Private Investments in Diversified Portfolios[*]

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Abstract

We study the impact of including private investment funds into diversified portfolios that otherwise hold only public stocks and bonds. Our analysis uses a large sample of 3,380 U.S. buyout, venture capital, and real estate funds to simulate portfolios from 1987 to 2018 that substitute part of the public equity allocation with private funds. We find that investing in private funds almost always increases average portfolio returns and reliably increases Sharpe ratios for portfolios with buyout and real estate funds. The results are robust to accounting for various practical considerations: higher costs of managing private assets, restrictions on the number of annual commitments, making only primary commitments, and the aggressiveness of building up to the target allocation. Our analysis allows for a better understanding of a variety of important characteristics of portfolios investing in illiquid private funds, including the range of possible performance outcomes, the deviation from target allocations over time, and the trade-offs between different risk-return profiles. Finally, we document a very pronounced risk-return trade-off between portfolios with buyout, real-estate, and venture capital funds.

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1 Introduction

> "The University's discipline of sticking with a diversified portfolio has contributed to the Endowment's market-leading long-term record. For the thirty years ending June 30, 2019, Yale's portfolio generated an annualized return of 12.6% with a standard deviation of 6.8%. Over the same period, the undiversified institutional standard of 60% stocks and 40% bonds produced an annualized return of 8.7% with a standard deviation of 9.0%. Yale's diversified portfolio produced significantly higher returns with lower risk."
>
> (The Yale Endowment, 2019 annual report)

Since the 1980s, sophisticated investors have increasingly allocated a portion of their portfolios to "private" funds that typically require investors to commit capital for periods in excess of 10 years. These funds are usually structured as closed-end vehicles where capital is committed at the outset and then drawn down over a multi-year investment period at the discretion of the manager (i.e., general partner, or GP). For example, many large university endowments, early adapters of limited partner (or LP) investing in private markets, allocate a substantial portion of their assets to private equity (PE), venture capital (VC) and other "alternative investments" funds (Lerner, Schoar, and Wang (2008) and Binfare, Brown, Harris, and Lundblad (2019)). While trailblazers like, the Yale Endowment, are credited for pioneering the way, a broad swath of institutional investors such as pension funds, sovereign wealth funds, and large foundations have steadily moved into private fund investing (Andonov, Hochberg, and Rauh (2018)). Today, most institutional investors have some holdings in private funds. A recent survey by Preqin documents that 75% of institutional investors have at least one private fund investment and the average institutional investor has an allocation target of 11% to PE, 9% to RE and 6% to private debt funds (Preqin (2020)). Furthermore, the majority of these investors plan to further increase their holdings to private funds in the near future.

Two intertwined trends have fueled the growth of private market funds: First, public markets in many developed economies have become less popular sources of raising equity (Doidge, Karolyi, and Stulz (2017)). For example, the number of listed firms in the United States has roughly halved since peaking in the late 1990s. Second, the swelling demand for perceived higher returns available from private funds has resulted in an unabating supply of capital to new private equity funds as well as a proliferation of new types of funds. Today, there are substantial assets committed to private market funds that invest in real estate (RE), infrastructure, and private debt, among others. By the end of 2019, global investors had committed the equivalent of more than 7 trillion USD to private funds of all types.[1] The growth in private funds has also sparked an increasing interest in private markets access for retail investors. In June 2020, the US Department of Labor (DOL) issued guidance that permits the inclusion of private-equity funds in diversified retirement funds such as 401(k) accounts.[2] In August 2020, the Security and Exchange Commission (SEC) voted to expand the definition of "accredited" investors, thus lowering the regulatory hurdle to invest in private market funds.[3] Under the new definition, certain credentials beyond wealth are considered for eligibility of investment, so that a broader spectrum of individuals can invest into private market funds. With almost $20 trillion USD of assets in 401(k) and IRA accounts and the potential for trillions more from newly accredited investors, the pool of assets making allocations to private funds is likely to grow substantially in coming years.

Despite the past and future potential growth in private fund investing, our understanding of the role of private investments in diversified portfolios remains quite limited. Detailed studies of long-run returns are severely hampered by a lack of data on institutional portfolios. A few studies that have looked at endowments and public pension funds (see Binfare et al. (2019) and cites therein), yet these studies often have limited information on the allocations to private funds and rarely have detailed cash flow data for the full portfolios. In some sense, the relatively rapid shift to investing in private funds has come with very little understanding of the impact of private funds on the re-

[1]The Burgiss data base documents cumulative commitments of about 3 trillion USD in global buyout funds, about 1.6 trillion USD in venture capital and other equity funds, about 1.5 trillion in real estate and other real asset funds, and about 0.9 trillion USD in private debt funds. Current net asset value (NAV) of active funds is about 2 trillion USD.

[2]See "U.S. Labor Department Allows Private Equity in 401(k) Plans", https://www.wsj.com/articles/u-s-labor-department-allows-private-equity-in-401-k-plans-11591229396, June 3, 2020

[3]On December 18, 2019, the SEC proposed amendments to the definition of "accredited investor" in the Commission's rules and the definition of "qualified institutional buyer" in Rule 144A under the Securities Act of 1933 (see "SEC Proposes Giving More Investors Access to Private Markets", https://www.wsj.com/articles/sec-proposes-giving-more-investors-access-to-private-markets-11576691685, December 18, 2019). On August 26, 2020, the board passed the proposal on a 3-2 vote, extending the potential list of institutional and individual investors that have the knowledge and expertise to participate in private capital markets (see "SEC Gives More Investors Access to Private Equity, Hedge Funds", https://www.wsj.com/articles/sec-gives-more-investors-access-to-private-equity-hedge-funds-11598452858, August 26, 2020)

turns and risks of diversified portfolios. Furthermore, traditional portfolio management techniques, from the seminal mean-variance analysis of Harry Markowitz (1952) to more recent dynamic models (Campbell, Chan, and Viceira (2003); Jurek and Viceira (2011)), are limited in handling the complexity introduced by illiquid private market funds. Existing approaches make assumptions that typically do not hold in private fund investing. Most basic is the ability to transact in the asset (and thus observe market prices). By their nature, there exists a very limited secondary market for private funds with almost no price transparency even when transactions occur (Nadauld, Sensoy, Vorkink, and Weisbach (2019)). The closest analog to price discovery comes from quarterly estimates of fund net asset values provided by fund managers which are obtained with a substantial lag and known to be systematically smoothed and biased (Brown, Gredil, and Kaplan (2019); Brown, Ghysels, and Gredil (2020b)).

Investing in private market funds also comes with a hefty price tag as fees and incentive compensation are typically substantially greater than in portfolios of publicly traded assets (Phalippou, Rauch, and Umber (2018)). Portfolio management costs are higher as well. Private fund investors typically undertake substantial due diligence on private fund managers which is generally more difficult and time consuming than research on public fund managers that have easily observable track records. Investors must have dedicated staff for private fund selection or rely on costly external consultants (or both). Back-office operations for private funds also require additional resources because reporting is not standardized in the same manner as for public securities. Even with these headwinds, empirical research suggests that private funds have indeed provided superior returns relative to public benchmarks.[4] However, other research has questioned these results, especially over more recent years (Phalippou (2020)). Furthermore, the difficulty in assessing the level (and types) of risk of private funds has further complicated the calculus as to the benefits they may provide in diversified portfolios.[5]

[4] Kaplan and Schoar (2005) find that average private equity fund returns (net of fees) equal those of its public benchmarks; they also introduce the public market equivalent (PME) as a measure of performance, a now widely used metric among academics and practitioners. However, recent work uses more comprehensive data of higher quality. Harris, Jenkinson, and Kaplan (2014) use Burgiss data and show that buyout and VC funds outperform the S&P 500 by an average of more than 3% annually between 1987 and 2010. Brown, Harris, Jenkinson, Kaplan, and Robinson (2015) demonstrate that while there is heterogeneity in the data provided by different databases, obtained performance metrics are similar in magnitude which strengthens confidence in the conclusions derived from recent studies. Brown and Kaplan (2019) update results from previous work and find continued outperformance of PE funds through 2018.

[5] Quite a few researchers have examined the risk characteristics of PE and VC funds. For example

These issues raise the basic question of whether private funds add value in a diversified portfolio. In this paper we undertake a large simulation analysis to shed light on this question as well as related issues. Specifically, we examine how diversified portfolios with allocations to private funds would have performed historically relative to portfolios with only publicly-traded assets. Using detailed net performance cash flow data for 3,380 U.S.-focused private market funds, we simulate portfolios that randomly allocate a part of a diversified portfolio of U.S. public market assets to private funds of different types. We assume a feasible benchmark portfolio with a 60% allocation to the Vanguard Total (U.S.) Stock Market Index Fund and 40% to the Vanguard Total (U.S.) Bond Market Fund (re-balanced quarterly). We then assume that an investor randomly selects private funds to replace a third of the 60% equity allocation (i.e., the investor targets a 20% overall allocation to private funds). We then track the cash flows and estimated net asset value of the overall portfolio to asses total performance, allocations over time, etc. We do this with a careful eye to the challenges of estimating risk-adjusted performance in a portfolio that contains assets without observed market prices. Namely, we use the econometric model of Getmansky, Lo, and Makarov (2004) to unsmooth observed returns and obtain serially-uncorrelated time-series of portfolio returns.

We find that investors almost always benefit from diversifying their portfolios with private market exposure. Allocating to buyout (BO) and real estate (RE) funds leads to both better returns and lower risk historically. The benefit of investing in buyout and RE funds is economically large, and the outperformance is robust to a variety of real-world considerations. For VC funds, the overall benefits are less obvious as both portfolio returns and volatility increase. Consequently, benefits are likely to be investor-specific depending on risk preferences and investment policy constraints. In addition, we do not find robust diversification benefits of investing across all three fund strategies (i.e. buyout, VC, and RE) as portfolios with just buyout funds dominate more diversified portfolios over our sample period.

