[17] From PROPOSITION 1, it also follows that if pricing errors exhibit autocorrelation, the expectation of the compounding error is non-zero. This is a relevant consideration given that Ang, Chen, Goetzmann, and Phalippou (2017) find autocorrelation in PE return residuals with respect to standard factor models. We acknowledge that by the law of iterative expectations, the Euler restriction has to hold at any horizon (see Chernov, Lochstoer, and Lundeby, 2018, (CLL) for discussion).

B. Compounding error correction

There are several ways to correct the compounding error bias. One way is to estimate θ at the frequency corresponding to the average duration of the sample PE funds. K-N implement this approach by using pseudo fund cash flows discussed in section IV.A. In K-N, the first step is (implicitly):

$$\mathbb{E}[NPV_i^B(\theta_{CB})] = 0 \tag{18}$$

where $NPV(\theta_{CB})_i^B$ is the vector of pseudo fund NPVs derived from investing in benchmark assets, $b = 1, ..., B$:

$$NPV_i^b(\theta_{CB}) = \sum_{t=s(i)}^{T(i)} C_{it}^b \prod_{\tau=s(i)}^{t} M_\tau(\theta_{CB}), \tag{19}$$

and C_{it}^b are obtained from utilizing equation (16) and assuming a particular mapping function $\tilde{\delta}_{it}$ in place of the unobserved δ_{it}. These are the pseudo fund NPVs introduced in section II.A.

One can think of the K-N method as jointly estimating the SDF parameters and correcting for the compounding error in the pseudo fund NPV estimates. The correction is embedded in θ_{CB} confounding the economic interpretation of these parameters. Simply put, θ_{CB} attempts to fit the compounding error embedded in the pseudo fund NPVs. While such θ might not even exist for some SDFs, it can also result in implausible estimates.[18]

B.1. Excess NPV

Because the compounding error in the NPV estimate is present for pseudo funds, $NPV^b(\theta^B)$ is informative about the relative size of the compounding error in $NPV(\theta^B)$ if the benchmark asset is a reasonable proxy for private equity returns. The presence of the compounding error implies that the finite sample estimate of $\mathbb{E}[NPV_i(\theta^B)]$ is shifted away from the true values just as the estimates of $\mathbb{E}[NPV_i^b(\theta^B)]$ so long as their pricing errors under a given SDF exhibit strong positive correlation. A simple way to mitigate the compounding error bias in PE fund NPV estimates is to subtract the estimates of $\mathbb{E}[NPV_i^b(\theta^B)]$ from those of $\mathbb{E}[NPV_i(\theta^B)]$ since both have the same direction of the bias arising due to the SDF history and the common component in the returns R_{it} and R_{it}^b:

$$\begin{gathered}\Delta NPV_i(\theta) = NPV_i(\theta) - NPV_i^b(\theta), \\ \mathrm{H_0}:\ \hat{\mathbb{E}}\big[\Delta NPV_i(\theta)\big] = 0 \qquad \mathrm{H_A}:\ \hat{\mathbb{E}}\big[\Delta NPV_i(\theta)\big] \neq 0.\end{gathered} \tag{20}$$

[18] For example, large and positive compounding error bias in the pseudo portfolio requires large and negative drift in the SDF to satisfy Equation 18. Such estimates imply unrealistic risk-free rates and push the present values of all cash flows towards zero leaving little power to reject $\mathrm{H_A}$ (see Appendix.A2).

Comparing this approach to the GPME statistic of K-N, *excess NPV* does not require the existence of θ^{CB} that satisfies equation (18) while allowing for more efficient estimates of SDF parameters using standard time series GMM (see PROPOSITION 2). Moreover, it does not require that the set of benchmarks used to identify SDF parameters be the same as those used to determine the bias correction related to the PE returns path. In fact, we argue that the set of benchmarks used to identify θ and the set of benchmarks used to correct for compounding error should be different (see PROPOSITION 3). This leads to excess NPV being a more efficient estimate of PE funds NPV than GPME, all else equal.

Conceptually, the excess NPV metric is close to the Risk-adjusted Profit (RAP) measure proposed in GvN, whereby the "budget feasible replicating portfolio of each fund" corresponds to NPV_i^b against a particular SDF derived from publicly tradeable factors. An important distinction is that GvN attempt to replicate the magnitudes of PE fund distributions, whereas we only match the dates (as do K-N). Because PE GPs have a discretion over fund cash flow, we argue that the timing of the distributions is a source of value added, especially when the SDF does not tightly correlate with the returns on publicly traded assets. Matching the dates of pseudo fund distributions to those of actual funds allows us to control for variation in a fund's exit conditions and the "softly binding" constraints on fund life.

B.2. Bootstrap

Since compounding error arises from a particular sequence of pricing errors, re-sampling the pricing errors offers a natural solution to gauge the effect of drawing a particular sequence of per-period pricing errors (equation 13). Thus, randomizing the sequence of pricing errors provides a baseline to construct a sample estimate of the compounding error.

Also, given that both tests (14) and (15) can be viewed as J-statistics for the overidentifying GMM restriction, a bootstrap is particularly attractive since it is known to improve the finite sample performance of this type of test (see, e.g., Hall and Horowitz, 1996). We however cannot observe the per period pricing error for PE funds, $e_t(\theta)$. Prior literature finds that PE fund reports of periodic returns are subject to appraisal bias (see, e.g., Ewens, Jones, and Rhodes-Kropf, 2013; Goetzmann, Gourier, and Phalippou, 2018). Nevertheless, we argue and provide simulation-based evidence that bootstrapping *feasible estimates* of PE fund $e_t(\theta)$ is effective in correcting the compounding error.

To implement this bootstrap correction, we construct a high-frequency proxy, $\tilde{r}_t$, of the unobserved PE fund returns, where $\tilde{r}_t$ is the residual from an ARMA(p,q) model of average quarterly returns of PE funds based on reported NAVs. Utilizing expression (16), we then construct pseudo PE funds from $\tilde{r}_t$ itself and from the resampled sequences of $\tilde{r}_t$ and evaluate the NPVs of those cash flows. This informs us of the sign and magnitude of the compounding error bias inherent in the feasible PE NPV estimate, as well. Importantly, the validity of this bootstrap procedure does *not* require that $\tilde{r}_t$ is a good proxy of the actual per period fund return, r_t, or that the covariance of $\tilde{r}_t$ with the SDF is a good proxy of the covariance of r_t and the SDF. The assumptions under which it reduces the compounding bias are much weaker — $\tilde{r}_t$ and r_t are at least weakly positively correlated on a per period basis and cointegrated on a cumulative basis. Nevertheless, the efficiency gains are partially driven by using information that is additional to the information contained in the PE cash flow data. Namely, we supplement the information set with fund NAV observations.

Specifically, denote the sample average NPV estimate across these *pseudo PE funds* for an estimate of θ as $N\hat{P}V^*(\theta)$. Our bootstrap procedure re-samples $\hat{e}_t = \tilde{r}_t - r_{b,t}$, the per period pricing errors of PE fund returns relative to a priced benchmark, to construct a bootstrap sample, $\{\tilde{r}_t^k\}$. Given $\{\tilde{r}_t^k\}$, we compute a new estimate of the sample average of the pseudo funds cash flows, $N\hat{P}V^k(\theta)$. Our sample estimate of the compounding error is then $N\hat{P}V^*(\theta) - \overline{NPV}^k(\theta)$ where $\overline{NPV}^k(\theta) = \frac{1}{K}\sum_{k=1}^{K} N\hat{P}V^k(\theta)$. Therefore, our bias-corrected estimate of PE fund NPV is given by:

$$NPV_{BC}(\theta) = NPV(\theta) - \left(N\hat{P}V^*(\theta) - N\bar{P}V^k(\theta)\right) \cdot \frac{\text{Duration}}{\text{Duration}^*}, \tag{21}$$

where the last term adjusts for any duration mismatch between the actual PE funds and the pseudo PE funds constructed using $\tilde{r}_t$.

One can view the excess NPV discussed before as a special case of bootstrap correction whereby $\left[N\hat{P}V^*(\theta) - N\bar{P}V^k(\theta)\right]$ is set to $NPV^b(\theta)$. Correspondingly, excess NPV only "nets-out" the part of pricing error sequencing effect that perfectly correlates with the benchmark asset. However, unlike equation 21, it also removes the effect of the sample SDF realizations. This makes both metrics useful, especially, for inference about PE subsamples that are short in vintage span, feature unresolved funds, or exhibit large pricing errors for benchmark assets.

C. *Implementation*

Our implementation proceeds as follows. First to estimate the parameter vector, we utilize the sample equivalent of Euler equation (12) for a set of benchmark assets $b = 1, ..., P \in B$ for which we observe non-overlaping quarterly returns R_t^b:

$$g(\theta; z_t) = \begin{bmatrix} \sum_{t=1}^{T} z_{t-1}^1 \cdot \left(R_t^1 M_t(\theta) - 1\right) \\ \vdots \\ \sum_{t=1}^{T} z_{t-1}^P \cdot \left(R_t^P M_t(\theta) - 1\right) \end{bmatrix}, \tag{22}$$

We obtain θ estimates using the standard GMM estimator:

$$\theta^* = \arg\min_{\theta} \quad g(\theta; z_t)' \, W \, g(\theta; z_t) \tag{23}$$

where W is a suitable weighting matrix, and $z := (z^1, ..., z^P)$ are instruments implementing the PE strategy. The system is just identified when the number of moments, P, equals the length of parameters vector θ. When P exceeds it, the system is overidentified, allowing for greater estimation efficiency and for testing whether $M(\theta)$ is an appropriate pricing kernel–given the instruments, z, and the returns of the benchmark portfolios, R^B.

Our preferred instruments are pseudo fund NAVs aggregated across the full sample of PE funds scaled by the public market capitalization as discussed in Section II.A and plotted in Figure 3 (Panel A, solid red). This implements the intuition embedded in K-N GPME estimation that the size of PE portfolios is a natural way to assess the importance weight for each period while ensuring that the series are stationary. These instruments also condition on the past benchmark returns, enabling a more robust parameter identification in spirit of CLL. We also note that the estimator per (22–23) can implement K-N GPME exactly, provided that the instruments z are modified accordingly (see PROPOSITION 2).

Next, based on equations (4–7), we derive the horizon-specific SDF changes since the idea behind both Habit and LRR models is that preferences are not time-separable while $b(i)$—the quarter of fund i start—seems a natural reference point. For LRR we get:

$$M(\theta)^{LRR}_{b(i):\tau} = \exp\Big\{a_\tau - r^{rf}_{b(i):\tau} - \gamma \sum_{t=b(i)}^{b(i)+\tau} \epsilon_{c,t} - f(\gamma) \sum_{t=b(i)}^{b(i)+\tau} \rho^{b(i)+\tau-t} \epsilon_{x,t}\Big\} \quad , \tag{24}$$

where $r^{rf}_{b(i):\tau}$ is the *ex ante* real risk free log return in the quarter corresponding to $b(i)$ and maturity τ; $\epsilon_{c,t}$ and $\epsilon_{x,t}$ are, respectively, consumption and the long-run risk innovation in quarter t rescaled to have standard deviations of σ and $\psi_e\sigma$. For the habit model we get:

$$M(\theta)^{Habit}_{b(i):\tau} = \exp\Big\{a_\tau - r^{rf}_{b(i):\tau} - \gamma\big(1 - \lambda(s_{b(i)})\big) \sum_{t=b(i)}^{b(i)+\tau} u_t\Big\} \quad , \tag{25}$$

where $\lambda(s_{b(i)})$ and u_t are the consumption surplus as of fund i inception and the consumption shock innovation as of quarter t. For greater comparability, we include ex ante risk free rate in the CAPM case as well:

$$M(\theta)^{CAPM}_{b(i):\tau} = \exp\Big\{a_\tau - r^{rf}_{b(i):\tau} - \gamma \sum_{t=b(i)}^{b(i)+\tau} r^m_t\Big\} \quad , \tag{26}$$

so that all three SDFs have the same conditional expectation, $a_\tau - r^{rf}_{b(i):\tau}$, albeit a_τ can be numerically different.

Given $M(\theta^*)_{b(i):\tau}$ for each cash flow and model of interest, we then evaluate NPV for PE funds and the pseudo funds using Equations (15)–(19) and compute excess NPV metric as described in Section V.B.1 as well as bootstrap-corrected NPV as described in Section V.B.2. For this analyses, our mapping function between per period returns and fund cash flows, $\tilde{\delta}_{it}$, is a simplified version of that from K-N (see Appendix.A2):

$$\tilde{\delta}_{it} = \begin{cases} C_{i,t} & \text{if } C_{i,t} < 0 \\ min(\frac{t-p(i,t)}{T'(i)-p(i,t)}, 1) & \text{if } C_{i,t} > 0 \\ 1 & \text{if } t = T'(i) \\ 0 & \text{otherwise} \end{cases}, \tag{27}$$

where $p(i,t)$ is the quarter of the previous distribution made by fund i, and $T'(i)$ is the life of the fund, modified to reflect longer effective life expectation for the sample funds that are far from being resolved, as discussed in Appendix.A3. We show that using last NPV in place of remaining distributions amounts to a very strong assumption for CBAPMs and suggests a greater reliance on the excess NPV metric (as opposed to bootstrap-corrected NPV).

For inference, we rely on a semiparametric bootstrap procedure that utilizes Hansen and Jagannathan (HJ) bounds to rule out economically implausible SDFs (see Appendix.A4).

VI. Main results

A. *SDF parameter estimates*

We apply the time series GMM estimator per equations (22–23) using. In a nutshell, we are trying to "fine-tune" the off-the-shelf SDFs to reduce the benchmark pricing errors relative to those in columns (5)–(7) of Table III. We estimate two parameters for three SDFs: (i) the SDF implied by CAPM, (ii) the SDF implied by the Long-run Risk model, and (iii) the SDF implied by the Habit formation model. Because we use quarterly returns, the definitions in Equations (24–26) simplify to the following:

$$log\big(M(\theta^*)_t\big) = \begin{cases} a^* - r^{rf}_t - \gamma^* \cdot r^m_t & \text{(i)} \\ a^* - r^{rf}_t - \gamma^* \cdot \epsilon_{c,t} - f(\gamma^*) \cdot \epsilon_{x,t} & \text{(ii)} \\ a^* - r^{rf}_t - \gamma^* \cdot u_t - \gamma^* \cdot u^h_t & \text{(iii)} \end{cases}$$

where t counts non-overlapping quarterly periods, $u_t^h = \lambda(s_t)u_t$, a and γ are the parameters of interest while other notation follows section V.C. For CBAPMs, we keep the remaining parameters fixed at the values from literature as they are either core assumptions embedded in the respective model and/or not identifiable from our data.

Table IV reports the parameter estimates, J-statistics with p-values, the average pricing errors for eight publicly traded assets[19], and the resulting SDFs' means and standard deviations. Panel A describes three estimations of the CAPM model, which only differ by weighting scheme of the quarterly pricing errors. The weights (i.e. the GMM instruments) are equal ($z_t = 1$) across periods in column (1), and proportional to the actual [pseudo] funds aggregate NAVs in columns (2) and (3). We see that applying PE activity weights results in higher relative risk aversion parameter estimates (albeit the standard errors are large). These weights also moderate the pricing errors such that J-statistic no longer rejects CAPM as a plausible SDF. At 2.5–3.1, these γ-estimates are notably lower than the 3.5 [4.1] levels (untabulated) that we obtain using the GPME procedure for our venture [buyout] sample. Regardless of the weighting scheme, we see high correlations between the intercept and risk aversion estimation errors, $\rho(a, \gamma)$, which is key to understanding the role of the intercept when ex ante risk free rates pin down the conditional expectation of the SDF as in Equations (26–24). The intercept essentially offsets the higher [lower] trend in $-\gamma \cdot f_t$ due to higher [lower] γ.[20]

Note that a lower intercept also relaxes the constraint on the Sharpe ratio of assets that a given SDF can price by increasing the ratio of SDFs' volatility to its mean (i.e., widening the HJ bounds). While there is an offsetting effect with a CAPM-like SDFs (due to the increase in the slope on the market return discussed above), the GMM procedure tends to pick very large negative intercepts (i.e., implying unrealistically high risk free rates) when SDF shocks and the returns of test assets are not tightly correlated. This is generally the case for CBAPMs and likely for any nontradeable SDF. In panel B and C, we report results for the two CBAPMs we considered. In both, we estimate γ for a grid of a-values and take the one that has a closest to zero while also satisfying the HJ bounds for the test assets.[21]

Column (1) of panel B shows that the LRR SDF, just like the CAPM one, can be rejected with equally weighted pricing errors. However, this SDF holds up quite well when the pricing errors are weighted by PE activity levels, as indicated by a relatively low J-statistics in columns (2) and (3). In particular, the pricing errors on small growth and the

[19] Only two of size&style portfolios are included in the estimation however: small growth and small value.

[20] Accordingly, the adjusted for inflation intercepts from GPME are 0.035–0.045 against 0.016–0.020 here.

[21] Again, typical asset pricing studies do not deal with multi-period cash flows that require the conditional SDF drift identification. Moreover, studies that express returns in excess of the risk-free rate do not encounter the problem of identifying a–see CLL for discussion.

broad market index decrease to just -3bps and 9bps per column (3) as opposed to over 400bps per Table III. Interestingly, this happens despite γ being quite close to the "off-the-shelf" level of 10.0 and not varying much across the weighting schemes. We note that the intercept, under the NAV-based weights, falls from already low -0.079 in column 1 to imply a quarterly effective risk free rate of approximately 3.3% (=1-0.9668) on top of the ex ante T-bill rate.

The pattern is quite similar for panel C, which reports results for the Habit SDF: estimates of γ are close to the off-the-shelf calibration (2.0) with most variation across weighting schemes coming from the intercept. We also observe notably tighter pricing errors and improved J-tests under the NAV-weighted specification. As with the LRR SDF, the unconditional SDF means are lower for the NAV-weighted estimations, supporting the measurement error explanation discussed in section IV.

For the analysis presented below we use the SDF parameter estimates from column (3) of each panel to adhere closer to the weighting scheme employed in the K-N GPME. In unreported results, we also estimate horizon-specific intercepts using horizon specific ex-ante risk free rates[22] and find that the intercepts' magnitudes increase with horizon, albeit not proportionally to the horizon. However it makes little difference for inference about fund NPVs. We therefore opt for a more simple approach and assume the conditional expectation of the SDF at horizon τ is given by $a \cdot \tau + \sum_{t=b(i)}^{b(i)+\tau} r_t^{rf}$ for all three models.

B. *PE fund NPVs*

Tables V, VI, and VII report results separately for venture funds, buyout funds, and generalists funds by vintage year cohort. The number of funds in the group denoted by the row's title is reported in column (1). In each table, panel A reports analysis assuming equally weighted cash flow (normalized by fund size) of all funds in the respective group so that the values from columns (2) through (12) can be interpreted as the expected NPV cents per dollar from investing in an average fund from that group. Panels B weight cash flows proportionally to the inflation-adjusted size of each fund, while panels C uses equally weighted cash flows but restricts the sample to only to substantially resolved funds only. A fund is considered substantially resolved if its inflation-adjusted cumulative distributions exceed the 4Q'18 NAVs by a factor of two.

Columns (2) and (3) report K-N GPME for, respectively, the log-utility (as in K-S PME) and unrestricted CAPM cases. The (a, b) parameters for the latter cases are estimated separately for three groups within each table×panel—before 2000 vintage, 2001 onwards,

[22] We use the method of Beeler, Campbell, et al. (2012) to estimate real yields for 3 month, 1 year, 5 year and 20 year maturities and linearly interpolate between. For the 5- and 20-year maturities, we use U.S. TIPS yields starting in, respectively, 2014 and 2003. See Internet Appendix for details.

and 'All'—and are reported in Internet Appendix. Columns (4) through (6) report bootstap-corrected NPV estimates[23] (section V.B.2) for, respectively, CAPM, LRR, and Habit SDFs estimated in section VI.A. The remaining columns, (7) through (12) report excess NPV estimates (section V.B.1) for each of the three SDFs and two pseudo funds—one invests in broad market, the other—in small growth (or value). The estimates significant at 5%(10%) confidence level are superscripted with $a(b)$. We use asymptotic standard errors of Korteweg and Nagel (2016) in columns (2) and (3) and p-values based on empirical distributions of the bootstrap samples as described in Appendix.A4. For selected fund cohorts, we also report bootstrap-based standard errors, in parentheses underneath the respective point estimates.

While the following subsections discuss these results in some detail, figures 4 and 5 summarize our key empirical findings. Both plot bias-corrected NPV by subtracting the NPV of pseudo funds that invest in broad market, and therefore reduce to minimum the effect of the particular SDF timing. From figures 4 it follows that, unlike under power utility CAPM, post-2000 venture funds did not destroy value according to both CBAPMs, and performed better than buyout and generalists in the full sample (1979–2008 vintages). the figure also highlights that that the NPV losses incurred in 2007–2008 cohort of buyout and generalist funds against both CBAPM SDFs stand in sharp contrast with the gains of similar magnitude observed for substaintially resolved 2007–2008 venture funds.

Meanwhile, figure 5 shows that variation of PE funds NPV across vintage years tends to be notably smaller under CBAPMs than under the CAPMs, especially with the LRR SDF. This supports the view that CBAPMs better capture the time variation in risk premia in private investment markets.

B.1. Venture

From Table V, we see that under log-utility CAPM (column 2) full sample venture NPV is positive at 18–34 cents, and marginally significant statistically. It falls notably for the post-2000 vintage to between zero and 6 cents, depending on the weighting sheme and the resolution rate. The NPV is significantly positive for 2007–2008 vintages and not especially sensitive to the weighting scheme. With power-utility CAPM per column (3), we observe negative NPVs of 14–20 cents albeit insignificant statistically, also largely insensitive to the weighting scheme. However, the NPVs in excess of small growth are positive but insignificant 4 cents for substantially resolved funds. Importantly, the NPVs turn sharply negative for post-2000 funds indicating 35–49 cent losses. The losses are higher, at 85 cents, and statistically significant for 2007-2008 vintages, without much difference across resolved and

[23] These use ARMA(1,1) to unsmooth feasible per period return estimates. Results are very similar with other models we considered as reported in Internet Appendix.

unresolved funds. We note that the bootstrap-corrected NPVs are generally concordant with KN GPMEs by broad vintage cohorts and on average. However they exhibit less extreme magnitudes than KN GPME, which suggests losses of 69 to 77 [124 to 127] cents for 2001–2008 [2007–2008] vintages for the sample that includes highly unresolved funds.

With the LRR SDF, the full sample venture NPV is positive and significant at 6–12 cents. We observe some NPV variation due to weighting scheme but the direction is inconsistent across metrics and resolution stage. The bootstrap-corrected NPVs are large and positive at 35–38 [51–55] cents for the post-2000 [2007–2008] venture vintages, albeit statistical significance is mixed. On the excess NPV basis however, post-2000 venture shows statistically significant but economically small losses of 4- to 8-cent, unless highly unresolved are excluded. In that case, post-2000 appear almost exactly zero regardless of the benchmark. However, there are positive and statistically significant 6 to 11 cents if 2007–2008 vintages examined separately. Interestingly, we note greater concordance with Log-utility CAPM than the power-utility one for the LRR model.

Turning to the habit model, for venture sample we observe positive and marginally significant NPVs at 36 to 62 cents on a bootstrap basis, but mostly negative readings of excess NPVs and with low empirical p-values. switch in sign with low p-values depending on whether unresolved funds are kept in the sample. This reflects very low utility the Habit SDF assigns to 4Q 2018 NAVs. Some variation due to weighting scheme but the direction is inconsistent across metrics âĂŞ bootstrapped NPVs look higher for pre- and post-2000 funds but excess NPVs look lower pre-2001. Bootstrapped NPV and excess NPVs for the largely-resolved funds suggest that post-2000 funds performed better than funds incepted before that. The 2007-08 largely resolved funds look best at 22 and 37 cents in excess of small growth and public market, respectively, albeit lack statistical significance. Overall, the NPVs look mostly concordant with those under LRR SDF.

B.2. Buyout

From Table V, we see that under log-utility CAPM (column 2) full sample buyout NPV is also positive at 19–27 cents, but more robust statistically than for the venture sample. At 55 cents significant statistically, the value is above the average for 2001–2004 vintage cohort. However it deteriorates sharply afterward to reach a 3 cent loss in the 2007–2008 cohort. The effect of weighting scheme depends on vintage âĂŞ higher NPVs for larger funds during the best-performing vintages, 1996âĂŤ2004, but mixed-to-zero effect for other groups.

Unlike for venture funds however, the full sample buyout fund NPVs stay positive and under the power-utility CAPM (column 4) at similar magnitudes of 12-to-20 cents but turn insignificant statistically. Largely same pattern as under the log-utility CAPM is observed across vintage groups and based on K-N GPME per column 3. Nevertheless there are episodes of "disagreement" within CAPMs even for the bias-corrected ones. One example is pre-1990 funds, where bootstrap-corrected NPV is negative 28 cents while excess NPV against CRSP-investing pseudo funds is positive 21 cents with both being significant statistically and KN GPME is almost exactly 0. This group however features only 54 funds. Another discrepancy is with respect to the far better populated 1996–2000 cohort. At 83–117 cents, it has the highest NPV on the bootstrap-basis and in excess of broad market investing pseudo funds. However, it comes across as the worst cohort against the small value, suggesting a 38- to 42-cent loss, also significant statistically. Meanwhile, the 2007-2008 vintages are significantly negative at 68 cents if bootstrap-corrected but show only a 2- to 11-cent loss to small value.

Buyout NPVs against the LRR SDFs are 27-36 cents with 8-cent standard errors but nevertheless above 10% empirical p-values. Full sample excess NPVs against broad index are smaller but statistically significant 5-7 cents if equally weighted but show zero to 6 cent loss if size-weighted or against small value. For mostly resolved post-2000 funds, the performance is positive regardless of the metric and better than for ealier funds, especially in excess of small value. Depending on the metric and resolution rate, the best-performing cohort is either 2001–2004 (excess NPVs) or 2005–2006 (bootstrap-corrected). While bootstrap-corrected NPVs are positive but insignificant for the 2007-08 vintages, excess NPVs are negative and statistically significant at 4–11 [3–7] cents versus market [small value].

For the habits SDF, we see very mixed results depending on the sample and the metric. Full sample bootstrap-corrected NPVs are positive 1-1.7 dollars which are significant based on empirical p-values but feature very large standard errors. In contrast, excess NPVs are near zero and slightly below zero if size-weighted. Dropping the unresolved funds improves excess NPVs to insignificant 14 and 5 cents versus, respectively broad index and small value. Because of the high sensitivity to the 4qâĂŹ08 NAV assumption, excess NPV analysis for the post-2000 vintages is likely more reliable with the nearly resolved funds. It suggests historically very good returns around 30-to-40 cents (yet under wide confidence intervals) through 2006 and a sharp drop for 2007-2008 vintage to statistically significant loss of 18 [31] cents against broad market [small value].

B.3. Generalists

By their self-declared strategy, generalist funds invest in both venture- and buyout-like deals. We see evidence of that in some of the NPV analysis per Table VII. In particular, the excess NPVs for the 2001–2008 vintage cohort appear roughly as a 40-60 mix between venture and buyout NPVs for all three SDFs.

This pattern does not hold for 1990-2000 funds, for which the excess NPVs of generalists appear virtually identical to those of buyouts, and during for the 2007-2008 where generalists significantly underperformed both venture and buyouts. In the latter cohort, the losses are particularly sharp under the habit model—tune of 54 cents even for nearly resolved funds for which the drag from low-valued 4Q 2018 NAVs are not that much of a drag. Nevertheless the bootstrap-corrected NPV for those funds remain significantly positive at 3.6% and are larger than for buyouts and venture. This indicates that the timing of funds launches and capital calls has been very positive, however the timing of the realizations and the level of excess cash flows have been lagging those of pseudo funds. A similar pattern for NPVs with LRR SDF, albeit less stark quantitatively, provides additional support for this explanation.

VII. Conclusion

This paper documents novel facts about PE fund performance. We compare inference about NPV of investing in various cohorts of venture, buyout and generalist funds derived from CAPM and the leading consumption-based asset pricing models. We show that latter may explain the continued interest to venture capital investments by university endowments and pension funds.

The methodology that we develop applies beyond these specific SDFs, enabling a construction and calibration of portfolio-specific discount factors reflecting non-tradeable assets and liabilities, unspanned by publicly-traded assets. In particular, one may use investor-specific shocks to consumption, including (but not limited) to realizations of liquidity risk. Whereas to background risk series (either forward looking persistent shock or backwards-looking habit) can be reflective of risks such as demographics, environmental, accumulated funding gap, etc.

There are limitations to our empirical results. While our methodology emphasizes robustness, other studies may find different results using conceptually same methods since this approach involves not explicitly observable discount factors. Furthermore, many funds in our sample are far from being resolved, especially those incepted after 2005. Thus, some results may change as post-2018 cash flows are added and SDF realizations are obtained.

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Table I

Summary statistics

This table reports summary statistics for data used in this study. Panel A describes the sample of venture, growth equity and buyout funds with cash flow data from Burgiss, for vintages between 1979 and 2008. We exclude funds with committed capital below $5 million in 1990 dollars. The means and quartile statistics are for the for Total Value to Paid-In capital of each fund. Panel B reports the summary statistics at annual frequency for the log changes in the discount factors (m) considered in this study (log-utility CAPM, Long-run Risk model, and the Habit formation model); the underlying data used to construct them (U.S. Price-dividend ratio, 3-month Treasury Bill rate, Moody's BBB to AAA credit spread, U.S. Consumption growth, U.S. Consumption to Output ratio); and the proxies for the marginal utility for investment returns of selected PE LPs, University Endowments and State Pension Plans (real growth rates of gifts and contributions respectively). The sample period is from 1979 through 2018 or earliest availability. Panel C reports yearly regression results of growth in university endowment gifts on the discount factors in which the dependent and explanatory variables are standardized and t-statics (reported in parentheses) adjusted for autocorrelation. Panel D reports the pairwise correlation for selected variables during years 1998-2018.