We also show how closely a simple primary-fund commitment strategy tracks an average target allocation. This lets us determine if an investor is likely to need to trade fund positions in the secondary market to stay within a target range. We document that

Korteweg and Nagel (2016) use a stochastic discount factor (SDF) valuation method to evaluate performance. For a detailed summary on the existing literature on risk of private market funds we refer to Korteweg (2019).

allocations to private funds can deviate substantially from target allocations in some cases. Most notably, the 20% target allocation to VC almost doubled during the dot-com bubble, only to plummet a few years later to less than a 10% allocation. Allocations to buyout and RE funds have much lower variation. As part of our analysis we develop a simple fund commitment model based on historical cash flows that significantly reduces (but does not eliminate) the deviations from target allocations. Finally, we show that in long-run results are not very sensitive to alternative strategies associated with the initial allocation build-up (such as quickening the pace of commitments in early years).

The paper proceeds as follows: In Section 2 we briefly discuss other related studies. Section 3 presents the data and our methodology. In Section 4, we define the commitment strategies to private market funds and discuss their implications on forming diversified portfolios. We present our results in Section 5 and conclude in Section 6.

2 Related Literature

To our knowledge, we are the first to provide a comprehensive assessment with detailed cash flows of the behavior of private funds in a diversified portfolio that includes public stocks and bonds. However, other research has addressed the challenge of portfolio choice with illiquid assets. Ang, Papanikolaou, and Westerfield (2014) show that risk-averse investors are willing to forgo on expected return by hedging against illiquidity risk stemming from the illiquid part of an diversified portfolio. Giommetti and Sørensen (2019) show that allocations to private equity substantially affect the optimal stock and bond allocations of a risk-averse investor. Using the model of Takahashi and Alexander (2002) to project future fund cash flows, the authors incorporate historical information to provide a base for forecasting NAVs and cash flows which impacts future commitment decisions. Our study is also related to Goetzmann, Gourier, and Phalippou (2020) who build a set of factors that capture variation in private market funds returns to estimate the nature of diversification benefits of private funds. The authors find evidence that private market funds are not entirely spanned by publicly available factors.

While these papers provide a theoretical framework for allocation strategies to private market funds, there is no comprehensive, empirical evaluation of historical portfolio performance. Our analysis takes a more direct approach to quantifying the performance benefits through an ex-post performance analysis of diversified portfolios using private market funds. We evaluate the feasibility of implementing such investment strategy by, for example, characterizing the performance and allocation differences that come from selecting different numbers of private funds per vintage year as well as risk-return trade-offs on varying commitments to different vintages to reduce tracking error around target allocations.

In public markets, selecting fund managers that reliably outperform the benchmark is very difficult. Performance persistence in private markets, in contrast, is well-documented. Harris, Jenkinson, Kaplan, and Stucke (2020) find significant persistence in performance of funds with the same GP for pre-2000 vintage funds but show that persistence of buyout fund performance has fallen considerably since then. Using a novel variance decomposition model, Korteweg and Sorensen (2017) find long-term return persistence. However, the authors conclude that investable persistence is low given the need of LPs need to observe an excessive number of past funds to identify PE firms with higher expected future returns with reasonable certainty. Braun, Jenkinson, and Stoff (2017) conclude that persistence of GPs has substantially declined as the private equity sector has matured and become more competitive. In contrast, skills of individual deal partners remains persistent and much of the decline in GP persistence is from talented deal partners leaving for other firms (including founding their own).

Buyout funds have a long track-record of outperforming the public markets. Harris et al. (2014) show that from 1984 to 2008, there are only four vintage years in which BO funds have underperformed the S&P 500 benchmark. The authors also show the outperformance is robust to using other benchmark indices such as NASDAQ index or the family of Russell Indices. Brown and Kaplan (2019) show that in contrast to popular notion that private equity returns have declined over time, the outperformance of buyout funds has increased again since the weak vintages just before the global financial crisis (GFC).[6]

The literature has also documented the return characteristics of other private fund strategies. For example, the cross-sectional distribution of VC returns is highly positively skewed (Robinson and Sensoy (2016)). In the time-series, return characteristics of VC

[6]The most recent studies provide performance characteristics for vintage years up to 2014. For more recent vintage years, it is difficult to obtain reliable measures of performance because most funds are still in their investment periods and few funds have distributed a significant portion of their capital.

funds have been highly cyclical: VC funds of vintages through the late 1990s provided strong performance but vintages in the early 2000s under-performed public markets Harris et al. (2014). More recently VC fund performance has improved again. However, the substantial cross-sectional skewness in VC fund performance means that top-performing funds consistently outperform public market benchmarks by a significant margin but the median fund underperforms. Outperformance of funds managed by top GPs is more reliably persistent even in more recent years (Harris et al. (2020)).

Fund performance data for private market funds investing in real estate is limited. The strategy began to have a large number of funds only in the early 2000s and historical returns of the strategy are greatly impacted by the 2008 GFC (Fisher and Hartzell (2016)). Nonetheless, Riddiough and Wiley (2019) document that that unlisted REITs underperform listed REITs and Pagliari (2020) shows that the fundamental risks associated with investing in RE funds vary substantially across fund type (e.g., commercial and residential) and strategy (e.g., core, value-add, and opportunistic).

Our analysis also relates to the literature on portfolio allocation with alternative assets. A natural starting point for an empirical assessment of private market funds in diversified portfolios are university endowments and foundations. Some evidence suggests that university endowments started to outperform public market benchmarks beginning in the early 2000s (Lerner et al. (2008)) which coincides with an increase of allocations to alternative assets in many portfolios. In particular, large endowments, which tend to have higher allocations to private markets, have achieved statistically higher risk-adjusted returns Binfare et al. (2019). Recent research examines endowments and foundations and finds conflicting evidence on outperformance (see Dahiya and Yermack (2020), Lo, Matveyev, and Zeume (2020)). Hochberg and Rauh (2013) give evidence that public state pension funds exhibit exhibit substantial home-state bias in private equity investments which in turns leads to lower portfolio performance.

Some studies provide evidence for diversification benefits of private market funds. Humphery-Jenner (2013) reports limited diversification benefits at the fund level but does not examine portfolio-level diversification. Ang et al. (2014) suggest that the cyclical but imperfect co-movement of PE, VC and RE fund returns can lead to diversification within such investment classes. The recent work of Goetzmann et al. (2020) indicates that private funds can expand the efficient frontier, due mainly to large buyout funds, according to their new classification of private market funds.

3 Data and Methodology

We develop a simulation model that captures important contractual features and practices observed in the market for private funds. The model enables us to, among other things, (1) provide a realistic evaluation of the performance of portfolios that allocate part of their assets to a variety of private funds, (2) estimate the historical range of outcomes based on reasonable (and varying) assumptions, and (3) examine the performance of different commitment strategies in hitting target allocations.

In this section, we first provide an overview of the institutional details of private fund investing. We next describe the data used in our analysis. We then outline the portfolio construction methodology including allocation and re-balancing strategies. Finally, we discuss methods for comparing portfolio performance on a risk-adjusted basis.

3.1 Economics of Investing in Private Funds

Private funds are typically investment vehicles structured as partnerships that raise money from limited partners to invest on their behalf. While there are many types of these funds that invest in both listed and unlisted securities, our analysis will focus on funds that primarily invest in unlisted assets such as private companies or real estate. The most common fund types are private equity buyout and venture capital funds, but the universe of private market funds also spans investments in broader areas such as natural resources, infrastructure, and private credit. Real estate private equity funds are also a large and fast growing segment of the private fund market.

A private fund is managed by an intermediary, the general partner. In the typical structure, the investment decisions (ranging from sourcing, executing, managing, and exiting investments) are delegated by the LPs to the GP. This delegation can give rise to well-known principal-agent conflicts; for example, frictions regarding the types and timing of investments. Investors contractually commit to a fund and can control the timing of the commitment; however, they do not control when the committed capital is called by the GP. Once called, the capital is deployed and locked up in the fund until distributed by the GP. The illiquid nature of private market funds thus makes this investment vehicle distinct from allocations to public market funds which often provide regular liquidity.[7]

When committing to private funds, investors have an obligation to provide funding within a short period after the capital is called. Therefore, it is standard practice for LPs to allocate part of their portfolio to fairly liquid assets to meet capital calls. This uncertainty is sometimes called "commitment risk." Equivalently, upon distribution of capital, a new investment opportunity with similar characteristics might not be available in private markets and hence the investor faces potential allocation uncertainty related to returned capital, which is often referred to as "reinvestment risk."

Further differences between private and public funds exist. Exposure to different risk factors can impact return characteristics which in turn result in potential diversification benefits for LPs. Also, reliable outperformance in public market investing is elusive. In private markets, however, some managers have proven-track records of consistently outperforming relevant benchmarks. Thus, private market fund managers that have an ability to invest in high performing assets have the viable opportunity to deliver above-market returns to investors. On the flip side, the same investors could potentially suffer from under-diversification – a typical private equity fund might invest in just a dozen or so portfolio companies over its lifetime. In the case of venture capital funds these might be very risky, young companies; or in the case of private equity, highly levered companies in a limited number of industries. An investment in a public equity fund, in contrast, would usually provide exposure to a larger set of less risky firms. As noted already, investments in private markets are highly illiquid so that in the case of a negative LP liquidity shock, the investor cannot sell shares in the fund. Although the emergence of a secondary market for private funds has evolved steadily over the last 20 years, LPs facing liquidity needs who are forced to sell fund stakes during times of market dislocation can typically only do so at a significant discount to NAV and can face reputational penalties with GPs as well.

As stated above, allocating to private market funds is inherently different from allocating to public market funds. For investments into public equity funds, the time between allocation and investment exposure (i.e. the time the asset appears on the books of the investor) is usually negligible. For private funds, the timing of investments is typically

[7]For example, in the U.S. most so-called "40-Act" funds provide daily liquidity. Hedge funds provide liquidity typically ranging from weekly to quarterly depending on the underlying asset liquidity though they often attempt to "lock-up" investor money for initial investment period and can gate redemptions under certain circumstances.

delegated to the GP and thus can occur any time during the years-long investment period. Sometimes not all committed capital is called, whereas occasionally more than the committed capital is called if additional investment opportunities arise (and the fund has "recycling" provisions). Private fund investments are also complicated by minimum/maximum commitment sizes, opportunities for co-investments, and complex fee structures. Following the investment phase, capital is returned to investors in the form of distributions; again, the decision to divest assets is almost always at the discretion of the GP during the normal life of the fund. Once all assets are sold, the fund is liquidated. However, it is not uncommon for funds to retain meaningful NAVs for longer than 15 years (i.e., "zombie" funds). Our analysis allows for these idiosyncrasies of private market investing and hence allows for a realistic ex-post evaluation of portfolio performances.

3.2 Data

We seek to understand the diversification and performance impact of private funds on diversified portfolios, net-of-fees. To this end, we start by constructing a benchmark portfolio that only invests in public equity and fixed-income funds. Our analysis is also restricted to the investable U.S. universe. The benchmark portfolio has allocations to the following two funds: the Vanguard Total Stock Market Index Fund (VTSMX) tracking the CRSP U.S. Total Market Index, and the Vanguard Total Bond Market Index Fund (VBMFX) tracking the Bloomberg Barclays U.S. Aggregate Bond Index.[8] Return data for these funds are publicly available and reported net-of-fees.

Historically, the limiting factor on evaluating ex-post performance of diversified portfolios is the availability of high-quality data on private funds. In this paper, we obtain fund-level data from Burgiss which covers a large sample of institutional-quality private funds. Burgiss sources its data directly from institutional LPs, and its data has been used in recent studies of fund performance (Harris et al. (2014); Brown and Kaplan (2019)). For a detailed discussion of the advantages and potential biases of this database, see Brown et al. (2015). We have access to the complete history of cash flows between LPs and the fund, as well as quarterly fund valuations. All data are net-of-fees and carried

[8]Because VBFMX was first offered to investors in April 1992 (after the beginning of our analysis), we extend back the fund return series to 1987 by using the Bloomberg Barclays U.S. Aggregate Bond Index minus a fixed management fee of 30 basis points annually. Although management fees for index mutual funds have declined significantly over the last decades, this level of fees is appropriate for this period and our results are insensitive to assuming reasonably higher or lower fees for these 5 years.

interest, and so represent the actual LP investment experience. Our sample consists of 3,380 U.S. buyout, venture capital, and real estate private equity funds with vintage years from 1987 to 2018, representing $2.2 trillion in total fund commitments. Table 1 provides on overview of our sample by vintage year and fund type.

[Insert Table 1 here]

Our sample of 3,380 private market funds is composed of 1,109 buyout funds (with $1.35 trillion in committed capital), 1,560 venture capital funds (with $409 billion in committed capital), and 711 real estate funds (with $431 billion in committed capital). There were relatively few funds available in the late 1980s and early 1990s, but investors could gain within-vintage diversification by the later-half of the 1990s. Focusing on the committed capital across strategies, we can show that fundraising activity is cyclical. Periods of high fund-raising, which is reflected in high committed capital, are followed by vintage years with less available capital. We observe cyclical peaks in the number of funds: at the turn of the millennium, VC funds experienced a heyday of new 119 funds in 2000. The number of buyout funds peaked just before the GFC in 2007 with over 70 funds. 2007 also experienced a peak in the number of RE funds. Although none of the strategies have returned to those annual levels of number of new funds, the total capital raised in 2018 is near the record 2007 level. Altogether private fund strategies have experienced substantial growth since the 1980s leading to increased competition among GPs for investment capital and deal flow as well as an increased stock of committed but un-called capital (or so-called "dry-powder") across the private fund industry.

Table 2 shows Kaplan-Schoar public market equivalents (PMEs), pooled by fund type for the funds in our sample.[9] PMEs are calculated using the S&P 500 as the benchmark index. The average PME is greater than one for all three strategies indicating that the typical fund for all three strategies outperformed similarly timed investments in the S&P 500. For all three fund types, the average fund PME exceeds the median fund PME, indicating positively skewed distribution of fund performances. Consistent with other studies, VC fund average performance is better than the market (PME of 1.18) while the median VC fund underperform public markets (PME of 0.88). VC funds also show the largest variation in PMEs. For example, the 5th to 95th percentile range of PMEs for

[9]See Kaplan and Schoar (2005) for a discussion of PMEs.

VC funds is 0.23 to 2.84 as compared to 0.48 to 1.95 for buyout funds. The distribution of RE fund PMEs is similar to the distribution for buyout funds: mean (median) PME is 1.06 (1.04) while the 5th to 95th percentile range is 0.39 to 1.79.

[Insert Table 2 here]

Given the results from Table 1 and 2, we expect our portfolios with private funds to have returns and time-series allocations that vary across fund type. For example, allocations to VC will be more volatile than allocations to buyouts since the significant dispersion in performance is persistent for a given fund over its life and cannot be easily rebalanced away. However, the precise performance of the diversified portfolio including any diversification benefits is harder to predict. If much of the performance variation in VC is idiosyncratic relative to the public portfolio, then overall portfolio variation could be less affected. This is precisely the type of question our analysis seeks to answer.

3.3 Portfolio Management

For our simulation analysis, we construct portfolios that allow for allocation to three distinct types of investments: a U.S. public equity fund, U.S. private market funds and a U.S. fixed-income fund. We define an all-public benchmark portfolio with a 60% target allocation to public equities (VTSMX) and 40% target allocation to public fixed income (VBFMX). At the end of each quarter, the investor re-balances her portfolio to maintain this 60-40 split: if she is over(under-)allocated to the fixed-income fund, she sells (buys) shares of the fixed-income fund and buys (sells) shares of the public equity fund. In our main analysis, we assume that an investor allocates a third of the equity allocation, or 20% of the total portfolio, to private fund assets. Thus the target allocation in the portfolio with private assets is a 40% target allocation to public equities (VTSMX), a 20% target allocation to private funds, and a 40% target allocation to public fixed income (VBFMX). When we conduct our analysis with private funds, only claims in public funds can be traded in rebalancing due to the assumed illiquidity of private funds. Consequently at the end of each quarter the portfolio is rebalenced to 40% public fixed income, and a mix of public equity and private funds determined by the value of the private funds represents the other 60% of the portfolio.

We begin our analysis in 1987 (i.e. the first year with a sufficient number of private market funds in the Burgiss data). The investor makes her first commitment to private funds with vintage year 1987 and then makes annual commitments at the beginning of each subsequent year in an attempt to reach, and stay close to, the 20% target allocation for private funds. In other words, the investor seeks to get to a steady-state"40-40-20" target allocation where 20% of the total portfolio is invested in private funds. The choice of 20% is ad-hoc but consistent with average allocations for some large institutional investors such as U.S. public pension funds.

In practice it is not possible to maintain exactly a 20% allocation to private funds because of the uncertainty in future capital calls, distributions, and net asset values of the private funds. Initially, most of the deviation from the target allocation comes from the need to get invested at all – it takes several years to build up the invested portfolio of private funds. The staggered commitments and idiosyncratic cash flow patterns of funds (e.g., the variable time lag between commitment and capital call) mechanically lead to two phases of the portfolios we examine: an initial "ramp-up" phase followed by a "steady-state" phase where existing funds are, on average, likely to distribute capital that are of similar magnitude to the capital calls of new fund commitments. In the ramp-up phase, the investor's goal is to increase the allocation until it reaches the target, but as we discuss below, the investor may also want to maintain diversification across vintages. For most of our analysis, we assume the ramp-up phase to be 1987-1996 and then do our analysis on performance during a mature phase of 1997-2018. While the typical investment period for a private fund is 5 years, it takes longer than that for the portfolio to get close to a target allocation because not all the committed capital is deployed in the first year. Intuitively, a steady-state portfolio diversified across vintage years should be committing about a fifth of its target allocation each year.[10] In our baseline analysis, we assume the investor makes constant (percentage of total portfolio) commitments each year.[11]

We further assume that the investor has knowledge of and access to the entire universe of private market funds in a given vintage year and commits to a random subset at the

[10]As we show subsequently, in practice it is more than one-fifth each year because funds do not call all committed capital, there are frequently distributions that occur before the end of the investment period, and the overall portfolio value tends to grow over time.

[11]For example, the target commitment level might be 4% of portfolio assets per year for a 20% target allocation in case of no over-commitment.

beginning of the year. Given the typical fundraising timeline for private funds, it is realistic to assume that investors often know which GPs will be raising new funds in the coming year.[12] In practice, not all investors have equal access to funds. For example, some top-performing VC funds are effectively closed to investors who have not invested in previous funds and we investigate the effects of these limitations in our robustness analysis.[13] The number of funds the investor chooses to commit to in each vintage year defines the level of overall private fund diversification. Results presented in this paper mostly focus on commitments to 10 funds per vintage (or the maximum number of funds in our data set for the vintage years in case there are fewer than 10 funds). However, in robustness analysis we also examine the effects of committing to fewer or more funds per vintage year.