Panel A: PE funds sample

Vintage year	Venture							Buyouts and Generalists						
	Fund count	Equal-weighted TVPI				Size-weighted		Fund count	Equal-weighted TVPI				Size-weighted	
		p25	p50	p75	mean	TVPI	$\frac{lNAV_r}{\sum D_r}$		p25	p50	p75	mean	TVPI	$\frac{lNAV_r}{\sum D_r}$
1979-85	109	1.205	1.669	2.378	1.894	1.889	0.02	19	2.141	2.719	3.814	4.076	4.823	0.00
1986	25	1.384	1.583	2.062	2.056	3.156	0.01	10	1.288	1.880	2.504	2.203	2.718	0.02
1987	31	1.154	1.927	2.795	2.206	2.453	0.01	15	1.532	2.119	4.059	2.840	2.213	0.13
1988	30	1.351	1.952	3.108	2.317	2.712	0.01	14	1.544	1.781	2.640	2.153	2.127	0.01
1989	32	1.161	1.888	2.947	2.521	2.700	0.01	16	1.294	2.381	4.151	2.690	3.156	0.01
1990	16	1.790	2.678	4.003	3.140	3.579	0.00	13	1.451	2.146	2.859	2.221	2.320	0.05
1991-92	26	1.547	2.022	3.324	3.348	3.089	0.01	28	1.572	2.701	3.220	2.770	2.827	0.01
1993	24	1.302	2.741	6.130	5.605	6.558	0.00	17	1.545	2.026	2.890	2.428	2.375	0.00
1994	22	1.308	2.966	7.250	6.104	9.914	0.01	40	1.308	1.949	2.728	2.202	3.028	0.03
1995	31	1.823	2.724	6.235	6.368	5.765	0.01	41	1.017	1.774	2.395	2.106	2.073	0.02
1996	22	0.872	2.251	8.116	8.013	10.962	0.01	36	1.084	1.701	2.275	1.763	1.851	0.03
1997	54	1.027	1.803	5.200	7.874	7.210	0.02	66	0.982	1.379	1.806	1.491	1.535	0.04
1998	57	0.604	1.108	1.744	2.290	2.439	0.07	81	1.007	1.504	2.004	1.582	1.528	0.02
1999	110	0.366	0.667	1.106	0.852	0.898	0.15	85	0.886	1.477	2.062	1.462	1.590	0.03
2000	141	0.546	0.764	1.308	0.952	1.026	0.22	113	1.256	1.878	2.471	1.923	2.146	0.03
2001	76	0.630	1.051	1.454	1.232	1.544	0.23	59	1.631	2.420	3.077	2.423	2.552	0.03
2002	28	0.806	1.115	1.580	1.197	1.210	0.11	49	1.688	2.279	2.798	2.343	2.438	0.03
2003	25	0.535	1.102	1.819	1.793	1.719	0.15	46	1.496	2.067	2.845	2.297	2.592	0.03
2004	44	0.591	0.960	1.882	1.629	1.667	0.61	88	1.332	1.881	2.329	1.951	2.000	0.02
2005	82	0.884	1.322	1.857	1.829	2.005	1.08	130	1.181	1.586	2.292	1.755	1.789	0.05
2006	107	0.820	1.436	1.913	1.645	1.669	0.69	173	1.051	1.544	2.012	1.720	1.583	0.11
2007	99	1.119	1.812	2.520	2.237	2.372	1.03	202	1.212	1.568	2.024	1.657	1.725	0.43
2008	90	1.024	1.556	2.838	2.380	2.469	1.32	169	1.204	1.586	2.088	1.636	1.828	0.28
All	1,281	0.800	1.381	2.179	2.377	2.159	0.45	1,510	1.212	1.692	2.326	1.881	1.888	0.15

Table I

Summary statistics—*Continued*

Panel B: "Off-the-shelf" SDFs and other data, YoY&levels

	Univariate Statistics:			Correlations with SDF from:		
	Mean	StDev	Skew	C-C Habit	B-Y LRR	CAPM
Δ.Consumption (log)	0.016	0.012	−0.453	−0.637	−0.797	−0.081
Consum-to-Output ratio	0.616	0.009	0.762	0.088	−0.337	−0.201
Risk Free Rate (real pp)	0.011	0.025	0.120	−0.213	−0.591	−0.245
BBB-AAA spread (pp)	0.010	0.003	1.277	0.364	0.351	0.170
Price-to-Dividend ratio	3.725	0.387	−0.296	−0.189	0.015	0.038
m(C-C Habit)	−0.077	0.462	0.904	1.000	0.691	0.347
m(C-C LRR)	−0.077	0.388	0.191	0.691	1.000	0.423
m(logU CAPM)	−0.077	0.160	1.151	0.347	0.423	1.000
Δ.SPP Contributions	0.033	0.052	−0.644	−0.307	−0.340	−0.069
Δ.UEd Gifts	0.069	0.035	0.494	−0.044	−0.110	0.041

Panel C: Regressions of Δ.UEd on log SDFs, June YoY

	(1)	(2)	(3)	(4)	(5)	(6)
CAPM	−0.508			−0.374	−0.359	−0.327
	(−2.19)			(−1.77)	(−1.53)	(−1.47)
C-C Habit		−0.480		−0.327		−0.177
		(−3.11)		(−3.13)		(−1.47)
B-Y LRR			−0.511		−0.365	−0.266
			(−3.49)		(−2.59)	(−1.59)
N	40	40	40	40	40	40
R^2	0.258	0.230	0.261	0.347	0.369	0.387

Panel D: Correlations post-1998, June YoY

	(1)	(2)	(3)	(4)	(5)	(6)
(1) Δ.UEd	1.000					
(2) Δ.SPP	0.081	1.000				
(3) Δ.Consumption	0.573	0.310	1.000			
(5) m(C-C Habit)	−0.701	−0.222	−0.803	1.000		
(4) m(B-Y LRR)	−0.708	−0.428	−0.805	0.829	1.000	
(6) m(CAPM)	−0.657	−0.135	−0.495	0.581	0.537	1.000

Table II

PE Fund NPVs with "Off-the-Shelf" SDFs

In this table, we conduct inference on the NPV of investing in PE funds against selected SDFs proposed in the literature, and compare the results with the inference on abnormal performance of public benchmarks during 1980–2018, which corresponding to the PE fund cash flows sample. 'K-S CAPM' and 'K-N CAPM' denote the results based on the SDF series assuming, respectively, the log-utility CAPM (as implied by the PME method of Kaplan and Schoar, 2005) and the unrestricted CAPM, as estimated in Korteweg and Nagel (2016). 'B-Y LRR' and 'C-C Habit' denote the results based on Long-run Risks and External Habit CBAPMs of, respectively, Bansal and Yaron (2004) and Campbell and Cochrane (1999). Columns (1) reports the NPV estimate as per equation (9) of an average venture fund, whereas column (2) weights fund-level NPV estimates by fund commitment size. Columns (3) and (4) do so for the buyout funds. Columns (5) through (7) report the average pricing errors (*PxErrs*) as defined in equation 10 at quarterly frequency for three publicly-traded portfolios: public market, small growth, and small value. Standard errors adjusted for autocorrelation are reported in parentheses. The PE fund sample and discount factors are described in Table I.

	PE funds NPVs				**Quarterly Pricing Errors (PxErrs)**		
	VC mean (1)	VC size-weighted (2)	BO mean (3)	BO size-weighted (4)	Public Market (5)	Small Growth (6)	Small Value (7)
K-S CAPM	0.269	0.177	0.245	0.194	0.000	-0.005	0.010
	(0.16)	(0.10)	(0.06)	(0.05)	–	(0.00)	(0.01)
K-N CAPM	-0.144	-0.170	0.105	-0.009	-0.013	-0.023	-0.004
	(0.12)	(0.13)	(0.14)	(0.15)	(0.01)	(0.01)	(0.01)
B-Y LRR	3.863	5.785	5.043	6.712	0.007	0.006	0.018
	(1.68)	(2.41)	(1.43)	(2.11)	(0.02)	(0.02)	(0.02)
C-C Habit	0.600	0.950	1.583	1.788	0.005	0.002	0.015
	(0.18)	(0.32)	(0.43)	(0.51)	(0.02)	(0.02)	(0.02)

Table III
Pseudo fund NPVs

In this table, we evaluate the NPV of hypothetical funds that mimic the actual PE fund cash flow schedules by investing in public benchmarks (i.e., *Pseudo funds*) constructed as in Korteweg and Nagel (2016) during 1980–2018 against selected SDFs proposed in the literature. 'K-S CAPM' and 'K-N CAPM' denote the results based on the SDF series assuming, respectively, the log-utility CAPM (as implied by the PME method of Kaplan and Schoar, 2005) and the unrestricted CAPM, as estimated in Korteweg and Nagel (2016). 'B-Y LRR' and 'C-C Habit' denote the results based on Long-run Risks and External Habit CBAPMs of, respectively, Bansal and Yaron (2004) and Campbell and Cochrane (1999). In both panels, column 1 [3] reports the NPV estimate of an average pseudo venture [buyout] fund investing in broad public market index, whereas column 2 [3]—in small growth [value] portfolio. Columns (5) through (7) report the average pricing errors (*PxErrs*) per equation (11) at quarterly frequency for those three publicly-traded portfolios, using the the net asset values of those pseudo funds (i.e., *Pseudo NAVs*) to weight the time series of the quarterly pricing errors. In Panel A, pseudo funds cash flows and pricing errors are weighted equally, whereas Panel B – by fund size adjusted for inflation. The NPVs and pricing errors are per one dollar invested. Standard errors adjusted for fund life overlaps and autocorrelation are reported in parentheses. The PE fund sample and discount factors are described in Table I.

Panel A: Equally weighted

	Pseudo fund NPVs				**"pseudo NAVs"-weighted PxErrs**		
	VC invest PubMkt (1)	VC invest SmGr (2)	BO invest PubMkt (3)	BO invest SmVal (4)	Public Market (5)	Small Growth (6)	Small Value (7)
K-S CAPM	0.000	-0.037	0.000	0.193	0.000	-0.004	0.009
	–	(0.03)	–	(0.08)	–	(0.01)	(0.01)
K-N CAPM	-0.113	-0.119	-0.127	0.141	0.001	-0.007	0.011
	(0.08)	(0.08)	(0.05)	(0.17)	(0.02)	(0.01)	(0.02)
B-Y LRR	2.777	2.823	3.752	3.906	0.027	0.028	0.037
	(1.33)	(1.35)	(1.24)	(1.19)	(0.02)	(0.02)	(0.02)
C-C Habit	0.900	0.924	1.536	1.707	0.018	0.016	0.028
	(0.39)	(0.45)	(0.50)	(0.48)	(0.02)	(0.02)	(0.02)

Panel B: Size-weighted

	Pseudo fund NPVs				**"pseudo NAVs"-weighted PxErrs**		
	VC invest PubMkt (1)	VC invest SmGr (2)	BO invest PubMkt (3)	BO invest SmVal (4)	Public Market (5)	Small Growth (6)	Small Value (7)
K-S CAPM	0.000	0.002	0.000	0.139	0.000	-0.002	0.010
	–	(0.01)	–	(0.06)	–	(0.01)	(0.01)
K-N CAPM	-0.066	-0.049	-0.120	0.087	-0.004	-0.010	0.006
	(0.09)	(0.09)	(0.06)	(0.14)	(0.02)	(0.02)	(0.02)
B-Y LRR	3.646	3.727	4.683	4.769	0.042	0.044	0.050
	(1.69)	(1.72)	(1.68)	(1.63)	(0.02)	(0.03)	(0.02)
C-C Habit	1.304	1.384	2.076	2.236	0.031	0.030	0.037
	(0.59)	(0.64)	(0.71)	(0.72)	(0.03)	(0.03)	(0.03)

Table IV
GMM estimation of SDF parameters

This table reports SDF parameter estimates and performance diagnostics via quarterly time series GMM as described in section VI.A for three asset pricing models—CAPM in Panel A, Long-run Risk model of Bansal and Yaron (2004) in Panel B, and the Habit formation model of Campbell and Cochrane (1999) in Panel C—for the PE fund cash flow sample: 1979–2018. Within each panel, the differences across columns derive from the different weights applied to the pricing error in the respective quarter. The weights are equal in column (1) and proportional to the PE activity level in the other two column. The PE activity is measured by the aggregate fund NAVs reported in column (2), and the pseudo fund NAVs (see section IV) in column (3). The test assets include returns on risk free rate, CRSP value-weighted index, and six size and style portfolios. Only four assets are included in the parameter estimation though: risk free rate, public market, small growth and small value. First two rows report the model J-statistic and the associated p-value. The last two rows report mean and standard deviation of the resulting SDF quarterly series, weighted accordingly. The penultimate eight rows report mean pricing error by each test asset in basis points per quarter, weighted accordingly. The remaining rows report the point estimates and robust for heteroskedasticity and autocorrelation standard errors (in parentheses) for the SDF intercept, a, and the relative risk aversion coefficient,γ.

	Panel A: CAPM			**Panel B:** LRR			**Panel C:** Habit		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
J-statistic	11.06	2.86	3.75	9.59	3.32	1.21	9.50	3.31	0.84
p-value	0.004	0.239	0.153	0.008	0.190	0.546	0.009	0.191	0.657
a^*	0.020	0.017	0.016	-0.079	-0.100	-0.096	-0.042	-0.080	-0.069
	(0.014)	(0.012)	(0.012)	(0.009)	(0.035)	(0.025)	(0.002)	(0.023)	(0.012)
γ^*	2.54	3.16	2.82	9.27	9.12	9.29	2.08	1.89	1.90
	(0.86)	(1.14)	(1.22)	(3.37)	(2.40)	(2.49)	(1.04)	(1.40)	(1.48)
$\hat{\rho}(a,\gamma)$	0.651	0.325	0.562	0.207	0.469	0.653	0.789	0.829	0.911
Risk-free rate	-40.9	-5.2	-29.4	-141.1	-164.1	-54.4	-136.8	-141.4	-5.2
Public Market	-0.3	-10.7	-25.5	23.6	20.8	-3.0	19.1	15.4	-11.9
Small Growth	-83.8	-42.4	-57.1	-12.1	40.2	9.3	-5.0	31.9	10.7
Small Neutral	76.6	63.6	86.3	108.0	108.7	76.8	101.2	99.4	49.6
Small Value	109.9	57.8	110.3	126.0	105.8	59.0	119.4	100.3	7.8
Large Growth	7.1	22.4	9.7	34.9	37.2	-13.8	30.6	32.9	-41.6
Large Neutral	21.5	-2.9	4.9	25.4	19.6	-4.4	18.2	10.7	-24.1
Large Value	41.0	-34.8	-30.7	54.5	25.5	6.1	43.4	24.1	0.2
$\hat{\mathbb{E}}[M]$	0.9969	0.9999	0.9951	0.9668	0.9454	0.9501	0.9900	0.9503	0.9613
$\hat{\sigma}(M)$	(0.309)	(0.42)	(0.352)	(0.223)	(0.199)	(0.208)	(0.262)	(0.206)	(0.218)

Table V

Venture funds performance: CAPM versus CBAPMs

This table reports Net Present Value estimates for venture fund cash flows described in table I, for the full sample (row 'All') and by selected vintage year groups. The number of funds in each group is indicated in column (1). The NPV is in dollars per dollar of capital committed. Panels A and C equally weight fund cash flows, Panel B – weights by the inflation-adjusted fund size. Panel C additionally restricts the sample to funds with inflation-adjusted distributions at least three times greater than the latest reported NAV. Columns (2) and (3) report K-N GPME for, respectively, the log-utility (as in K-S PME) and unrestricted CAPM cases. The rest of the columns report bias-corrected NPV estimates against SDFs implied by three models—CAPM, Long-run Risk, and External Habit—estimated as per section V.C. Columns (4)–(6) apply bootstrap correction (section V.B.2), while other columns subtract the NPV or pseudo funds investing in CRSP value-weighted index (columns 7–9) and Fama-French *small growth* (columns 10–12). The estimates statistically significant at 5% [10%] are superscripted with ^{a} [^{b}], see Appendix.A4 for detail.