In practice, it is also true that private funds have minimum and/or maximum commitment thresholds, which tend to correlate with the total fund size. Given the within-vintage heterogeneity in private fund sizes, it is thus unlikely that an investor can follow an equal-weight commitment strategy (i.e. commit the same amount to each fund). We therefore assume that the investor value-weights her commitments across funds, with no restrictions on actual fund commitment sizes.[14]

Portfolio allocation weights are market values for the public assets and reported NAVs of the private funds [15]. For simplicity, we assume all cash flows happen at the end of the quarter (i.e. capital calls and distributions). Therefore, at the end of each quarter, the investor will liquidate (purchase) shares in the U.S. public equity funds (in addition to the re-balancing to maintain 40% fixed-income allocation described above) as needed to respond to the cash flows and NAV marks of the private funds. For example, if capital calls exceed distributions, the investor is forced to sell shares of the public equity fund to meet his contractual obligation to the GP. The effects of the re-balancing are twofold: first, we ensure comparability of the diversified portfolio to the benchmark portfolio and

[12]In addition, we measure vintage year using the date of the first capital call which typically occurs after the first fund closing date and well after the first steps in raising a new fund.

[13]Furthermore, personal connections and networks can matter for allocations to private equity and venture capital funds (Binfare et al. (2019)).

[14]Large institutional investors usually have investment policies that limit annual commitment size to any one single fund. Because investors in our analysis commit to funds every year, they will always have a well-diversified portfolio of funds in the steady-state period where we conduct our performance analysis.

[15]In practice, there is a lag between the end of the quarter and the time when funds report NAVs to LPs. Results are essentially unchanged if we use lagged NAVs.

second, we ensure that most of the variation in risk-return characteristics is a result of the asset substitution from public to private market funds. In our analysis, we normalize the starting portfolio value to $100 and then simulate the portfolio management process 1,000 times. For each simulation, we record the annual commitments, quarterly cash flows and quarterly portfolio values which in turn allows us to calculate quarterly returns for each portfolio. Tabulated results Section 5 are based on the distribution of time-series portfolio returns.

3.4 Measuring Portfolio Performance

The dual goals of this paper are to: first, quantify the risk and return characteristics of portfolios with private market funds, and second, investigate the cross-sectional and time-series variation in diversified portfolio allocations. While investors prefer portfolios that generate higher risk-adjusted returns over those with lower risk-adjusted returns, estimating risk and returns with periodic portfolio returns is complicated by a lack of market prices for private funds. Instead, most investors rely on NAVs of private market funds that are directly reported on a quarterly basis by GPs. The appraisal-based valuation is subject to substantial reporting biases by GPs (Barber and Yasuda (2017)). And critical for our analysis is the well-documented fact that private fund NAVs are greatly smoothed relative to public market valuations of similar assets. Hence, estimates of private equity fund volatility based on cash flows and NAV changes are much lower than for comparable publicly traded assets. This, in turn, leads to smoothed portfolio returns and imposes an upward bias on reported Sharpe ratios (our preferred measure of risk-adjusted returns) for portfolios with private fund allocations.[16]

The distortion of fundamental returns through appraisal-based NAVs introduces auto-correlation in the time-series of portfolio returns. We utilize the method of Getmansky et al. (2004) to obtain estimates of true, economic returns that are not auto-correlated. In their model, the observed return at time t is a weighted average of the true returns

[16] GPs have additional incentives to "game" NAVs. For instance, strategic reporting of NAVs is used in attempts to demonstrate good performance at times of fundraising for subsequent funds (Brown and Kaplan (2019)).

over the most recent $k{+}1$ periods:

$$R_t^{obs} = \sum_{i=0}^{k} \theta_i R_{t-i}^{true} \tag{1}$$

where the loading θ_i represents how much information of fundamental return R_{t-i}^{true} is put into the observed return R_t^{obs}. The additional restriction that the weights sum to one, i.e. $\sum_{i=0}^{k} \theta_i = 1$, implies that all information driving portfolio performance in period t is fully reflected in the observed return, although it can take up to $k{+}1$ periods. Hence, $\theta_i = 1$ represents the case with no smoothing. We assume no specific smoothing profile but estimate the number of MA-lags using the AIC criterion. We allow for a maximum lag of three quarters (i.e. smoothing only occurs within the last year) which is sufficient for generating returns without significant auto-correlation.

In the results presented in Section 5, we refer to the unsmoothed performance measures as those obtained from the Getmansky et al. (2004) method as "adjusted". Overall realized returns are not altered by the unsmoothing process. However, adjusted volatilities are strictly greater than the unadjusted volatilites if $\theta_0 < 1$. The same argument holds for our measure of risk-adjusted returns, the Sharpe ratios. Using this indirect procedure to obtain unsmoothed portfolio return distributions allows us to directly compare the risk and return characteristics of diversified and benchmark portfolios. Applying the standard Markowitz diversification criteria, our analysis allows us to speak to potential diversification benefits of private funds. Specifically, there are diversification benefits from private funds to a risk-averse investor if either (or both):

1. A positive allocation to private funds increases the return of a diversified portfolio while the adjusted standard deviation of returns is at most that of the benchmark portfolio
2. A positive allocation to private funds decreases the adjusted standard deviation of returns of a diversified portfolio while the return is at least that of the benchmark portfolio.

However, previous research indicates that investors also care about the higher statistical moments of return distributions (Scott and Horvath (1980); Harvey, Liechty, Liechty, and Peter (2010)). For example, investors like positive performance skewness since this implies higher probability of out-sized positive returns.[17] Investors dislike return kurtosis and also care more about downside return risk than overall risk. In this analysis, we do not make specific assumptions about the investor's preferences for higher return distribution moments but instead report the several relevant statistics: return standard deviation, semi-deviation, skewness, and kurtosis as well as the Sharpe ratio. We typically report the means of these statistics across all simulated portfolios.

We assume an uninformed investor, i.e., an investor who does not have fund selection skill and instead randomly commits to a subset of available funds each vintage year.[18] Consequently, our simulation analysis allows us to determine, for example, how likely it would have been for an unskilled private fund investor to outperform the all-public benchmark portfolio. To characterize this unskilled performance, we report the percentages of simulations in which the diversified portfolios have (1) a return at least as great, (2) an adjusted volatility at least as low, and (3) an adjusted Sharpe ratio at least as great as the benchmark portfolio. We conduct 1,000 simulations for each diversified portfolio. Assessing these percentages allows us to make inference about the statistical significance of improving on the benchmark portfolio.

Most institutional portfolios have investment policy statements that specify allowed ranges for each asset type. Violations of allocations ranges are typically viewed as undesirable and can even lead to forced rebalancing of illiquid assets.[19] For instance, a pension plan might have an investment policy that allows for an allocation to buyout funds of 10% to 20% of overall portfolio value, but if the allocation exceeds 20% the plan would be forced to sell some fund assets in the secondary market to get the portfolio back into compliance. To characterize deviations from allocation targets, we also measure the mean and standard deviation of the difference between actual and target allocations of the private fund portfolio.

[17] Similarly, investors dislike negative skewness which would be an indication of asymmetric downside risk.

[18] Therefore, the results can be interpreted as a lower bound of benefits since the portfolio of an investor with superior selection ability could perform better (assuming access to all funds).

[19] In some cases this is a matter of law, e.g., some public pension plans have asset allocations specified by state legislation.

4 Commitment Strategies

We examine two general commitment strategies: a simple, fixed commitment strategy that commits the same percentage of total portfolio value to private funds each year; and a dynamic commitment strategy that conditions commitments based on current private fund NAVs and expected fund cash flows. For both, we must make several methodological assumptions. First, we assume (in our base case) that the investor can choose among all available funds in a given vintage. Second, commitments to funds for each vintage year are made at the beginning of that year. Third, all cash flows to funds (contributions) and out of funds (distributions) accrue at the end of the quarter. With the end-of-quarter rebalancing in the public funds, in effect, contributions are financed out of the public equity account while distributions are funneled back into the public equity account. This methodology has three distinct features that provide a fairly clean identification of the effects of private market funds: substitution effect of public and private market equity investments, interpretation of total return measures and minimization of complications arising from using a separate cash account for fund contributions and distributions (e.g., liquidity shortfall where a dedicated cash account may be insufficient for funding capital calls). In essence, portfolio characteristics can be linked to the substitution of private equity funds for and public market funds. Our analysis allows us to attenuate concerns about commitment and re-investment risk, as the spread in returns between public and private equity funds is much narrower than the spread between private market funds and cash assets.

Our base-case analysis assumes that portfolio management takes place in a mostly frictionless setting. For example, the investor can buy and sell public market funds without costs and does not face tax responsibilities. Initially, we assume that the investor incurs no additional cost for investing to private market funds (i.e no additional capital or human resources are needed to manage private funds) though we relax this assumption in a robustness analysis. Finally, we assume that the investor does not face any internal or external agency concerns that would affect portfolio management decisions. Likewise, we assume that the portfolio does not need to generate cash outflows to satisfy external funding requirements (as is typically the case for public pension funds, university endowments, and charitable foundations).

4.1 Fixed Commitment

An intuitive starting point for understanding the affect of different investment strategies is to commit a constant percentage of total portfolio value to private market funds each vintage. We call this approach the *fixed commitment* strategy. Effectively, it assumes that the investor does not use information from past NAVs or cash flows in making commitment decisions. In addition, the investor also possesses no desire to condition commitments on current private fund portfolio allocations. The calculation of the size of the annual commitment is straight-forward: The investor chooses a target allocation to private market funds, e.g. 20% in our base case, and defines a period after which the allocation is to be reached, e.g. 5 years. A naive fixed commitment strategy would hence allocate 20%/5=4% of portfolio value each year. This calculation assumes that the future NAV of each fund is roughly equal to the amount of committed capital and appreciation of the private fund portfolio is comparable to that in the overall portfolio. However, as discussed above, a simple 4% annual commitment will typically leave the investor under-allocated to private funds since not all capital is called and often some capital is returned before the end of the investment period. This is common knowledge among private fund investors who generally follow "over-commitment" strategies. In our analysis, we utilize empirically-motivated fixed over-commitment strategies that generate allocations close to 20%, on average, for each fund type.

4.2 Dynamic Commitments

In practice, private fund investors do not follow a fixed commitment strategy but instead condition commitments based on their current information set of economic conditions, capital at hand or expectations about future states. Most importantly, investors know their current allocations to private funds and observed cash flows from prior year commitments. Based on these observations, they can form expectations: how much capital will be called and distributed this year per prior vintage-year commitment, how are assets from past commitments performing, and what is the overall state of the economy moving forward. For example, an investor targets an end-of-year NAV of 20% of portfolio value, observes the current allocation to be 16%, knows that capital calls over the last two years were below expectation, and investments made five years ago are performing well. In this case, the investor decreases commitments for the current year (relative to prior vintages) to achieve portfolio weights closer to the target weights as asset appreciation and capital calls from past commitments offset the need to deploy too much capital this year. Effectively, this approach takes into account the superposition of J-curves of different vintage years. To model this behavior we allow the investor to condition annual commitments on current portfolio weights and the realization of cash flows and NAV over the past five years. Information about cash flows and NAVs is geometrically weighted with higher weights on more recent years. Such a weighting scheme allows us to capture the cyclical nature of private market fund performance.

Initially, the investor has prior beliefs about the distributions of cash flows and NAVs for each year of the fund life-cycle. Capital is called at a constant rate over the first five years of the fund (i.e. 20% of commitment is called in each of the first five years), and no capital calls occur after the fifth year. Likewise, capital is distributed at a constant rate over the last five years of fund life (years 6 to 10) with no distributions in the first five years. For simplicity, we assume that once capital is invested by the GP there is no growth in the underlying assets and so NAV grows and declines at a constant rate with no residual NAV after 10 years. While these are clearly over-simplifying assumptions, they are sufficient for significantly reducing deviations from target allocations.[20]

$$\mathrm{c}_t = \frac{w_{t+1} * (E_t[NAV_{t+1}^{PuE}] + E_t[NAV_{t+1}^{FI}] + E_t[\Delta NCF_{t+1}^{new_c}] + E_t[\Delta NAV_{t+1}^{past_c}] + E_t[\Delta NAV_{t+1}^{new_c}]) - (E_t[\Delta NAV_{t+1}^{past_c}] + E_t[\Delta NAV_{t+1}^{new_c}])}{E_t[NAV_{t+1}^{past_c}] - w_{t+1}(E_t[\Delta NCF_{t+1}^{past_c}] + E_t[NAV_{t+1}^{past_c}]} \tag{2}$$

The commitment c_t, made at the beginning of the year, is a function of the targeted private fund allocation w_{t+1} and the the expectations about future asset values and cash flows in both public and private markets. For simplicity, we assume that an investors expects his public equity account to appreciate with the long-run return of 8% annually (from the beginning of the year t to the end the year $t+1$), so that the end of year public equity wealth equals $E_t[NAV_{t+1}^{PuE}]$. Similarly, the fixed income part of the portfolio appreciates with an long-run return of 4% annually, $E_t[NAV_{t+1}^{FI}]$.

Expectations about private asset values and cash flows are formed using the geometric weighting scheme outlined above. After the first year of investing in private funds, the

[20] Using more realistic assumptions about contributions, distributions, and NAVs has a relatively small impact on our results.

investor can update his prior using one set of observations: capital calls, distributions, and NAVs of the first year. Hence, the prior is only updated with respect to first year cash flows and NAVs, while beliefs for fund years two and onward remain unchanged. After the second year, the LP observes two additional set of observations: cash flows and NAVs for first and second year commitments for a total of three information sets. The investor uses two observations to update beliefs about first year cash flows, and one observation to update year two cash flows. The process is iterative and after one vintage life cycle, the formerly uninformed investor has a full set of cash flow and NAV data for private market funds. This information set is then used to form expectations about future portfolio allocations. In cases where public market valuations decline significantly relative to private fund valuations, the dynamic commitment strategy can result in LPs wanting to make negative commitments, so we restrict the commitment to be non-negative. Moreover, in our analysis we also define ranges of possible commitments for the dynamic strategy by defining minimum and maximum commitments.

5 Results

We now turn to examining the results of our simulations and begin by providing results for fixed commitment strategies. We then discuss how portfolio allocations and performance are affected by the use of dynamic commitment strategies. Finally, we provide analysis of portfolios that seek to keep overall portfolio risk constant as well as other robustness tests.

5.1 Risk and Return Characteristics of Fixed Commitments

Figure 2a shows an example of results from our simulations with the fixed commitment strategy to buyout funds. The shaded regions represent the average time-series of allocations. The buyout allocation gradually increases toward the 20% target allocation during the ramp-up phase (1987-1995) and then remains close to 20% during the steady phase (1996-2018). The quarterly rebalancing ensures that the fixed-income allocation remains near 40% throughout the investment horizon. These features result in a shrinking public equity allocation over the ramp-up period and a more constant allocation afterwards (i.e., close to 40%). Figure 1b shows the distribution in allocations to private market funds over time for the 1,000 simulations. As expected, the variation in allocations increases over time as the number of funds per vintage year increases (i.e., portfolios are less similarly invested). Nonetheless, the range of allocations is not extreme even in more recent years with about 90% of portfolios having buyout fund allocations within a 4% range. This suggests that even the simple fixed commitment strategy to buyout funds can result in long-run allocations that are reasonably stable. This is not generally the case; other strategies, such as VC, exhibit much greater variation in performance across funds.

[Insert Figure 1 here]

To understand the full range of diversified portfolio characteristics, Table 3 present results of simulations with fixed allocation strategies for portfolios that include buyout, venture capital, and real estate funds separately as well as all strategies combined. As discussed above we adjust the annual fixed allocation amounts so that portfolio allocations to private funds average about 20% during the mature portfolio period. Specifically, the simulations make 5.5%, 4.7%, and 5.7% fixed annual commitments to buyouts, venture capital, and real estate funds, respectively, for the portfolios with just one fund type. For a portfolios that includes all three fund types we use a fixed annual commitment of 5.2%. These commitment strategies all lead to average annual allocations to private funds of between 20% and 21%.

[Insert Table 3 here]

Panel A of Table 3 reports the first four moments of the return distributions and other performance characteristics of simulated portfolios. The first column provides portfolio characteristics for the benchmark all-public portfolio for the mature portfolio period of 1995 to 2018. The all-public portfolio has an annual return of 6.58% and a standard deviation of 9.83%. We also adjust the standard deviation of the all public portfolio using Getmansky et al. (2004) to facilitate comparison with the other portfolios [21].

The results reveals that average returns are higher for portfolios that include all three private markets strategies (buyout, VC, and RE) as compared to the all public benchmark return. The average return for portfolios with buyout funds increases 1.14% to 7.72%.

[21] In expectation, we would assume that there is no effect of un-smoothing return series of publicly-traded assets. We report very small increase of 0.34% to 10.17%, which can be due to the limitation of 1,000 simulations or other sources of spurios auto-correlation

VC investments increase returns by 2.14% over the benchmark for a total return of 8.72%, while real estate funds provide the smallest average return increase of 0.34% to 6.92%. The strategy that invests in all three fund types generates annual returns of 7.80%,or an increase of 1.22% over the all public benchmark. In general, these findings are consistent with the pooled PMEs greater than 1.0 reported in Table 2.

Of course, the question of whether these higher returns benefit investors hinges on the changes in overall portfolio risk. Average adjusted standard deviations are presented in the third row of Table 3 and show that the three strategies have distinct effects on risk. Buyout and RE funds reduce overall portfolio volatility from 10.17% for the all public benchmarkto 9.14% and 8.22%, respectively. VC funds, however, increase portfolio volatility to 12.94%. For the portfolio with investments to all three strategies, the average adjusted standard deviation falls to 9.80% – just slightly below the volatility of the all-public portfolio. These results reflect the underlying risks associated with each strategy: the high-risk, high-return nature of VC fund investments into new companies is the riskiest, leveraged investments in more mature firms by buyout funds generates risk on par with public equities, and investments in real estate by real estate private equity funds are the lowest risk.

Table 3 also presents other estimates of portfolio risk. Adjusted semi-deviations provide estimates of down-side volatility.[22]. The semi-deviation for the all-public portfolio is 12.06%. Including either buyout fund or real estate funds lowers the average portfolio semi-deviation to 11.16% and 10.50%, respectively. Portfolios with VC funds have a higher average semi-deviation of 13.49%. Portfolios with all three fund strategies have a semi-deviation of 11.63% which is again quite similar in magnitude to the all-public benchmark.

Examining the differences in skewness and kurtosis of portfolio return distributions provides further insight into the risks of portfolios with private fund investments. The baseline portfolio exhibits negative skewness (-0.56) and a positive excess kurtosis (0.43). Values are quite similar for the portfolio with buyout funds (skewness of -0.59 and kurtosis of 0.64) indicating that higher moments of return distributions for public equities and buyout funds are similar. In sharp contrast, the average portfolio with VC funds exhibits positive return skewness of 0.56 with much fatter tails (excess kurtosis of 4.49). This is

[22]Semi-deviation only measures variation below the mean with the intuition that investors are more concerned about downside risk Harry Markowitz (1952)

consistent with the wide dispersion in VC fund returns shown in Table 2 and specifically the existence of a few VC funds with exceptionally strong performance. Portfolios with RE funds also have a higher excess kurtosis (2.30) but a negative skewness of -1.04 suggesting inferior higher-moment characteristics of RE relative to other private funds and the all-public portfolio. While the volatility of real estate investments, in general, is lower than for other equity investments, RE funds in our sample invest disproportionately in "value-add" and "opportunistic" assets which often result in more extreme performance than investments to "core" real estate which is more commonly owned by publicly-traded REITs (as well as directly by many large institutional investors).