		GPME		NPV			ΔNPV(mkt)			ΔNPV(ff6)		
Vintage	Funds	KS	KN	CAPM	LRR	Habit	CAPM	LRR	Habit	CAPM	LRR	Habit
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
				Panel A: All funds equally weighted								
<1990	227	-0.12^{b}	-0.40^{b}	-0.44^{a}	-0.20^{a}	-0.07^{a}	-0.06^{a}	-0.04^{b}	-0.05^{a}	0.03	-0.02	0.01
		(0.07)	(0.21)	(0.11)	(0.01)	(0.02)	(0.02)	(0.01)	(0.01)	(0.03)	(0.00)	(0.00)
1990-00	503	0.74^{b}	0.39	0.05	-0.08^{a}	-0.07^{a}	0.07	0.24^{a}	0.17^{a}	0.06	0.24^{a}	0.17^{a}
		(0.43)	(0.47)	(0.32)	(0.05)	(0.08)	(0.11)	(0.04)	(0.06)	(0.12)	(0.04)	(0.06)
2001-08	550	0.00	-0.77^{a}	-0.49^{a}	0.38^{a}	1.35	-0.08^{a}	-0.06^{a}	-0.37^{a}	-0.10^{a}	-0.08^{a}	-0.44^{a}
		(0.03)	(0.34)	(0.27)	(0.07)	(4.77)	(0.03)	(0.01)	(1.85)	(0.03)	(0.01)	(2.13)
1990-95	119	1.08^{a}	0.14	-0.24	-0.06^{a}	-0.11^{a}	0.29	0.19^{a}	0.23^{a}	0.37	0.21^{a}	0.27^{a}
1996-00	384	0.63^{a}	0.47	0.13	-0.09^{a}	-0.06^{a}	0.00	0.26^{a}	0.15^{a}	-0.04^{b}	0.25^{a}	0.14^{a}
2001-04	172	-0.13	-0.25	-0.17^{a}	0.14^{a}	0.60	-0.16^{a}	-0.06^{a}	-0.08^{a}	-0.19^{a}	-0.07^{a}	-0.10^{a}
2005-06	189	-0.04	-0.74^{b}	-0.46^{a}	0.50^{a}	2.08^{b}	-0.10^{a}	-0.10^{a}	-0.47^{a}	-0.13^{a}	-0.12^{a}	-0.55^{a}
2007-08	189	0.16^{b}	-1.27^{a}	-0.86^{a}	0.51	1.43	0.02	-0.03	-0.53^{a}	0.00	-0.06^{a}	-0.65^{a}
All	1,280	0.27	-0.20	-0.24	0.07^{a}	0.47	-0.02	0.06^{a}	-0.10^{a}	-0.02	0.06^{a}	-0.12^{a}
		(0.16)	(0.14)	(0.26)	(0.05)	(2.07)	(0.05)	(0.02)	(0.81)	(0.06)	(0.02)	(0.94)
				Panel B: All funds size-weighted								
<1990	227	-0.04	-0.37^{b}	-0.43^{a}	-0.16^{a}	-0.07^{a}	-0.06^{a}	-0.04^{a}	-0.06	0.02	-0.03	-0.01
1990-00	503	0.41^{b}	0.29	0.04	-0.08^{a}	-0.06^{a}	-0.10^{a}	0.14^{a}	0.08^{a}	-0.14^{a}	0.13^{a}	0.07^{a}
2001-08	550	0.06	-0.69^{b}	-0.48^{a}	0.38	1.47	-0.04^{b}	-0.04^{b}	-0.30^{a}	-0.07^{a}	-0.06^{a}	-0.37^{a}
1990-95	119	1.33^{a}	0.23	-0.22	-0.06^{a}	-0.11^{a}	0.35	0.21^{a}	0.26^{a}	0.42	0.24^{a}	0.29^{a}
1996-00	384	0.29	0.29	0.06	-0.08^{a}	-0.05^{a}	-0.16^{a}	0.13^{a}	0.05^{a}	-0.22^{a}	0.11^{a}	0.04^{a}
2001-04	172	-0.06	-0.05	-0.15	0.12^{a}	0.53	-0.07^{a}	-0.04^{a}	0.00	-0.10^{a}	-0.05^{a}	-0.01
2005-06	189	0.00	-0.75^{b}	-0.46^{a}	0.51	2.27^{b}	-0.09^{a}	-0.08^{a}	-0.41^{a}	-0.13^{a}	-0.11^{a}	-0.51^{a}
2007-08	189	0.22^{b}	-1.24^{b}	-0.85^{a}	0.54	1.71	0.05	0.01	-0.47^{a}	0.03	-0.01	-0.59^{a}
All	1,280	0.18^{b}	-0.15	-0.23	0.12^{a}	0.62^{b}	-0.07^{a}	0.03^{b}	-0.12^{a}	-0.09^{a}	0.02	-0.16^{a}
		(0.10)	(0.24)	(0.29)	(0.05)	(3.08)	(0.05)	(0.01)	(0.90)	(0.06)	(0.01)	(1.05)
				Panel C: Nearly resolved funds, equally weighted								
2001-08	355	0.02	-0.48^{a}	-0.35^{a}	0.35	1.36^{b}	-0.02	0.01	0.07	-0.06^{a}	-0.01	0.00
2001-04	153	-0.13	-0.27^{a}	-0.13^{a}	0.13^{a}	0.57	-0.13^{a}	-0.05^{a}	-0.04^{b}	-0.16^{a}	-0.06^{a}	-0.05^{a}
2005-06	119	0.06	-0.41^{b}	-0.35^{b}	0.52	2.24^{a}	0.02	-0.01	0.01	-0.02	-0.03^{b}	-0.08^{a}
2007-08	83	0.24^{b}	-0.97^{b}	-0.84^{a}	0.55^{b}	1.82^{b}	0.12	0.15^{a}	0.37	0.08	0.11^{a}	0.22
All	1,069	0.34^{a}	-0.07	-0.14	0.02	0.36^{a}	0.04	0.11^{a}	0.10	0.04	0.11^{a}	0.09^{a}
		(0.17)	(0.11)	(0.25)	(0.05)	(2.43)	(0.05)	(0.02)	(0.13)	(0.07)	(0.02)	(0.08)

Table VI

Buyout fund performance: CAPM versus CBAPMs

This table reports Net Present Value estimates for buyout fund cash flows described in table I, for the full sample (row 'All') and by selected vintage year groups. The number of funds in each group is indicated in column (1). The NPV is in dollars per dollar of capital committed. Panels A and C equally weight fund cash flows, Panel B – weights by the inflation-adjusted fund size. Panel C additionally restricts the sample to funds with inflation-adjusted distributions at least three times greater than the latest reported NAV. Columns (2) and (3) report K-N GPME for, respectively, the log-utility (as in K-S PME) and unrestricted CAPM cases. The rest of the columns report bias-corrected NPV estimates against SDFs implied by three models—CAPM, Long-run Risk, and External Habit—estimated as per section V.C. Columns (4)–(6) apply bootstrap correction (section V.B.2), while other columns subtract the NPV or pseudo funds investing in CRSP value-weighted index (columns 7–9) and Fama-French *small value* (columns 10–12). The estimates statistically significant at 5% [10%] are superscripted with ^{a} [^{b}], see Appendix.A4 for detail.

		GPME		NPV			ΔNPV(mkt)			ΔNPV(ff6)		
Vintage	Funds	KS	KN	CAPM	LRR	Habit	CAPM	LRR	Habit	CAPM	LRR	Habit
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
				Panel A: All funds equally weighted								
<1990	54	0.35	0.01	-0.38^{a}	-0.12^{a}	-0.08	0.21^{a}	0.06^{a}	0.10^{a}	0.16^{a}	0.05^{a}	0.05^{a}
		(0.23)	(0.06)	(0.18)	(0.01)	(0.03)	(0.06)	(0.01)	(0.01)	(0.04)	(0.01)	(0.01)
1990-00	350	0.34^{a}	0.50^{a}	0.79^{a}	-0.03^{b}	-0.03^{b}	0.62^{a}	0.06^{a}	0.03^{b}	-0.33^{a}	-0.04^{a}	-0.06
		(0.14)	(0.19)	(0.29)	(0.02)	(0.08)	(0.08)	(0.01)	(0.01)	(0.10)	(0.01)	(0.04)
2001-08	706	0.19^{a}	-0.18	-0.19	0.51	1.80^{b}	0.20^{a}	0.05^{a}	0.06	0.16^{a}	0.03	-0.02
		(0.05)	(0.36)	(0.25)	(0.10)	(8.40)	(0.04)	(0.01)	(0.35)	(0.04)	(0.01)	(0.42)
1990-95	96	0.11^{b}	-0.08	-0.12^{a}	-0.07^{a}	-0.13^{a}	0.05^{a}	0.03	0.03^{b}	-0.09	0.01	0.01
1996-00	254	0.43^{a}	0.72^{a}	1.17^{a}	-0.01	0.01	0.83^{a}	0.07^{a}	0.03^{b}	-0.42^{a}	-0.06^{a}	-0.08
2001-04	194	0.55^{a}	1.17^{a}	0.31^{b}	0.31	0.94	0.62^{a}	0.24^{a}	0.43	0.50^{a}	0.18^{a}	0.37
2005-06	234	0.14	0.11	-0.02	0.69	2.88^{a}	0.15^{a}	0.07^{a}	0.28	0.13^{a}	0.06^{a}	0.24
2007-08	278	-0.03^{b}	-1.35^{a}	-0.76^{a}	0.54	1.62^{b}	-0.04^{b}	-0.09^{a}	-0.40^{a}	-0.06^{a}	-0.11^{a}	-0.52^{a}
All	1,110	0.24^{a}	0.48	0.16	0.28	1.03^{a}	0.33^{a}	0.05^{a}	0.05	0.00	0.01	-0.03^{b}
		(0.06)	(0.30)	(0.25)	(0.07)	(5.37)	(0.05)	(0.01)	(0.22)	(0.01)	(0.01)	(0.26)
				Panel B: All funds size-weighted								
<1990	54	0.27	-0.01	-0.35^{a}	-0.11^{a}	-0.07^{a}	0.17^{a}	0.06^{a}	0.06^{a}	0.14^{a}	0.05^{a}	0.01
1990-00	350	0.42^{a}	0.62^{a}	0.89^{a}	-0.02	-0.01	0.72^{a}	0.07^{a}	0.04^{a}	-0.32^{a}	-0.05^{a}	-0.06^{a}
2001-08	706	0.14^{a}	-0.32	-0.18	0.55	1.98^{a}	0.07	0.00	-0.09^{a}	0.02	-0.03	-0.18^{a}
1990-95	96	0.16^{b}	-0.04	-0.09^{a}	-0.07^{a}	-0.12^{a}	0.07^{a}	0.04^{b}	0.04^{b}	-0.07	0.02	0.02
1996-00	254	0.48^{a}	0.77^{a}	1.15^{a}	-0.01	0.02	0.87^{a}	0.07^{a}	0.05^{a}	-0.38^{a}	-0.06^{a}	-0.07^{a}
2001-04	194	0.61^{a}	1.16^{a}	0.32	0.29	0.88	0.62^{a}	0.26^{a}	0.49	0.50^{a}	0.20^{a}	0.44
2005-06	234	0.06	0.01	0.09	0.73	3.07^{a}	-0.03^{b}	-0.01	0.12	-0.06^{a}	-0.03	0.07
2007-08	278	0.00	-1.27^{a}	-0.68^{a}	0.57	1.71^{b}	-0.09^{a}	-0.11^{a}	-0.54^{a}	-0.11^{a}	-0.13^{a}	-0.67^{a}
All	1,110	0.19^{a}	0.30	0.12	0.36	1.30^{a}	0.23^{a}	0.02	-0.06^{a}	-0.06^{a}	-0.03^{b}	-0.14^{a}
		(0.05)	(0.34)	(0.27)	(0.08)	(7.01)	(0.06)	(0.01)	(0.41)	(0.04)	(0.01)	(0.72)
				Panel C: Nearly resolved funds, equally weighted								
2001-08	648	0.22^{a}	-0.09	-0.15	0.52	1.83^{a}	0.25^{a}	0.09^{a}	0.19	0.20^{a}	0.06^{a}	0.11
2001-04	191	0.57^{a}	1.10^{a}	0.32^{b}	0.31	0.94	0.64^{a}	0.24^{a}	0.44	0.51^{a}	0.19^{a}	0.38
2005-06	222	0.16	0.13	-0.01	0.69^{b}	2.90^{a}	0.17^{a}	0.09^{a}	0.37	0.16^{a}	0.08^{a}	0.33
2007-08	235	0.01	-1.26^{a}	-0.73^{a}	0.55	1.70^{a}	0.00	-0.04^{a}	-0.18^{a}	-0.02	-0.07^{a}	-0.31^{a}
All	1,049	0.27^{a}	0.49^{b}	0.20	0.27	1.02^{a}	0.37^{a}	0.08^{a}	0.14	0.02	0.03	0.05
		(0.05)	(0.27)	(0.26)	(0.07)	(5.74)	(0.05)	(0.02)	(0.58)	(0.01)	(0.01)	(0.34)

Table VII

Generalist fund performance: CAPM versus CBAPMs

This table reports Net Present Value estimates for cash flows funds classified as 'generalists' (see table I), for the full sample (row 'All') and by selected vintage year groups. The number of funds in each group is indicated in column (1). The NPV is in dollars per dollar of capital committed. Panels A and C equally weight fund cash flows, Panel B – weights by the inflation-adjusted fund size. Panel C additionally restricts the sample to funds with inflation-adjusted distributions at least three times greater than the latest reported NAV. Columns (2) and (3) report K-N GPME for, respectively, the log-utility (as in K-S PME) and unrestricted CAPM cases. The rest of the columns report bias-corrected NPV estimates against SDFs implied by three models—CAPM, Long-run Risk, and External Habit—estimated as per section V.C. Columns (4)–(6) apply bootstrap correction (section V.B.2), while other columns subtract the NPV or pseudo funds investing in CRSP value-weighted index (columns 7–9) and Fama-French *small value* (columns 10–12). The estimates statistically significant at 5% [10%] are superscripted with ^{a} [^{b}], see Appendix.A4 for detail.

		GPME		NPV			ΔNPV(mkt)			ΔNPV(ff6)		
Vintage	Funds	KS	KN	CAPM	LRR	Habit	CAPM	LRR	Habit	CAPM	LRR	Habit
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
				Panel A: All funds equally weighted								
<1990	20	0.05	-0.25	-0.46^a	-0.14^a	-0.01	0.00	-0.02	-0.02	-0.07^a	-0.04^b	-0.08^a
		(0.04)	(0.16)	(0.15)	(0.00)	(0.03)	(0.02)	(0.00)	(0.00)	(0.00)	(0.01)	(0.02)
1990-00	170	0.23^b	0.25^a	0.83^b	-0.04^b	-0.04^b	0.24^a	0.04^b	0.03	-0.88^a	-0.07^a	-0.07
		(0.14)	(0.12)	(0.28)	(0.02)	(0.07)	(0.03)	(0.01)	(0.01)	(0.16)	(0.02)	(0.05)
2001-08	210	0.06	-0.26	-0.22	0.70^a	2.63^a	0.08^a	-0.04^a	-0.21^a	0.04	-0.06^a	-0.29^a
		(0.04)	(0.26)	(0.24)	(0.09)	(8.68)	(0.02)	(0.01)	(0.74)	(0.02)	(0.01)	(1.26)
1990-95	43	0.35^a	-0.03	-0.19^a	-0.04^a	-0.10^a	0.11	0.05^a	0.06^a	0.00	0.04^b	0.04^b
1996-00	127	0.19^a	0.34^a	1.17^a	-0.04^b	-0.02	0.29^a	0.03^b	0.01	-1.18^a	-0.10^a	-0.11
2001-04	48	0.38^a	0.78^a	0.38	0.35	1.18	0.46^a	0.16^a	0.42	0.34^a	0.11^a	0.37
2005-06	69	0.15	0.09	0.04	0.85	3.63^a	0.15^a	0.07^a	0.37^a	0.13^b	0.06^a	0.33^b
2007-08	93	-0.17^a	-1.05^a	-0.76^a	0.80^a	2.75^a	-0.17^a	-0.23^a	-0.97^a	-0.18^a	-0.25^a	-1.09^a
All	400	0.13^a	0.11	0.26	0.31^a	1.25^a	0.15^a	-0.01	-0.10^a	-0.36^a	-0.06^a	-0.19^a
		(0.05)	(0.15)	(0.25)	(0.05)	(4.58)	(0.03)	(0.01)	(0.39)	(0.06)	(0.01)	(0.68)
				Panel B: All funds size-weighted								
<1990	20	0.09	-0.20	-0.42^a	-0.10^a	0.01	0.03	-0.01	0.01	-0.05^a	-0.03^b	-0.06^a
1990-00	170	0.42^a	0.56^a	0.97^a	-0.03	-0.01	0.65^a	0.06^a	0.07^b	-0.56^a	-0.06^a	-0.04^b
2001-08	210	0.05	-0.43	-0.23^b	0.73	2.91^b	-0.01	-0.10^a	-0.56^a	-0.07^a	-0.13^a	-0.68^a
1990-95	43	0.81^a	0.22	-0.16	-0.05^a	-0.11^a	0.24	0.11^a	0.12^a	0.14	0.10^a	0.11^a
1996-00	127	0.35^a	0.62^a	1.18^a	-0.02	0.01	0.72^a	0.05^a	0.06	-0.69^a	-0.09^a	-0.06^a
2001-04	48	0.43^a	0.95^a	0.41	0.24	0.78	0.49^a	0.14^a	0.47	0.30^b	0.07^a	0.40
2005-06	69	0.18^b	0.30	0.35	0.89^b	3.57^a	0.07	0.09^a	0.65	0.05	0.09^a	0.67
2007-08	93	-0.18^a	-1.40^a	-0.87^a	0.86^a	3.49^b	-0.26^a	-0.30^a	-1.68^a	-0.29^a	-0.33^a	-1.91^a
All	400	0.20^a	0.29	0.30	0.37^a	1.54^b	0.25^a	-0.03^b	-0.29^a	-0.27^a	-0.10^a	-0.41^a
		(0.08)	(0.32)	(0.31)	(0.06)	(4.96)	(0.07)	(0.01)	(2.22)	(0.03)	(0.02)	(2.71)
				Panel C: Nearly resolved funds, equally weighted								
2001-08	170	0.16^a	-0.08	-0.04	0.78	3.12^a	0.19^a	0.04^a	0.14	0.14^a	0.02	0.05
2001-04	44	0.43^a	0.71^a	0.38	0.37	1.24	0.54^a	0.19^a	0.49	0.42^a	0.14^a	0.44
2005-06	62	0.23	0.12	0.20	0.95^b	4.16^a	0.22^a	0.12^a	0.59^a	0.19^b	0.11^a	0.53^b
2007-08	64	-0.08^b	-0.81^a	-0.58^a	0.94	3.65^a	-0.08^a	-0.12^a	-0.54^a	-0.10^a	-0.15^a	-0.68^a
All	357	0.19^a	0.16	0.43	0.31^a	1.34^a	0.21^a	0.04^b	0.08	-0.34^a	-0.02	-0.01
		(0.04)	(0.11)	(0.26)	(0.06)	(5.56)	(0.04)	(0.01)	(0.40)	(0.06)	(0.00)	(0.19)