To help understand risk-return trade-offs, we examine the Sharpe ratios for the simulated portfolios. For each portfolio with private funds, the average adjusted Sharpe Ratio is higher than for the all-public benchmark. The highest adjusted Sharpe ratio is for buyout funds (0.789), followed by RE funds (0.781), and finally VC funds (0.636). When considering the portfolio that invests in all three fund types, the average adjusted Sharpe Ratio is 0.746 which is slightly lower than for the portfolios with just buyout and RE funds. These results highlight the effects of exposure to private market funds and the potential ability to expand the efficient frontier beyond the limits of of portfolios with just public market investments. Our results are generally consistent with the finding of Goetzmann et al. (2020) who show that large buyout funds provide the greatest diversification benefit, while the benefit from VC funds is only marginal.

Panel B of Table 3 shows the average allocations across asset classes. For all diversified portfolios, average allocations are close to target allocations. In Panel C of Table 3 we examine the mean and standard deviation of private fund tracking error across the 1,000 simulations. We find substantial tracking error differences across simulated portfolios despite the fact that the fixed commitment strategy is identical across all portfolios. This variation from target allocations is driven by the idiosyncratic performance of individual funds and is greatest for VC funds (7.6% standard deviation relative to the target allocation) and least for RE (2.8% standard deviation relative to the target allocation). The standard deviation for buyout (all) funds is 3.0% (3.9%).[23]

In Panel D, we show the range of outcomes for returns, adjusted standard deviations, and adjusted Sharpe ratios of the simulated portfolios. We report the frequency of

[23]As discussed subsequently, the results for portfolios with VC are heavily impacted by allocations around the tech bubble; however, allocations remain more volatile in the following two decades as well.

characteristic outcomes relative to the all-public benchmark statistic. A portfolio with PE funds always generates a higher return with lower volatility, and thus has a higher Sharpe Ratios than the benchmark portfolio. The same is true for portfolios with RE funds. Portfolios with VC funds, in contrast, always generate higher returns but never have lower volatility. On average, the higher risk appears to be more than fairly compensated for with higher returns as shown by higher Sharpe Ratios in 97.1% of simulations. The volatilities of portfolios diversified across the three different private market fund strategies are lower in 80.0% of simulations but returns and risk-adjusted returns always exceed those of the all-public benchmark.

The results on the distribution of outcomes is easiest to see graphically. Figure 2b plots the histograms of returns for the diversified portfolios relative to the performance of the all-public benchmark. There is no variation in the benchmark return of 6.58%. Most portfolios with real estate funds have simulated returns within the range from 6.75% to 7.25%, or a range of about 0.5%. Investing in buyout funds increases average returns but also the range of returns to about 1%, wheres portfolios with venture funds have still higher returns and a wider dispersion in returns: portfolios generate returns within a range of 7.7% to 9.6%. Clearly there has been a historical risk-return trade-off with private fund strategies.

[Insert Figure 2b here]

5.2 Risk and Return Characteristics of Dynamic Commitments

Table 4 reports the results for our simulated portfolios following dynamic commitment strategies. As noted above, these commitment strategies allow investors to update their beliefs about future cash flows and asset values based on information from prior investments. The main objective is two reduce the tracking error in private funds allocations. In these simulations annual commitments are constrained to be between 2% and 10% of total portfolio value (and we discuss other ranges below). Panel A shows that return and risk characteristics of the diversified portfolios remain qualitatively similar to those for the fixed allocation strategy (i.e., returns increase for all fund types and risk declines except for the portfolios with VC). However the quantitative results differ so that adjusted Sharpe ratios are lower in all cases and the average adjusted Sharpe ratio for portfolios with VC are lower than for the all public benchmark. Panel B shows somewhat higher average allocations to private funds. Importantly, Panel C shows that as compared to the fixed commitment strategy, the volatility around the target allocation is reduced for portfolios with buyout funds, VC funds and all fund types but actually increases RE. The reductions are somewhat modest suggesting that either benefits from dynamic commitment strategies are limited or different strategies need to be considered.

[Insert Table 4 here]

Panel D shows the range of outcomes across simulated portfolios. Comparison with Panel D of Table 3 makes it apparent that historically there has been a trade-off associated with managing allocation tracking error and risk-adjusted return. For the dynamic commitment strategies, portfolios with buyout funds no longer have superior Sharpe ratios in all simulations. This trade-off is more evident for portfolios with VC funds where only 0.4% now have higher Sharpe ratios. The diversified portfolio with RE funds and all three fund types outperform the all public benchmark portfolio in 80.4% and 82.3% of all simulations, respectively. Consequently, we cannot conclude that such portfolios statistical significantly outperforms an all-public portfolio. These somewhat counter-intuitive results are mainly driven by commitments made around 1998-2002 when conditioning commitments on the cash flows of prior years effectively reduced commitments (in some cases zero) to some of the best performing vintage years and especially to VC funds. The findings mirror those of Brown, Harris, Hu, Jenkinson, Kaplan, and Robinson (2020a) which conclude that it is difficult to use available information to profitably time commitments to private equity funds (and especially VC funds).

Given that end-of-quarter allocations to private funds can vary significantly across investors following similar allocation strategies, we also seek to understand the time-series characteristics of variation in allocations.

[Insert Figure 3 here]

Figure 3 plots time-series allocations for the three portfolios with specific fund types, as well for the portfolio with allocations to all three fund types. As in Figure 2a the graphs show the range of allocations across simulated portfolios (5th, 25th, 50th, 75th, and 95th percentiles). In each case, the ramp-up phases evolve similarly so that it takes about 8 years to reach the 20% target allocation. However, allocations during the steady-state phase (after 1996) demonstrate that portfolio allocations vary markedly over time depending on fund type. For example, Panel (a) shows that the median allocation to buyout funds hovers around the target of 20% and rarely does the inter-90% range not include the 20% target. The range of allocations does not increase significantly after the mid-1990s. Furthermore, large market shocks such as around the dot-com bubble and the GFC do not significantly impact the allocations to PE funds. These results all indicate that buyout fund allocations are fairly stable over long horizons.

In stark contrast to buyout funds, Panel (b) of Figure 3 shows that the allocations to VC funds swing dramatically during the steady-state phase. In particular, the median VC allocation spiked to 36% during the dot-com bubble only to plummet shortly after to 12% in 2003. After bottoming out, median allocations bounced back to 27% by 2009. So while the average allocation is close to the target allocation, the inter-90% range includes the target allocation of 20% in only in few quarters. Moreover, there are quarters where the allocation to private VC funds exceeds the allocation to public equities.

Panel (c) of Figure 3 plots results for portfolios with real estate funds. Recall from Table 1 that there are few RE funds in the early years of our sample. Therefore, the variation in allocations to RE funds is mechanically low through the early 2000s since simulated portfolios are holding similar funds. Overall, the median allocation to RE funds is close to the target allocation until 2008. In the aftershocks of the GCF, the NAVs for RE funds dropped more than the value of public equity causing the median allocation to fall to about 16% in 2010. During this period many RE funds invested in distressed or opportunistic assets and as commercial real estate came back into favor, the NAVs of RE funds increased sharply driving simulated allocations to around 27% by 2012. By the end of the sample period, the median RE fund allocation had returned to just below the 20% target. With the exception of the years highly impacted by the GFC, the inter-90% range of RE fund allocations includes the 20% target for most of the steady-state period.[24]

Given that most large institutional investors hold broadly diversified private portfolios, the portfolio that is invested in all three private fund types is perhaps the most interesting. Given that there are distinct patterns for buyout, VC and RE returns, we should expect there to be diversification benefits to investing in all three fund types. The time series allocations are plotted in Panel (d) of Figure 3 and, as expected, allocations inherit some variation attributable to each of the individual fund types. For example, VC funds

[24]In future work we plan to explicitly compare performance for RE funds to a public benchmark with REITs where the RE fund allocation replaces REIT investments.

introduce variation around the dot-com bubble. We note that the inter-90% range is wider than for the individual fund types because there is more heterogeneity in the randomly selected private portfolios. Still, The inter-90% range includes the 20% target allocation only about half of the time during the steady-state period. As we discuss subsequently, these results suggests a potential need to properly allocate and diversify across strategies by investing in the right mix of strategies and a fairly large number of funds each vintage year.

We now examine the effects of how characteristics of specific dynamic commitment strategies affect the time-series deviations from target allocations and the ramp-up period. We look specifically at the effect of varying the floors and caps on annual commitment amounts and the speed of commitments in the ramp-up period. These highlight two important factors that need to be considered by investors: vintage-year diversification and organizational conflicts.

By construction, the fixed commitments strategy delivers the highest vintage-year diversification but it results in fairly high tracking error of actual allocations relative to target allocations. However, our results shown above indicate that there can still be substantial time-series variation even with a dynamic commitment strategy and there is no guarantee of lower time-series risk or higher return in the overall portfolio.[25]

Consequently, deviating from the fixed strategy with the aim of committing less when, in expectation, future allocations will be high, can unintentionally lower diversification in a way that harms the realized risk-adjusted returns. To better understand the effect of different constraints on the time-series of allocations, we consider three different allowed ranges for annual commitments to private funds (in terms of overall portfolio NAV), 0%-12%, 2%-10% (as discussed above), and 4%-8%. The results for buyout funds are plotted in Figure 4. Panel (a) shows the time-series median of allocations for each different range of constraints. As would be expected, the least constrained strategy (0%-12%) is generally closer to the target allocation for the latter half of the sample, though the differences are not large. However, early in the steady-state period (1996-2000) the least constrained strategy is farther from the target allocation. Panel (b) of Figure 4 plots the annual allocations and explains why–during the ramp-up phase (prior to 1996) the least

[25] Also, Robinson and Sensoy (2016) show that variation in quarterly cash flows can be reduced by more than 50% for PE and VC funds for a similar sample period via alternative commitment strategies, so cash flow variability represents another possible risk characteristic. Cash flows are baked into our analysis and so we do not examine them separately.

constrained strategy makes much more varied allocations across vintage years and several times hits the 0% and 12% constraints. Thus it is less diversified and overall allocations are more affected by idiosyncratic variation in NAVs. Later in the sample period all of the strategies converge on similar annual allocations.

[Insert Figure 4 here]

Figure 4 also shows that commitment schedules with different constraints are highly correlated and share a high degree of cyclicality, particularly in the ramp-up phase. For most institutional investors, there is another important issue associated with commitments bands related to relationships with GPs. As shown in panel (b), when the commitment is constrained by the 0% and 12% boundaries, there are four years in when the LP would make no commitments at all to private equity funds (and in the case of 1995 and 1996 even two consecutive years). However as a practical matter top GPs expect consistency in allocations across fund offerings. In other words, it may be hard for an LP to skip a fund offering from a given GP and then get access to the next fund. This is especially true for the most desirable VC funds. Consequently, the LP is incentivized to deviate from this strategy. Overall, it appears that any potential gains from widening constraints on annual commitment are limited and possibly offset with institutional or reputational costs.

[Insert Figure 5 here]

Another aspect of the dynamic allocation strategy is the speed of the initial portfolio investment ramp-up. Figure 5 shows the effects on private fund allocations of decreasing (increasing) the ramp-up period to 4 (10) years. Panel (a) shows that a 4-year ramp-up period achieves the 20% target about 2 years before the 7-year ramp-up period, and 3 years before the 10-year ramp-up period but also has a higher tracking error in early years. Panel (b) shows why this is the case as there are higher levels for the annual commitments in the first few years and thus less vintage year diversification. The number of years targeted for ramp-up has little effect on allocations after about 15 years (i.e., after 2002). This is not surprising given that the average fund life-cycle is 10-15 years.

[Insert Figure 6 here]

A fundamental question about the inclusion of private funds in diversified portfolios if (and how) they extend the mean-variance efficient frontier (see Harry Markowitz (1952)). The results above for 20% target allocations suggest private funds do push out the frontier but potentially in different ways. Figure 6 demonstrates how a steady increase in the allocation to each private market fund type plots relative to the all-public portfolios efficient frontier.[26]. Our results show that there are economically sizeable diversification benefits for both buyout and real estate funds which consistently result in both higher returns and lower risk as the weights of each in the portfolio increase.[27] Diversified portfolios with VC funds improve returns but also lead to substantially higher (adjusted) volatilities. Therefore, the value of diversification benefits depends on investor's risk preferences. Similar to Goetzmann et al. (2020) we find that buyout funds have a positive diversification benefit. In contrast to their results, our findings indicate that investors can also benefit from investing in unlisted real estate. Economically, both results seem reasonable. Private equity funds give investors exposure to small firms, value firms, and lately also mature growth firms; all of which tend to stay private longer or postpone the going-public decision indefinitely. Private real estate firms differ from their public counterparts as well: for example, following the GFC, many RE funds bought distressed residential properties and benefited from the recovery of the U.S. housing market, while most public REITs focus on core or value-add commercial properties.

5.3 Other Results and Robustness Checks

Given the results just discussed concerning how private funds affect the efficient frontier, we undertake an alternative analysis related to overall portfolio risk. Table 5 shows the results from a portfolio analysis that exposes investors to the same level of market risk taken with the all-public benchmark portfolio. We construct these "constant-beta" portfolios from an altered rebalancing strategy: at the end of each quarter, the investor

[26] In practice, most investors will not target an equity allocation that is all private equity, but we plot the full range of 0% to 60% target private funds.

[27] We recognize that lower overall volatilities also imply that the "true" market betas of private funds is less than one. In our case the definition of the 'true market' is somewhat ambiguous because we are considering assets not available to many investors. For private real estate funds, the result is not unpalatable: by our calculations (not reported) the long-run beta of public REITs is about 0.6. For private equity funds, this result is more controversial. Although there have been contrasting findings on the market beta of buyout and VC funds, the consensus among academics is that both should have a beta that is greater than one relative to public markets. Consequently, it is interesting that we find a beta for buyouts somewhat below 1.0 for the market portfolio that includes private funds.

allocates to public equities and fixed income so as to keep an estimate of the total systematic risk constant. While there are potentially many ways to estimate portfolio systematic risk, we simply assume that the public fixed income has a beta of zero, and the equity and fund asset have betas consistent with the extant literature, specifically: We assume that $\beta_{PuE} = 1$, $\beta_{FI} = 0$, $\beta_{BO} = 1.3$, $\beta_{VC} = 1.6$, and $\beta_{RE} = 0.6$. the results indicate that, as would be expected, average allocations to fixed income increase when the private fund portfolio has a beta greater than 1.0 (all cases except RE) and vice versa (RE). Results for buyouts, RE, and all funds together, almost always provide superior Sharpe ratios whereas portfolios with VC provide higher Sharpe ratios in only about half of simulations. Overall, the results provide further evidence that portfolios with buyout funds, real estate funds, and highly diversified mix of funds generate superior risk-adjusted performance.

[Insert Table 5 here]

A clear drawback of investing to private funds is that investors can not easily diversify by creating a portfolio of all available assets. In public markets, investors can diversify their portfolio across all publicly-traded equities by simply buying index funds. In private markets, such investable private equity indexes do not exist. Instead, LPs tend to make a relatively small number of annual commitments because of the per-fund costs associated with due diligence, monitoring/compliance, and reporting. In our main analysis we assume 10 LP commitments a year which would be consistent with a large and active investor in private funds. Furthermore, capital committed to a private fund manager is invested across a limited set of investment opportunities (maybe 10-20 underlying investments depending on the type of fund) and thus any given fund is fairly undiversified and generates concentrated idiosyncratic risk exposures. An investor can significantly reduce overall idiosyncratic risk by increasing within-vintage diversification, i.e., committing to a larger number of funds in each vintage.

[Insert Figure 7 here]

Figure 7 reports the distributions of simulated total portfolio returns with investments to 2, 5, 10, and 20 private funds per vintage year. As should be expected, the mean long-run portfolio returns are largely unchanged by the number of funds committed to in each vintage year. However, there is a substantial effect on the dispersion of portfolio mean returns. While 99% of simulated portfolios with 2 buyout funds per vintage year have long-run mean returns between 8% and 10%, this dispersion declines 8.75%–9.5% once an investor commits to 10 buyout funds per vintage. In general, doubling the number of funds per vintage roughly halves the spread in long-run returns. Our finding is consistent with the empirical evidence that most large institutional investors make between 5 to 15 private equity investments per year. This level of diversification can be rationalized: there exists a reasonable trade-off between diversification and additional costs from managing the allocation to private funds. LPs spend resources on establishing and maintaining relationships with GPs which are increasing in the number of commitments per year.

In untabulated results, we show that relaxing the assumption of managing private funds without incurring additional costs is not crucial. If we assume additional fees of 50 basis points on assets in private market funds the overall return distributions are shifted but volatilities are unchanged. The long-run mean return is reduced by about 10 basis points (when the target allocation is 20%). A reduction in expected returns of this magnitude does not change the statistically significant outperformance of diversified portfolios versus the benchmark portfolio,

Finally, we show results that can help explain the time-series variation in private fund allocation. A fixed commitment strategy outlined in Section 4 makes the implicit assumption that across vintage years, the evolution of asset values is homogeneous. In particular, following the textbook example of a private fund life-cycle, the maximum net asset value is reached shortly before the fund starts to enter the divestment phase around the beginning of year 6. In practice, this assumption does not hold: there is significant heterogeneity in the age when private funds reach their maximum NAV, both within and across vintage years as well as across strategies. This heterogeneity leads to variation in the allocation to private funds. Figure 8a shows the median fund quarter of maximum NAV per vintage, by fund strategy.

[Insert Figure 8 here]

For private fund strategies, the time to account the maximum NAV over a fund life-cycle is heterogeneous. On average, RE funds start to distribute the earliest as shown by a long-run average of about 16 quarters (or 4 years) to maximum NAV. Excluding the early part of our sample (where only few RE funds exist) and the time around the GFC, the variation in NAVs and cash flow pattern is relatively constant. BO and VC funds are more volatile: for example, VC funds with vintages prior to the dot-com bubble had maximum NAVs within the next three years, while those funds raised during or after the dot-com bubble took 7 years to account for maximum NAV. The time-series variation is similar for BO funds, although variation is less severe. The level of quarters to maximum NAV for BO and VC funds is significantly higher than for RE funds: in the long-run, both fund strategies' NAVs peak around quarter 24, or after (6 years). Figure 8a confirms the importance of controlling for vintage-year cyclically in asset values, and across asset classes. Our dynamic commitment strategy can capture some of this variation in maximum NAVs. By over-weighting more recent vintage cash flows and NAVs, the dynamic commitment strategy can capture some the cyclicality and hence adjust for the level of quarters to reach maximum NAV; however, the strategy does not capture the slope of the graphs shown in Figure 8a. Our results presented in Tables 3 to 5 also include a portfolio that invests across all three fund strategies (value-weighted). The variation and cyclicality in fund NAVs and cash flows can help explain why there is only a limited diversification benefit of investing across asset classes per vintage (i.e., the adjusted Sharpe ratio for portfolios that invest in BO, VC, and RE jointly is only slightly higher than the average for the three fund types separately).

6 Conclusion

Institutional investors have diversified their portfolios by allocating to private market funds for several decades. Furthermore, growing interest among policy makers and regulators in expanding the set of investors with access to private markets could soon eventuate in a boom of investing into private market funds by retail investors. Although there is anecdotal evidence that these funds hold favorable characteristics for investors, such as higher returns and low correlation with public markets, we are the first to provide a comprehensive ex-post performance analysis of such diversified portfolios. Our results confirm the wide-spread notion that most private market funds have generated diversification benefits and improved risk-adjusted returns historically. We also document the allocation and risk characteristics of diversified portfolios using only primary commitment strategies.