Figure 1. PE fund sample

This figure plots the aggregate net asset values reported by 2,750 private equity funds incepted between 1979 and 2008. See table I for sample description.

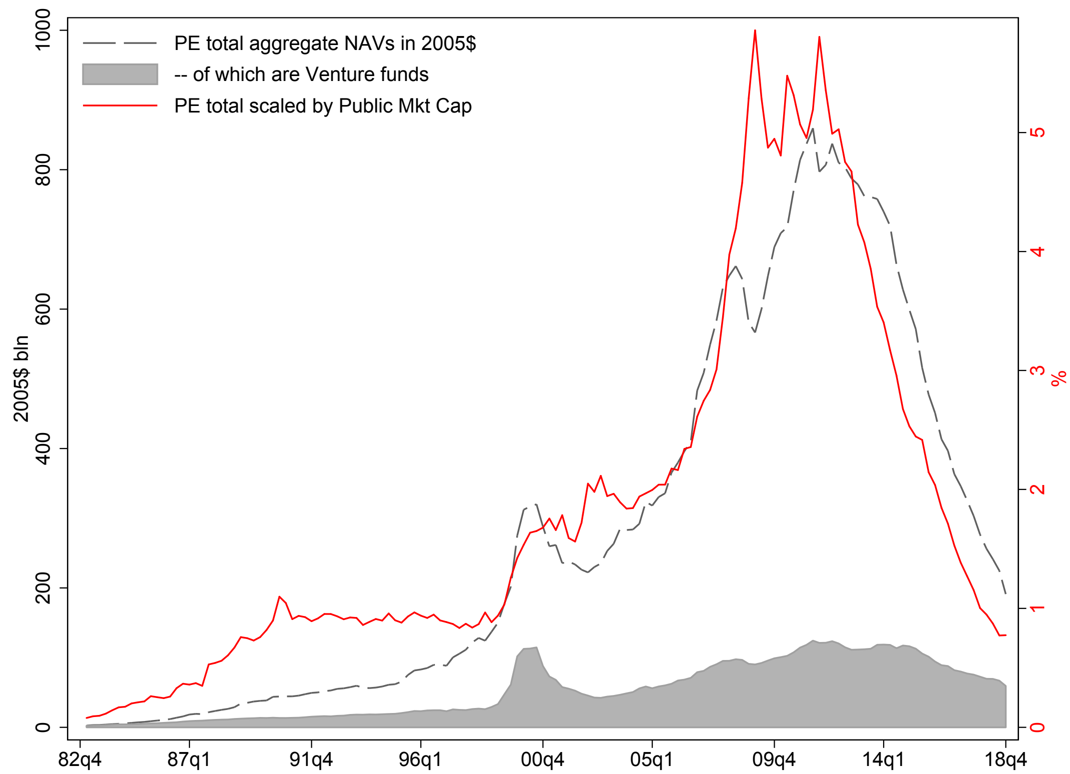


Figure 2. Long-term changes for off-the-shelf SDFs

This figure plots rolling 6-year log changes in the SDFs implied by models calibrated as in the literature. Panel A graphs the series at a semiannual, Panel B breaks down the returns in the CBAMP SDFs by source of innovation.

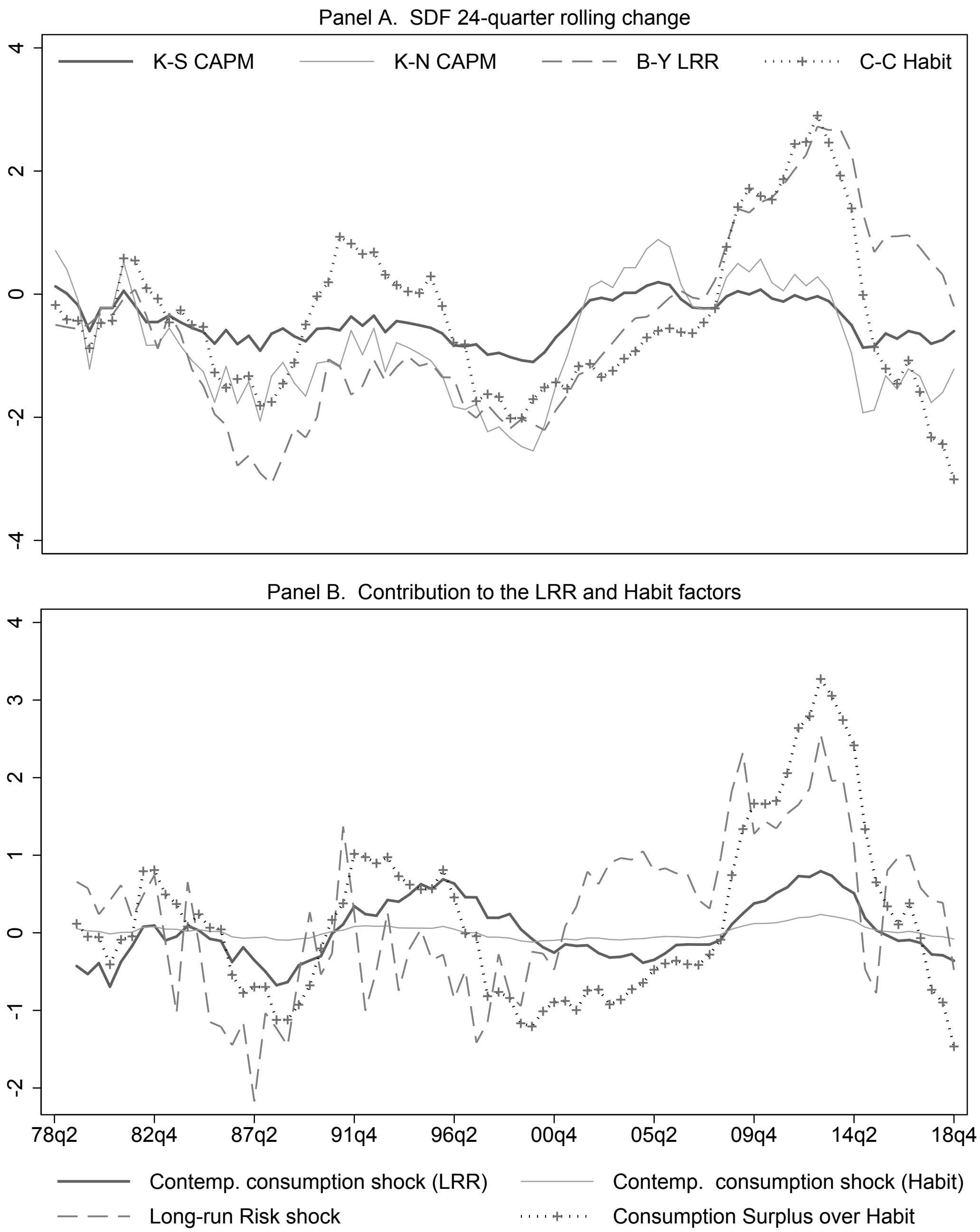


Figure 3. PE pseudo funds

This figure describes the size and cash flow dynamics of 2,750 PE funds incepted between 1979 and 2008 (see table I for details). Panel A plots the aggregate market value of PE mimicking portfolios (i.e., *pseudo funds*), constructed as in Korteweg and Nagel (2016), in inflation-adjusted US dollars and scaled by CRSP index market value. The dates of all cash flows and the size of capital calls of each pseudo fund are set equal to those of the respective actual fund. Unlike the actual funds, the pseudo funds invest in public market index. The size of a pseudo fund distribution is a fraction of its asset value right before the distribution. This fraction is determined by a fixed rule that reflects the time elapsed since the previous distribution and the fund age. Panel B plots the aggregate distributions of the pseudo funds and the actual funds, as well the aggregate capital calls and the last reported NAV (assumed to be the terminal distributions).

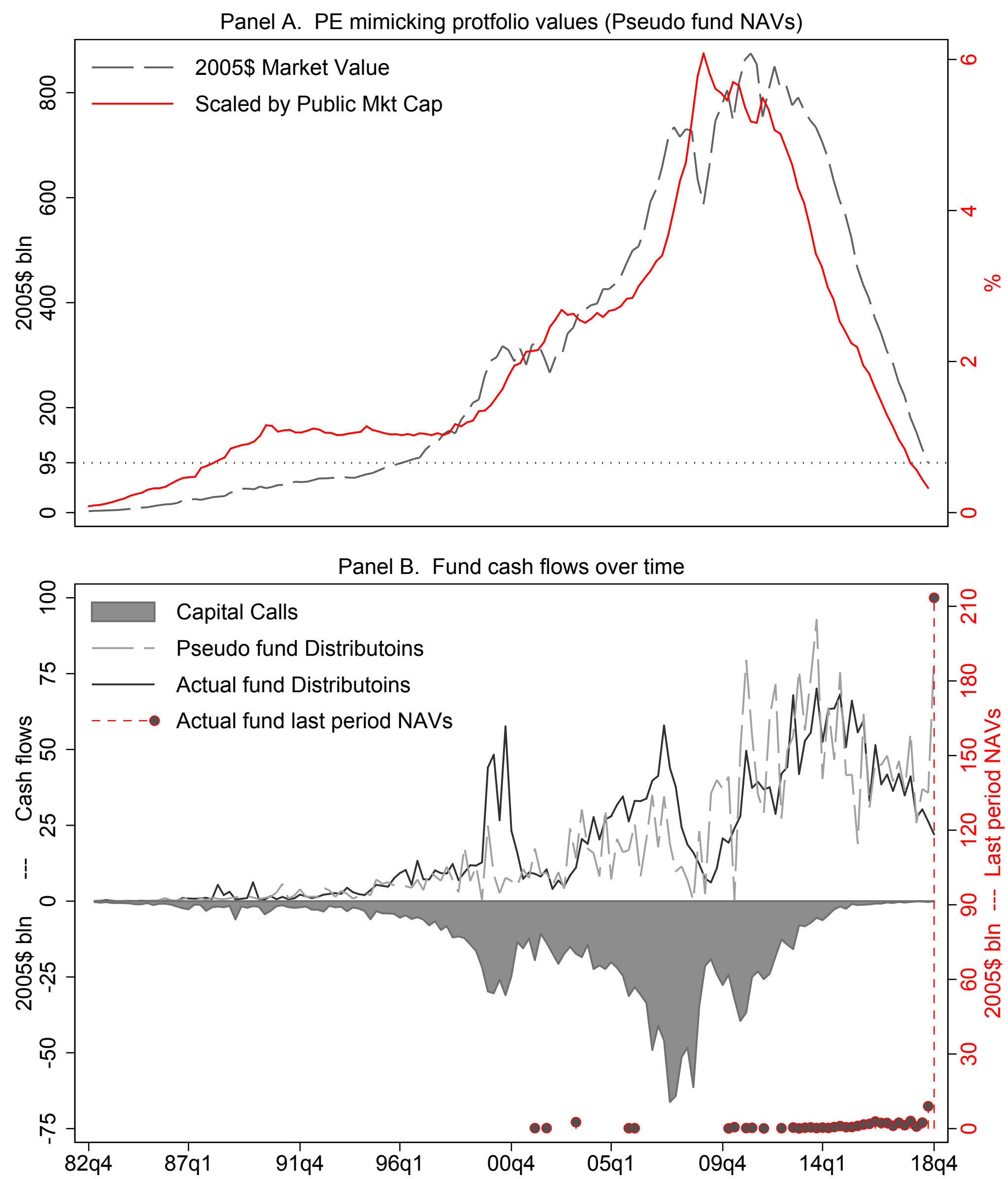


Figure 4. Key findings

This figure graphs the main findings of this study. The bars are the estimates of expected NPV per dollar invested in, respectively, venture, generalist and buyout funds according to two types of asset pricing models: CAPM and consumption-based ones. *Log Utility* CAPM corresponds to the assumption implied in the computation of PME of Kaplan and Schoar (2005), whereas *Unrestricted* CAPM builds on the method of Korteweg and Nagel (2016). Long-run Risk and External Habit formation models are of Campbell and Cochrane (1999) and Bansal and Yaron (2004) respectively. The *NPV is bias-adjusted using pseudo funds* investing in public markets. The sample is described in table I and includes *substantially resolved funds only*. See panel C of tables VII, VI, and VII for additional detail. The estimates reported in this figure correspond to those in columns (2) and (7) through (9).

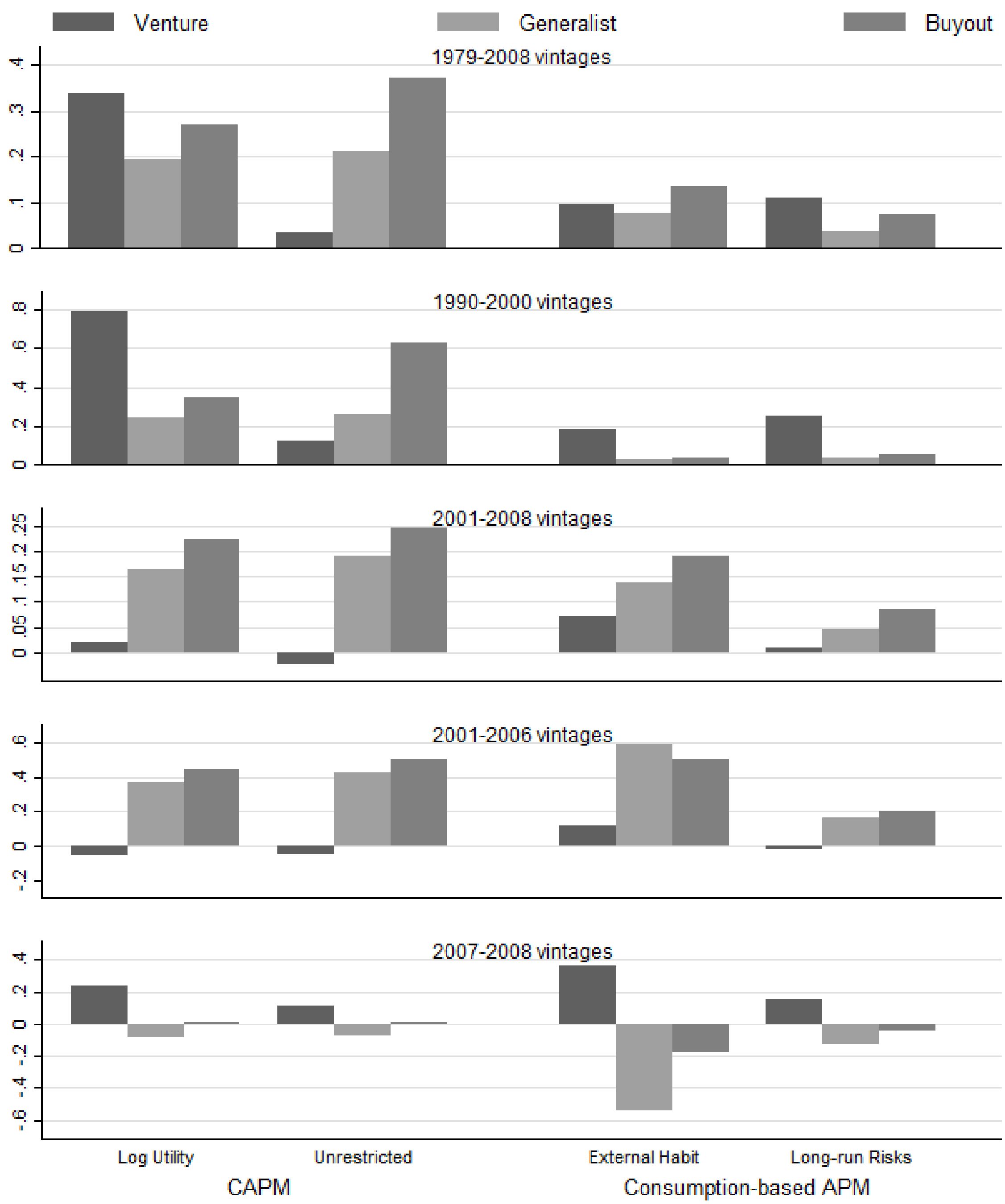
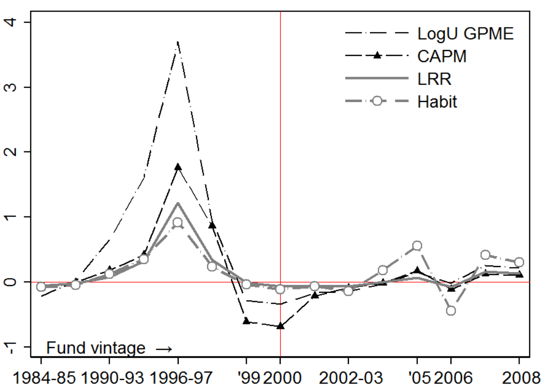


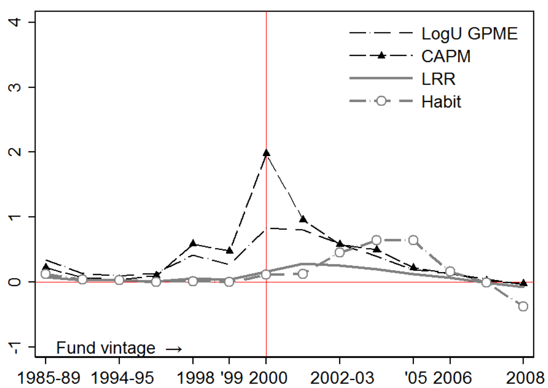
Figure 5. PE fund performance in excess of public equity

This figure plots PE fund Net Present Values by vintage year against the log-utility GPME (i.e., K-S PME expressed as difference rather than ratio), the discount factors implied by unrestricted CAPM, the Long-run Risk, and the Habit formation models adjusted for the NPV of similarly timed public investments for the sample of nearly resolved venture and buyout funds—ie., with inflation-adjusted distributions at least three times greater than NAV reported as of Q4 2018. Panel A1 [A2] reports results net of value-weighted CRSP index for venture [buyout], Panel B1 [B2] - Fama-French small *growth* [*value*] stocks. NPVs are in dollars per dollar of commitment capitall and equally weighted across funds. See table I for sample description. Each vintage group features at least 40 funds.

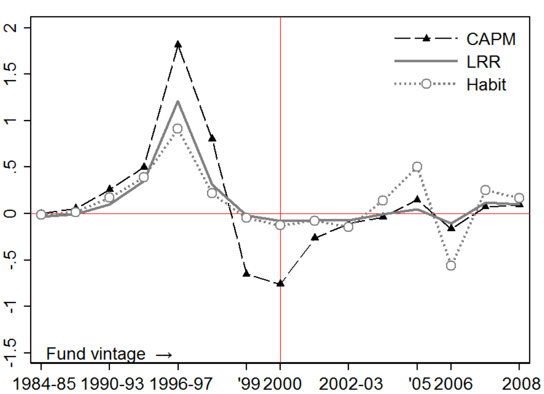
Panel A1. Venture *VS* public market



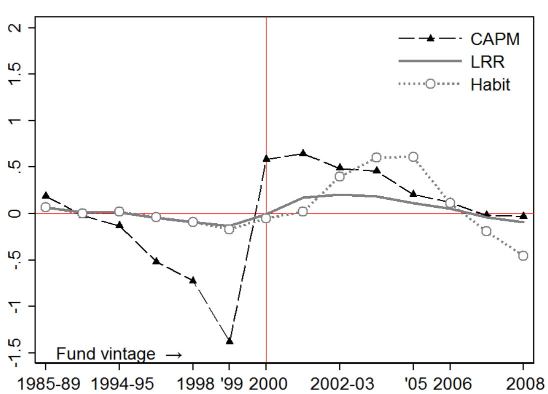
Panel A2. Buyout *VS* public market



Panel B1. Venture *VS* small growth



Panel B2. Buyout *VS* small value



Appendix A. Additional details on methodology

Appendix A1. Proofs

PROPOSITION 1 (Compounding Bias): *The NPV-based test per 15 is biased relatively to the returns-based test 14 even if the SDF is exponentially affine. The magnitude and direction of the bias are not explained by the duration of the funds alone but also depend on the compounding of pricing errors.*

Proof. Consider a population of 3-period funds with perfectly overlapping lives that make a single capital call of \$1 in the beginning of the first period, earn R_{i1}, R_{i2}, and R_{i3} gross return on their assets during the first, second, and third, and distribute all capital in the end of the third period . The SDF realization are, respectively, M_1, M_2, and M_3.

Plugging $z_0 = z_1 = z_2 = 1$, $\delta_{i0} = -1$, $\delta_{i1} = 0$, $\delta_{i2} = 0$, $\delta_{i3} = 1$ in equations (14–15) and dropping i superscript to reduce notation clatter, we get the following difference between the statistics of the tests (15) and (14):

$$\begin{aligned}
NPVbias_{T=3} :=& \mathbb{E}[R_1R_2R_3M_1M_2M_3 - 1] - \frac{1}{3}\big(\mathbb{E}[R_1M_1 - 1] + \mathbb{E}[R_2M_2 - 1] + \mathbb{E}[R_3M_3 - 1]\big) \\
=& \mathbb{E}[R_1M_1 \cdot R_2M_2 \cdot R_3M_3] - \frac{1}{3}\sum_{k=1}^{3}\mathbb{E}[R_kM_k] \\
=& \mathbb{E}[(e_1+1)(e_2+1)(e_3+1)] - \frac{\sum_{k=1}^{3}\mathbb{E}[e_k]}{3} - \frac{3}{3} \\
=& \mathbb{E}[e_1e_2e_3 + e_1e_2 + e_1e_3 + e_1 + e_2e_3 + e_2 + e_3 + 1] - \frac{\sum_{k=1}^{3}\mathbb{E}[e_k]}{3} - 1 \\
=& \frac{2}{3}\sum_{k=1}^{3}\mathbb{E}[e_k] \ + \mathbb{E}[e_1e_2e_3] \ + \mathbb{E}[e_1e_2] + \mathbb{E}[e_1e_3] + \mathbb{E}[e_2e_3].
\end{aligned}$$

Similarly, for a 4-period funds, the inference bias is:

$$\begin{aligned}
NPVbias_{T=4} :=& \frac{3}{4}\sum_{k=1}^{4}\mathbb{E}[e_k] + \\
&+ \mathbb{E}[e_1e_2e_3e_4] \ + \mathbb{E}[e_1e_2] + \mathbb{E}[e_1e_3] + \mathbb{E}[e_1e_4] + \mathbb{E}[e_2e_3] + \mathbb{E}[e_2e_4] + \mathbb{E}[e_2e_5] + \\
&+ \mathbb{E}[e_1e_2e_3] + \mathbb{E}[e_1e_2e_4] + \mathbb{E}[e_2e_3e_4] + \mathbb{E}[e_2e_3e_5]
\end{aligned}$$

, and for a more general case of T-periods funds,

$$\begin{aligned}
NPVbias_T :=& \frac{(T-1)}{T}\sum_{k=1}^{4}\mathbb{E}[e_k] + && \text{–}\textit{duration difference} \qquad \text{(A.1)} \\
&+ \mathbb{E}[\prod_{k=1}^{T} e_k] \ + \frac{1}{2}\mathbb{E}[\sum_{k=1}^{T}\sum_{j\neq k}^{T} e_k e_j] \ + \mathbb{E}[\sum_{n=3}^{T-2} C^e(T-1, n)] && \text{–}\textit{compounding error}
\end{aligned}$$

, where $C^e(T, t)$ demotes a permutation term, e.g., $C^e(5, 4) = e_1e_2e_3e_4 + e_1e_2e_3e_5 + e_1e_2e_4e_5 + e_1e_3e_4e_5 + e_2e_3e_4e_5$.

With not fully overlapping fund lives and interim distributions, e_k-terms simply get scaled by the cross-sectional expectation of δ_{it}, which can also be autocorrelated and cross-autocorrelated with e_k. See Table AI for simulation-based evidence. □

PROPOSITION 2 (Cash flow-based moments equivalence): *The benchmark NPV restriction per 18 is equivalent to the benchmark return restriction 12, in which the instrument, z_t, is either less efficient or correlated with the pricing error, e^B_{t+1}.*

Proof. Using the mapping definition per 18, rewrite the expectation of $NPV(\theta_{CB})^b_i$ as follows:

$$\begin{aligned}\mathbb{E}[NPV_i^B(\theta_{CB})] =& E\Big[\sum_{t=s(i)}^{T(i)} C^b_{it} \prod_{\tau=s(i)}^{t} M_\tau(\theta_{CB})\Big] \\ =& \mathbb{E}\big[z'_{t-1}(R^b_t \cdot M_t(\theta_{CB}) - 1) \cdot cM^-_{t-1}(\theta_{CB}) \cdot \tilde{\delta}_t\big] \\ =& \mathbb{E}\big[z'_{t-1}e^b_t(\theta_{CB})\big] \quad \cdot \mathbb{E}[\tilde{\delta}_t] \cdot \mathbb{E}[cM^-_t(\theta_{CB})] + \\ & + cov\big(z'_{t-1}e^b_t(\theta_{CB}) \ , \ \tilde{\delta}_t\big) \cdot E[cM^-_{t-1}(\theta_{CB})] + cov\big(z'_{t-1}\tilde{\delta}_t \cdot e^b_t(\theta_{CB}) \ , \ cM^-_{t-1}(\theta_{CB})\big)\end{aligned} \tag{A.2}$$

, where z'_t is the average NAVs of pseudo funds which life spans over period t ($\equiv I(t)$):

$$z'_t = \sum_{i}^{i \in I(t)} \Big(- C^b_{i0} \prod_{\tau(i)=1}^{t} R^b_{\tau(i)}(1 - \tilde{\delta}_{\tau(i)})\Big)/|I| \quad , \tag{A.3}$$

and $cM^-_t(\theta_{CB})$ is the cumulative SDF innovation through period t since the average sample fund inception:

$$cM^-_t(\theta_{CB}) = \sum_{i}^{i \in I(t)} \Big(\prod_{\tau=s(i)}^{t} M_\tau(\theta_{CB})\Big)/|I| \quad . \tag{A.4}$$

From expression A.2 it follows that, given a valid instrument z'_t (i.e., which does not condition on the information set when pricing errors are realized), the '*pseudo funds*'-based identification scheme exhibits a loss of efficiency relatively to expression (12) because it features the covariance terms of the pricing error of the pseudo funds with their distribution intensity and the SDF history. The identification of θ via expression (12) is equivalent to that via (18) for $z_t = z'_t \cdot m(\theta_{CB})^-_t \cdot \tilde{\delta}_{t+1}$ which potentially fails the orthogonality condition, $\mathbb{E}[z_t, e(\theta_{CB})_t] = 0$, and exhibits additional variation that is theoretically uninformative of θ. Therefore, θ_{CB} is not equal to θ_B due to either noise or bias, whereby the latter can be present even asymptotically for some $\tilde{\delta}_t$ or $m(\theta_{CB})^-_t$ that result in non-zero covariance with the pricing error. In other words, for a given z'_t, one cannot improve upon the sample counterpart of expression (12) in estimating SDF parameters, so θ_{CB} is at most as good an estimate of true θ as θ_B but strictly worse at least for some choice of $M_t(\theta)$ and $\tilde{\delta}_t$. See Table AII for simulation-based evidence. □

PROPOSITION 3 (Benchmark choice conflict): *Given a mapping function $\tilde{\delta}_{it}$, the excess NPV test (20) performed at θ_B is less biased and more efficient than the NPV-test (15) or the excess NPV test (20) at θ_{CB}.*

Proof. The intuition is that Korteweg and Nagel (2016) GPME is a weighted average *excess* NPV over the selected *pseudo fund* NPVs. To see this, consider an overidentified system whereby number of benchmark exceeds the dimension of SDF parameter vector θ, and one of the *pseudo funds* invests in risk-free rate, rf:

$$GPME(\theta) = NPV(\theta) \; - \; \big(w_{rf} \cdot NPV(\theta)^{rf} + \sum_{b \neq rf} w_b \cdot NPV(\theta)^b\big)$$
$$\text{, where } \sum_{b \neq rf} w_b + w_{rf} = 1 \tag{A.5}$$

and parameters θ are chosen so that the squared NPVs of *pseudo funds* are jointly minimized, while each being affected by the *compounding error* as discussed in PROPOSITION 1. This implies that the bias correction through GPME puts non-zero weight on $NPV(\theta)^{rf}$. The latter reflects pricing error series $\epsilon(\theta)_t^{rf}$ which are the least correlated with those of the actual PE funds, $\epsilon(\theta)_t$, and, thus, are more likely to exhibit a different *compounding error* than that in PE funds' NPV estimates (provided that SDF innovations, returns of risky assets and PE funds, indeed have a significant common factor). Meanwhile, the fact that θ_{CB} is further away from true parameter values than θ_B (PROPOSITION 1) adds additional noise to inference about *excess* NPV (see Table AIII). □

Appendix A2. Simulations

Setup. Our data generating process nests the one considered in Korteweg and Nagel (2016) when the SDF persistence parameter, ρ, equals zero and the benchmark assets are the risk free rate ($r_{f,t} = (1-\rho)r_f + \rho f_{t-1}$) and the risk factor(s) f_t that drive SDF itself:

$$\begin{aligned} f_t &= (1-\rho)\big(r_f + \frac{\gamma\sigma^2}{1-\rho} - \frac{\sigma^2}{2(1-\rho)}\big) + \rho f_{t-1} + \sigma\varepsilon_t \\ &= r_{f,t} + \gamma\sigma^2 - \frac{\sigma^2}{2} + \sigma\varepsilon_t, \quad \text{and} \qquad \varepsilon_t \sim \mathcal{N}(0, 0.15^2 \text{ p.a.}) \end{aligned} \tag{A.6}$$

Assuming that the log-SDF, $m_t = a_t + bf_t$, perfectly prices f_t and $r_{f,t}$, such that $\mathbb{E}_{t-1}[\exp(r_{f,t})\exp(m_t)] = 1$ and $\mathbb{E}_{t-1}[\exp(f_t)\exp(m_t)] = 1$, one can solve for SDF parameters $a = \mathbb{E}[a_t]$ and b ($a, b \in \theta$):

$$\begin{aligned} b = \gamma \quad , \qquad a_t &= (\gamma-1)r_{f,t} + \frac{\gamma\sigma^2}{2}(\gamma-1) \\ \mathbb{E}[a_t] &= (\gamma-1)\left(r_f + \frac{\rho\gamma\sigma^2}{1-\rho} - \frac{\rho\sigma^2}{2(1-\rho)}\right) + \frac{\gamma\sigma^2}{2}(\gamma-1)\ . \end{aligned} \tag{A.7}$$

To model non-tradeable SDFs, we consider risky benchmarks that are affine in the risk factor: $r_{bt} = f_t + u_{bt}$, where $u_{bt} \sim \mathcal{N}(0, \sigma_u^2)$.[A-1] We assume that the first risky benchmark has a σ_u of 0.1 per year which can be interpreted as the idiosyncratic return relatively to the factor. If additional benchmarks are used in estimation, their σ_u are equal to 0.15 per year. As in Korteweg and Nagel (2016), r_f is set 2.5% per year and the PE funds return process is defined similarly to that if the benchmarks' with $\sigma = 0.25$ per year and is 0.1-correlated across funds.

We consider several hypothetical fund samples: (i) *Very Large*: 2,500 funds equally spread over 500 vintage years to gauge the asymptotic properties, (ii) *Large*: 2,500 funds equally spread over 50 vintage years as considered in the simulations by Korteweg and Nagel (2016), (iii) *Realistic*: 30

[A-1] Notably under the non-treadable SDF, the NPV-estimate of investing is public benchmark is not necessarily equal to zero due to a sampling error in u_{bt}, even for the just-identified case with $\rho = 0$.

vintages unequally (i.e., “sparsely”) populated consistent with the actual sample of venture funds as reported in Table I. The simulated funds make on average 20 distributions of (future values of) the equal fraction of capital invested in quarter 0. The distributions are uniformly distributed over a maximum of 44 quarters. This implicitly defines the returns-to-cash flow mapping function δ_{it} (equation 16) as the fund since-inception returns scaled by a random number.

To implement the feasible bootstrap procedure (section V.B.2), we assume that feasible high-frequency PE pricing errors represent either a 1-period or 4-period moving averages of the true pricing errors. This assumptions is consistent with the time-series of PE fund returns based on the as-reported NAVs exhibiting significant time-series persistence even after standard “unsmoothing techniques” are applied (see, e.g., Goetzmann, Gourier, and Phalippou, 2018). Unless explicitly stated otherwise, we use the risky benchmark with the smallest idiosyncratic return relative to the factor—thus. by construction, relative to the PE funds returns as well—for *excess NPV* computation (section V.B.1). The intuition here is that one can ex-ante determine a publicly traded asset that is most related to the PE group of funds of interest (e.g., ’small growth’ equities for venture, ’small value’ for buyouts). At the same time, we seek to mimic the reality where other publicly traded assets are observable and informative of the SDF process, even though they are less related to the return generating process of PE funds.

In-line with CBAPMs’ data limits, we simulate and conduct estimations at quarterly frequency, assume $\rho = 0.2$ per annum for autoregressive SDF cases, and assume that the measurement error is MA(2):{0.50,0.25} for the SDF with measurement error cases.[A-2]

Discussion. Table AI illustrates PROPOSITION 1. Each panel reports median, mean, and root mean squared errors for NPV estimates across the simulated PE fund samples when SDF parameters are estimated consistently. Panel A reveals that, regardless of whether the SDF is spanned (i.e., tradeable) or unspanned by the public benchmarks, the NPV-based test (15) reported in column (1) is biased even when the sample is practically infeasible (2,500 funds over 50 years). The median bias of negative 3 cents per dollar (i.e., on the same order as the point estimates in the literature) switches to positive 5 with a 3-fold increase in RMSE (from 15-19 to 42) if there is positive autocorrelation in the measurement error on the SDF, arguably present with macro time series.[A-3] The remaining columns of Panel A indicate a substantial improvement in both the bias and efficiency of the estimates once the suggested bias-correction methods are applied. In particular, both the *excess NPV* and *Bootstrapped-NPV* estimates are centered much closer to the true values while exhibiting significantly smaller variance in the estimates. The improvements in RMSEs are particularly notable with the bootstrap method where they drop by 40-65%. It also follows that bootstrap performance is not sensitive to assumptions regarding how smooth the feasible time series of PE pricing errors are: columns 2 and 3 are virtually identical.

Meanwhile, Panels B and C of Table AI show that (i) the magnitude of the compounding bias increases as the sample shrinks; (ii) all else equal, the bias is stronger when the pricing errors are not centered at zero in expectation; but (iii) the proposed bias correction methodologies continue to work. Finally, the RMSE-superior *Bootstrapped NPV*s appear less robust to misspecification of the drift in the SDF (i.e. Moving Average of the true innovations) than *excess NPV*s. This is intuitive since *excess NPV* differences out the “purely SDF-related” error. Nonetheless, this advantage is reduced when PE returns exhibit abnormal performance relative to the benchmark (Panel C).

Table AII illustrates PROPOSITION 2 by contrasting parameter identification via time series

[A-2] We find similar effects in unreported analysis of different specifications between MA(1) and MA(3).

[A-3] In untabulated analysis, we find the magnitude of compounding bias increasing to 7-13 cents at a single vintage level, depending on vintage size and SDF type.

GMM (i.e., θ_B per equation 12) in columns (1) with that using the pseudo fund cash flows (i.e., θ_{CB} per equation 18). The latter also requires taking a stand on the mapping function between the benchmark returns and pseudo fund cash flows ($\tilde{\delta}_{it}$ per equation 19). In column (2), we assume the true mapping function (i.e. fund distribution rule) is known to econometrician, hence, $\tilde{\delta}_{it} = \delta_{it}$ for all i and t. Nonetheless, Panel A suggests that asymptotically—unlike in column (1)—the SDF slope parameter per column (2) is not centered at the true value even if the risk-factor underlying the SDF is a known tradeable portfolio. Moreover, as column (4) indicates, the approximate mapping function proposed by Korteweg and Nagel (2016) (henceforth, K-N $\tilde{\delta}_{it}$[A-4]) notably outperforms the true one in terms of both central tendency and the variance of the bias. Equation (A.2) reveals the reason for this surprising result—even though using true mapping function better matches the cash flow durations of pseudo funds with those of PE (untabulated), K-N $\tilde{\delta}_{it}$ reduces the correlation between the distribution size and the period t pricing error, thus, mitigating the $cov\big(z'_{t-1}e^b_t(\theta_{CB}), \tilde{\delta}_t\big)$-term introduced by the instrument that the cash flow based identification of θ implies. To highlight this effect further, consider column (3), in which we set $\tilde{\delta}_t$ to equal the per period benchmark return whenever the distribution is non-zero—the upward bias increases, especially when the fund sample is shorter and less balanced as panels B and C of Table AII demonstrate. Notably, the asymptotic bias (Panel A) in the slope estimate switches to being negative and large with the autoregressive SDFs. This is due to the $cov\big(z'_{t-1}\tilde{\delta}_t \cdot e^b_t(\theta_{CB}), cM^-_{t-1}(\theta_{CB})\big)$-term in equation (A.2). Interestingly, autocorrelation in the SDF appears to mitigate the $\tilde{\delta}_t$-related bias in finite samples (Panels B and C). Still RMSEs under the θ_{CB}-approaches are at least twice as high with realistic samples than under the θ_B-approach. In other words, the GMM instrument implied by the pseudo fund cash flows is inferior relative to the optimal instrument.[A-5]

Table AIII illustrates PROPOSITION 3 by comparing *excess NPV* estimates under θ_B with those under θ_{CB} for a *Realistic* sample across different mapping functions. The key pattern that emerges in each panel (i.e., regardless of the mapping function) is that the error on ΔNPV^{rf} is always smaller under θ_{CB} while ΔNPV against the risky benchmark tends to be smaller (and better centered) under θ_B with exception of RMSEs with autoregressive SDFs. However, Figure AI provides insights into why focusing on 'RMSE'-alone can be misleading in those cases. From Panel A, the estimates from the θ_{CB}-approach are much more likely to result in very large estimates of the intercept as GMM struggles to price the compounding bias in pseudo-fund NPVs towards zero. Consequently, all cash flows tend to have present values biased towards zero, as do the differences thereof plotted in Panel B. This panel shows that *excess NPVs* tend to be small for those implausibly large intercept values. While this trend suppresses RMSEs, the power of θ_{CB}-based NPV estimates suffers as we explore in the Internet Appendix.

Comparing across panels of Table AIII, we note that $\Delta NPV(\theta_B)$ is not particularly sensitive to misspecification in the mapping function δ_{it}—K-N $\tilde{\delta}_{it}$ returns similar estimation errors to the true mapping. We propose a slight modification to K-N $\tilde{\delta}_{it}$ which amounts to dropping the first component (i.e., "the return accumulated since the last cash flow date") per the description in footnote A-4 and results in equation (27). Results based on this $\tilde{\delta}_{it}$ are reported in Panel C. In unreported analysis, we find that this mapping function exhibits better robustness under more general DGPs, larger pricing errors on benchmark assets, and for *bootstrapped-NPVs* (Table AI).

[A-4] p.1446: "If fund i makes a payout at $t + h(j)$, then we assume that the benchmark funds also make a payout equal to the sum of two components. The first component is equal to the return accumulated since the last cash flow date, $t + h(j-1)$. The second component pays out a fraction $\pi(j)$ of the capital that was in the benchmark fund after the last cash flow at $t + h(j-1)$ occurred. The payout ratio is determined by $\pi(j) = min\big((h(j) - p)/(10 - p)\big)$, where p is the time ... of the most recent payout prior to $t + h(j)$."

[A-5] The DGP considered in this appendix implies that the optimal instrument is a constant.

Table AI

Simulation Evidence—NPV compounding bias

This tables illustrates PROPOSITION 1 by reporting summary statistics for the NPV estimation error across 5,000 independent simulations. In each simulation, the error is defined as a difference in cents per dollar of capital committed between the estimate and the true life-time fund NPV (0 in panels A and B, 19 cents in panel C). Each panel reports median, mean and mean squared root error for three SDF types 'Tradeable','Non-Treadeable' (so that benchmarks do not contain SDF itself), and 'MA(2) Meas.Error' (so that SDF is observed with an error that exhibits persistence). In each case, SDF parameters are estimated consistently (equation 12, θ_B). Columns (1) report NPV estimates based on (equation 16), and hence not adjusted for *compounding error*, columns (2) and (3) perform feasible bootstrap correction (section V.B.2) under different assumption about time-series properties of the observed PE returns. Columns (4) report excess NPV estimates (section V.B.1) as an alternative correction method for the *compounding error* and general misspecification of the SDF. The seed values are fixed across and within panels. Appendix.A2 describes the simulation setup and discusses results.

Panel A. True NPV is 0, Large sample: 50 balanced vintage years

		Raw	Feasible Bootstrap		Excess
		NPV	$e_t(\theta)$ MA(1)	$e_t(\theta)$ MA(4)	NPV
		(1)	(2)	(3)	(4)
Tradeable SDF	Median Error	-2.74	-0.27	-0.20	-0.79
	Mean Error	-1.68	0.47	0.50	0.09
	RMSE	14.69	8.70	8.63	10.46
Non-Tradeable SDF	Median Error	-2.81	0.01	-0.00	-0.90
	Mean Error	-1.93	1.25	1.29	0.04
	RMSE	18.76	10.91	10.81	14.75
MA(2) Meas.Error SDF	Median Error	5.02	2.57	2.66	0.74
	Mean Error	10.67	4.60	4.64	0.90
	RMSE	42.25	14.63	14.58	24.20

Panel B. True NPV is 0, Realistic sample: 30 vintages, sparse as venture

		Raw	Feasible Bootstrap		Excess
		NPV	$e_t(\theta)$ MA(1)	$e_t(\theta)$ MA(4)	NPV
		(1)	(2)	(3)	(4)
Tradeable SDF	Median Error	-4.11	-0.77	-0.74	-0.64
	Mean Error	-2.36	0.64	0.67	0.40
	RMSE	21.38	10.43	10.37	13.33
Non-Tradeable SDF	Median Error	-3.37	0.12	0.11	-0.62
	Mean Error	-2.47	1.90	1.93	0.97
	RMSE	24.21	12.67	12.56	17.67
MA(2) Meas.Error SDF	Median Error	2.20	2.61	2.67	0.68
	Mean Error	10.05	5.65	5.68	2.36
	RMSE	68.60	24.99	24.49	30.76

Panel C. True NPV is 19 cents, Realistic sample: 30 vintages, sparse as venture

		Raw	Feasible Bootstrap		Excess
		NPV	$e_t(\theta)$ MA(1)	$e_t(\theta)$ MA(4)	NPV
		(1)	(2)	(3)	(4)
Tradeable SDF	Median Error	-4.88	-2.55	-2.67	-2.13
	Mean Error	-2.62	-0.80	-0.92	0.13
	RMSE	26.72	14.59	14.51	17.56
Non-Tradeable SDF	Median Error	-3.95	-2.47	-2.51	-1.57
	Mean Error	-2.74	0.50	0.39	0.70
	RMSE	30.24	18.34	18.21	22.31
MA(2) Meas.Error SDF	Median Error	2.72	0.34	0.29	0.90
	Mean Error	12.85	5.11	4.99	5.16
	RMSE	88.38	38.35	37.70	46.49

Table AII

Simulation Evidence—SDF parameter identification: θ_B versus θ_{CB}

This tables illustrates PROPOSITION 2 by reporting summary statistics for the SDF slope parameter estimation error across 5,000 independent simulations. In each simulation, the error is defined as the difference between estimated γ and the true value (equal to 2.0) multiplied by 100. Each panel reports median, mean and mean squared root error for three SDF types ('Tradeable','Non-Treadeable','Autoregressive') and four estimators—one is based on time-series GMM of benchmark returns (equation 12, θ_B), and three are based on cash flows (equation 18, θ_{CB}) constructed from benchmark returns. θ_{CB} estimates depend on the mapping function (equation 16) with (δ_{it}) representing the true percentage of capital distributed and $(\tilde{\delta}_{it})$ being the capital distributed assumed in the estimation. The seed values are fixed across and within panels. Appendix.A2 describes the simulation setup and discusses results.

Panel A. Very large sample: 500 balanced vintage years

		θ_B	θ_{CB}		
		$z_t = 1$	$\tilde{\delta}_{it} = \delta_{it} \propto R_{it}$	$\tilde{\delta}_{it} = R_{it} \mid \delta_{it} > 0$	K-N $\tilde{\delta}_{it}$
		(1)	(2)	(3)	(4)
Tradeable SDF	Median Error	-0.28	3.96	4.55	3.10
	Mean Error	0.28	6.80	8.15	4.20
	RMSE	30.34	40.75	42.82	36.34
Non-Tradeable SDF	Median Error	0.59	3.32	3.96	2.73
	Mean Error	0.48	7.49	8.98	5.01
	RMSE	36.29	48.36	51.01	43.77
Autoregressive SDF	Median Error	-0.22	-15.83	-16.50	-10.78
	Mean Error	0.30	-13.49	-13.76	-9.48
	RMSE	30.34	35.35	36.10	34.20

Panel B. Large sample: 50 balanced vintage years

		θ_B	θ_{CB}		
		$z_t = 1$	$\tilde{\delta}_{it} = \delta_{it} \propto R_{it}$	$\tilde{\delta}_{it} = R_{it} \mid \delta_{it} > 0$	K-N $\tilde{\delta}_{it}$
		(1)	(2)	(3)	(4)
Tradeable SDF	Median Error	1.62	47.97	59.51	28.77
	Mean Error	1.60	91.93	113.46	48.10
	RMSE	97.56	237.10	273.70	161.91
Non-Tradeable SDF	Median Error	3.27	45.91	53.72	31.57
	Mean Error	3.81	106.37	134.15	65.99
	RMSE	117.88	306.44	368.69	227.89
Autoregressive SDF	Median Error	1.80	20.67	26.80	20.09
	Mean Error	1.74	58.68	69.21	41.38
	RMSE	97.69	192.69	209.55	154.04

Panel C. Realistic sample: 30 vintage years, sparse as venture

		θ_B	θ_{CB}		
		$z_t = 1$	$\tilde{\delta}_{it} = \delta_{it} \propto R_{it}$	$\tilde{\delta}_{it} = R_{it} \mid \delta_{it} > 0$	K-N $\tilde{\delta}_{it}$
		(1)	(2)	(3)	(4)
Tradeable SDF	Median Error	0.63	88.87	103.34	55.03
	Mean Error	6.10	147.84	187.43	75.17
	RMSE	107.48	338.80	419.71	214.31
Non-Tradeable SDF	Median Error	3.68	78.82	90.06	56.45
	Mean Error	5.68	150.67	202.80	92.84
	RMSE	129.08	821.74	629.40	295.12
Autoregressive SDF	Median Error	1.08	54.64	60.72	47.04
	Mean Error	6.26	106.19	125.68	69.94
	RMSE	107.72	278.58	319.48	204.86

Table AIII

Simulation Evidence—NPV inferences: θ_B versus θ_{CB}

This tables illustrates PROPOSITION 3 by reporting summary statistics for *excess NPV* estimation error across 5,000 independent simulations. In each simulation, the error is defined as a difference in cents per dollar of capital committed between the estimate and the true life-time fund NPV (set to zero across all panels). Each panel reports median, mean and mean squared root error for three SDF types 'Tradeable','Non-Treadeable' (so that benchmarks do not contain SDF itself), and 'Autoregressive' (so that true SDF exibits time series persistence). In columns (1) and (2), SDF parameters are estimated using overidentified time series GMM (equation 12, θ_B). In columns (3) and (4), the SDF parameters are estimated using cash flow based restriction instead (equation 18, θ_{CB}). Columns (1) and (3) [(2) and (4)] report *excess NPV* relatively to the risky [risk free] benchmark. The seed values are fixed across and within panels. In panel A, the mapping function (equation 16) assumed for estimation is set to match the true proportion of capital distributed, while in panel B [C] it is set to be [approximately] as in Korteweg and Nagel (2016). Appendix.A2 describes the simulation setup and discusses results.

Panel A. True δ_{it}

		θ_B		θ_{CB}	
		ΔNPV^b	ΔNPV^{rf}	ΔNPV^b	ΔNPV^{rf}
		(1)	(2)	(3)	(4)
Tradeable SDF	Median Error	-1.00	4.88	-0.91	-0.72
	Mean Error	-0.01	3.54	0.09	0.32
	RMSE	12.57	18.19	12.59	13.79
Non-Tradeable SDF	Median Error	-0.06	4.53	0.18	1.61
	Mean Error	0.52	4.01	1.47	3.26
	RMSE	17.55	20.18	17.02	19.72
Autoregressive SDF	Median Error	-1.10	3.69	-0.91	-0.69
	Mean Error	-0.05	1.67	0.09	0.36
	RMSE	14.71	22.24	12.65	13.85

Panel B. Korteweg-Nagel $\tilde{\delta}_{it}$

		θ_B		θ_{CB}	
		ΔNPV^b	ΔNPV^{rf}	ΔNPV^b	ΔNPV^{rf}
		(1)	(2)	(3)	(4)
Tradeable SDF	Median Error	-0.91	4.32	1.29	1.84
	Mean Error	0.34	3.19	2.65	3.24
	RMSE	12.79	17.79	13.90	14.77
Non-Tradeable SDF	Median Error	-0.57	4.02	1.51	3.45
	Mean Error	0.64	3.64	2.75	4.94
	RMSE	17.08	19.87	17.45	18.40
Autoregressive SDF	Median Error	-0.92	3.91	1.09	1.91
	Mean Error	0.47	2.04	2.54	3.38
	RMSE	15.03	22.27	14.21	15.15

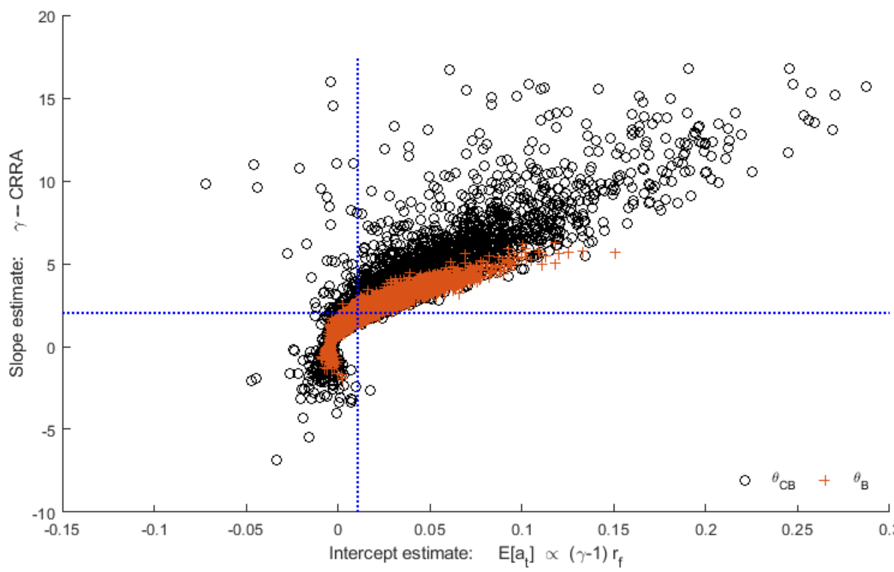
Panel C. Simplified Korteweg-Nagel $\tilde{\delta}_{it}$

		θ_B		θ_{CB}	
		ΔNPV^b	ΔNPV^{rf}	ΔNPV^b	ΔNPV^{rf}
		(1)	(2)	(3)	(4)
Tradeable SDF	Median Error	-0.84	5.50	-1.35	-1.01
	Mean Error	-0.08	3.83	1.25	1.61
	RMSE	12.61	18.88	20.25	21.03
Non-Tradeable SDF	Median Error	0.31	5.43	1.08	2.87
	Mean Error	0.45	4.33	7.77	9.95
	RMSE	17.93	20.81	36.30	37.67
Autoregressive SDF	Median Error	-1.08	4.04	-1.61	-1.21
	Mean Error	-0.37	1.57	0.62	0.97
	RMSE	14.82	23.29	18.13	19.05

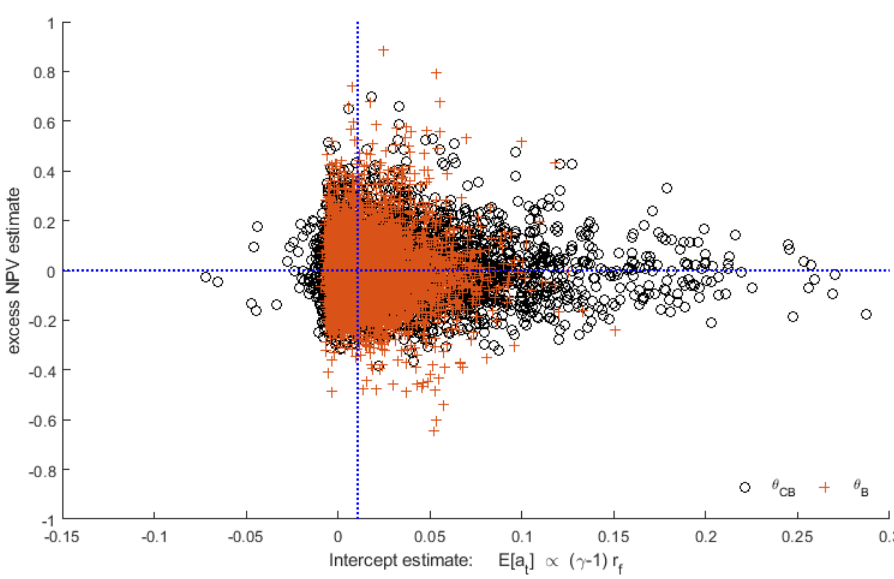
Figure AI. Simulation Evidence—the low RMSE puzzle under θ_{CB}

This figure plots estimates across 5,000 simulations corresponding to those reported Table AIII (Panel A, line 'Autoregressive SDF'). In these simulations, θ_{CB} results in a lower RMSE of ΔNPV^b relative to that under θ_B despite the latter is a more efficient and consistent estimates of SDF parameters. Both panels of the figure report intercept estimates on x-axis while panel A [B] reports the estimates of γ [excess NPV]. The blue dotted lines are the true values of each parameter. Appendix.A2 describes the simulation setup and discusses results.

Panel A: SDF Parameter estimates



Panel B: SDF Parameters versus NPV estimates



Appendix A3. Handling unresolved funds

As shown in section II.A, a significant fraction of PE fund assets remain unresolved as of the forth quarter of 2018, even after 10 years in operation. The usual approach in the literature (and amongst practitioners) has been to consider the last most NAV reported as the terminal distribution (see, e.g., Harris, Jenkinson, and Kaplan, 2014, for discussion). Formally, this implies to the following assumption, where $L(i)$ is the quarter of the last-most available NAV report for fund i:

$$\mathbb{E}\Big[\sum_{\tau=0}^{T} C_{i,\tau} \cdot M_{s(i):\tau}\Big] = \mathbb{E}\Big[\sum_{\tau=0}^{L(i)} C_{i,\tau} \cdot M_{s(i):\tau}\Big] + \mathbb{E}\big[NAV_{L(i)} \cdot M_{s(i):L(i)}\big] \quad , \tag{A.8}$$

which, in turn, implies that:

$$\mathbb{E}\Big[\sum_{\tau=L(i)+1}^{T} \delta_{i\tau} R_{i\tau} M_{L(i):\tau} \cdot \prod_{t=L(i)+1}^{\tau} \big(R_t(1-\delta_{it})\big)\Big] \approx 1 \tag{Res NAV Asmp1}$$

$$\mathbb{E}\big[NAV_{L(i)}\big] \perp\!\!\!\perp \mathbb{E}\Big[\sum_{\tau=L(i)+1}^{T} \delta_{i\tau} R_{i\tau} M_{L(i):\tau} \cdot \prod_{t=L(i)+1}^{\tau} \big(R_t(1-\delta_{it})\big)\Big] \tag{Res NAV Asmp2}$$

as it follows by plugging the cash flow-to-return mapping equation (16) into $\mathbb{E}\big[\sum_{\tau=L(i)}^{T} C_{i,\tau} \cdot M_{L(i):\tau}\big]$ and replacing $-C_{i0}$ and $s(i)$ with, respectively, $NAV_{L(i)}$ and $L(i)$.

The first assumptions seems plausible for SDFs that are inversely proportional to the traded benchmark return (i.e. $M_{t:t+\tau} \propto \exp\{-\sum_{j=t+1}^{t+\tau} r_j^b\}$), such as implied with K-S PME, since the variance of idiosyncratic return R_{it}/R_{bt} is likely to be moderate for mature funds. However it appears much less plausible with CBAPM SDFs which, as evident from panel B of table II, exhibit large variance of the pricing errors for those public benchmarks.

Meanwhile, the fact that $L(i)$ happens to be 4Q'18 for almost all funds, makes the second assumption especially particularly vulnerable since it has to also hold conditionally and the period, as shown in figure 2, is characterized by relatively low marginal utility levels, especially, as per the habit model. If PE fund managers add value by timing the distributions than the second assumption fails. Under this hypothesis, the reason venture funds are so much unresolved is precisely that investors exhibit very low utility for payouts in year 2018.

Accordingly, we take two simple steps to mitigate the likely failure in both assumptions for our context. First, we conduct analysis on a subsample of mostly resolved funds in addition to the full sample. Second, for the funds that have inflation adjusted NAVs of at least 25% of real distributions of fund size as of 4Q, we assume the life of pseudo funds to be the maximum of the respective actual fund life and 50 quarters (i.e., fund managers elected the optional life extension). This slows down the distribution pace of the pseudo funds so that there is less discrepancy in the scale of 4Q'18 NAVs between actual and pseudo funds than as depicted in figure 3, in which pseudo fund life has a 40 quarter limit (as in K-N).

Appendix A4. Inference

For our estimate of θ, we construct the standard errors as follows. Given the estimate of θ, θ^*, we calculate pricing errors for each test asset i at time t

$$e_{i,t}(\theta^*) = R_{i,t} M_t(\theta^*) - 1. \tag{A.9}$$

From this sample of pricing errors, we draw a new sample of pricing errors, $e_{i,t}^k$ using a block bootstrap procedure with block length 4. We then form a new series of returns

$$R_{i,t}^k = (1 + e_{i,t}^k)/M_t(\theta^*). \tag{A.10}$$

θ^k is then the GMM estimator which minimizes the sample estimate of the pricing errors centered at $g(\theta^*)$ and $\text{var}\big(\theta^*\big)$ is the sample estimate of the covariance matrix of the K bootstrap replications of θ^k.

We can also measure the performance of our choice of SDF in pricing public market returns using a J statistic when the dimension of g exceeds the dimension of θ. The J statistic is

$$J = g(\theta^*)' \, S^{-1} \, g(\theta^*) \ , \tag{A.11}$$

where S is the estimate of the covariance matrix of the moment conditions.

To perform inference on PE fund NPVs, we rely on a semiparametric bootstrap procedure that takes advantage of the fact that PE cash flows, and hence, NPV estimates do not affect estimates of the SDF parameters. Specifically, for each bootstrap sample, we draw

$$\theta^k \sim \mathcal{N}(\theta^*, \text{var}\big(\theta^*\big)). \tag{A.12}$$

For each θ^k, we first check its permissibility relative to three criteria: (1) γ, the coefficient of relative risk aversion, must be positive; (2) the quarterly average log SDF must be less than 1.03 in each of the periods 1980-1995, 1990-2005, and 2000-1015; and (3) the quarterly average Sharpe ratio must exceed 0.10 in each of the periods 1980-1995, 1990-2005, and 2000-1015. These three criteria help ensure that θ^k is economically plausible by requiring that investors be risk-averse, that the SDF discounts rather than grows future cash flows, and that θ^k does not imply a SDF that violates the Hansen-Jagannathan bounds, respectively. Figure AII provides a visual example of the draws of θ^k that would be excluded over the full sample 1980-2018. We can then obtain a bootstrap estimate of the PE fund NPVs, $NPV^*(\theta^k)$. Correspondingly, the bootstrap estimate of the standard error is the standard deviation of the K bootstrap replications $NPV^*(\theta^k)$. Our simulations suggest that, as a nonlinear function of θ, the NPV estimate tends to have leptokurtotic sampling distribution, and as such, we use the percentile method to identify the critical values for the hypothesis test. Specifically, we find the upper $\alpha/2$ and lower $\alpha/2$ quantiles of the bootstrap estimates and reject H_0 if the bootstrap-adjusted NPV estimate falls outside this region.

Figure AII. Permissible Draws of θ^k

Black cross denote permissible combinations, red diamonds (blue circles) fail the risk-free rate (Sharpe-ratio) criteria.