Diversification benefits vary by private market fund strategy. Historically, benefits are large and statistically reliable for buyout funds, while the increased return associated with VC funds is countered by a significant increase in portfolio volatility. Real estate funds provide diversification benefits as well, although less than buyout funds. Our results do not rely on selection skill but do assume that investors have access to all funds. Our results are robust to other practical consideration, such as investing in just a limited number of funds per vintage year and additional costs for managing private market funds.

We also provide an empirical analysis of practical commitment strategies that can allow investors to allocate to private market funds. Using primary commitments, it is feasible to maintain an allocation to buyout and RE funds that is usually within a reasonable allocation range (e.g., within plus or minus 5%). In contrast, VC funds experience substantial volatility in allocations. Other tests suggest that a long-run horizon helps reduce allocation tracking risk: commitment strategies with different parameters for modeling beliefs about future cash flows, limiting annual commitments or years to reach target allocation converge after approximately 10 years. Hence, the long investment horizon of endowments and pension funds is a natural match for private market funds. A more granular analysis of the interrelationship of investment horizon and diversification benefit is left to future research.

A logical next step in this research is to expand the set of private market funds, both geographically (funds with a geographic focus outside the US) and with respect to other strategy (private credit, infrastructure, natural resource and hedge funds). In additional, further practical considerations of commitment and liquidity structures could help understand the role of secondary markets, restricting the set of investable funds or considering risk preferences of heterogeneous investors.

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Table 1 Burgiss Fund Universe

This table shows the number of funds and the market capitalization (in $ millions) by vintage year using the Burgiss data. Only funds with a US geographical focus are included. If there are less than five funds per vintage year, we cannot disclose market capitalization in light of confidentiality.

Vintage	Buyout		Venture Capital		Real Estate		All	
	Cap.	Funds	Cap.	Funds	Cap.	Funds	Cap.	Funds
1987	8,371	8	1,567	29	****	4	11,048	41
1988	6,065	9	1,967	29	1,667	6	9,699	44
1989	3,129	11	3,403	29	904	8	7,436	48
1990	2,306	8	828	13	****	3	3,589	24
1991	****	4	522	7	****	2	2,343	13
1992	4,015	9	1,430	17	****	3	6,424	29
1993	4,026	7	1,914	20	691	6	6,630	33
1994	5,446	17	1,479	16	2,726	8	9,652	41
1995	16,971	27	3,688	27	3,309	10	23,967	64
1996	4,756	17	2,359	18	3,488	9	10,603	44
1997	24,869	26	6,223	47	6,325	18	37,417	91
1998	35,427	40	9,420	51	12,776	27	57,623	118
1999	32,220	34	29,957	95	5,603	15	67,780	144
2000	54,647	48	41,271	119	5,120	12	101,037	179
2001	22,288	30	22,568	64	4,293	15	49,149	109
2002	14,640	20	5,983	21	4,910	16	25,533	57
2003	21,559	24	5,427	23	7,164	15	34,150	62
2004	33,219	38	10,611	42	8,531	26	52,361	106
2005	51,365	61	19,050	67	19,314	43	89,729	171
2006	140,172	66	26,419	80	26,958	38	193,548	184
2007	120,382	71	24,191	77	55,078	57	199,651	205
2008	103,037	70	20,302	66	14,526	30	137,865	166
2009	19,306	23	11,260	27	10,843	15	41,410	65
2010	19,702	30	10,764	33	10,166	18	40,633	81
2011	60,284	47	13,405	46	30,387	26	104,076	119
2012	64,269	51	20,198	65	16,632	34	101,099	150
2013	75,286	51	12,727	52	26,556	44	114,570	147
2014	67,861	61	21,322	74	24,897	38	114,080	173
2015	58,164	41	20,199	88	46,597	45	124,960	174
2016	88,111	64	16,262	68	20,404	42	124,777	174
2017	77,107	42	22,611	80	27,636	42	127,354	164
2018	115,091	54	19,863	70	31,042	36	165,996	160
1980s	17,565	28	6,937	87	2,571	18	28,183	133
1990s	130,036	189	57,820	311	34,918	101	226,028	601
2000s	580,615	451	187,082	586	156,737	267	924,433	1,304
2010s	606,173	411	146,587	543	224,151	307	976,912	1,261
Total	1,355,267	1,109	409,190	1,560	431,732	711	2,196,189	3,380

Table 2 Fund Performance

This table shows the distribution of fund performance, measured by the public-market equivalent (PME). See Kaplan and Schoar (2005) for further discussion. The S&P 500 is used as the benchmark in calculations of the PME.

	N	Mean	Min	1%	5%	25%	50%	75%	95%	99%	Max
BO	1,109	1.12	0.00	0.15	0.48	0.87	1.06	1.31	1.95	2.49	4.37
VC	1,560	1.18	0.00	0.10	0.23	0.62	0.88	1.21	2.84	9.11	23.12
RE	711	1.06	0.00	0.11	0.39	0.84	1.04	1.27	1.79	2.21	2.98

Table 3 Fixed Commitment Strategy

This table shows the portfolio characteristics for a diversified portfolio if an investor follows a fixed (over-)commitment strategy. Over-commitment factors are chosen such that, on average, the average portfolio allocation to private funds is 20%. Adjusted values follow the unsmoothing procedure outlined by Getmansky et al. (2004). We run 1,000 simulations and report average simulation outcomes for the steady state (1995-2018). In panel A, we report the first four moments of the return distribution of simulated portfolios as well as additional performance characteristics. In panel B, we report the average portfolio allocations and in panel C the first two moments of deviation from target. Panel D reports the percentages in which the diversified portfolio outperformed the benchmark portfolio.

	Bench	BO	VC	RE	BVR
Panel A: Risk-Return Characteristics					
Return	6.58%	7.72%	8.72%	6.92%	7.80%
Standard Deviation	9.83%	8.13%	10.11%	7.05%	8.37%
Standard Deviation (adjusted)	10.17%	9.14%	12.94%	8.22%	9.80%
Semi-Deviation (adjusted)	12.10%	11.16%	13.48%	10.50%	11.63%
Skewness	-0.564	-0.590	0.555	-1.036	-0.384
Kurtosis	0.432	0.641	4.494	2.298	1.271
Sharpe Ratio	0.618	0.888	0.814	0.911	0.872
Sharpe Ratio (adjusted)	0.597	0.789	0.636	0.781	0.746
Panel B: Average Allocations					
Fixed Income	40.0%	39.7%	39.4%	39.8%	39.7%
Public Equity	60.0%	39.6%	39.7%	40.1%	39.4%
Private Fund	-	20.6%	20.9%	20.1%	20.9%
Panel C: Deviation from Target Allocation					
Average	-	0.6%	0.9%	0.1%	0.9%
Standard Deviation	-	3.0%	7.6%	2.8%	3.9%
Panel D: Outperformance of Benchmark					
Return	-	100.0%	100.0%	100.0%	100.0%
Standard Deviation (adjusted)	-	100.0%	0.0%	100.0%	80.0%
Sharpe Ratio (adjusted)	-	100.0%	97.1%	100.0%	100.0%

Table 4 Dynamic Commitment Strategy

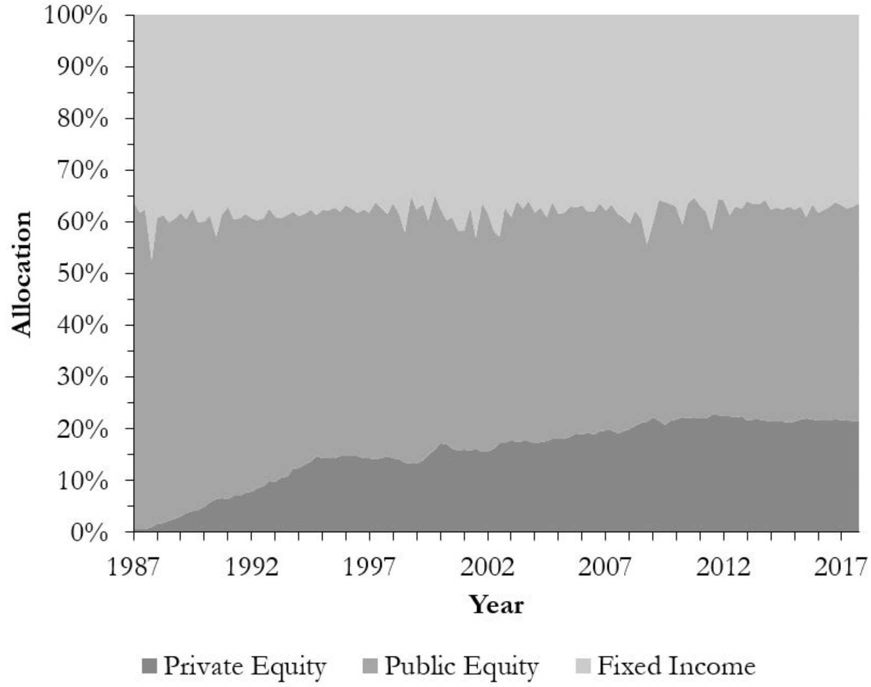
This table shows the portfolio characteristics for a diversified portfolio if an investor follows a dynamic commitment strategy as described in Section 4. Weighting follows a geometric weighting scheme that uses cash flow and net asset values of the last five years that, on average, the average portfolio allocation to private funds is 20%. Adjusted values follow the unsmoothing procedure outlined by Getmansky et al. (2004). We run 1,000 simulations and report average simulation outcomes and report average simulation outcomes for the steady state (1995-2018). In panel A, we report the first four moments of the return distribution of simulated portfolios as well as additional performance characteristics. In panel B, we report the average portfolio allocations and in panel C the first two moments of deviation from target. Panel D reports the percentages in which the diversified portfolio outperformed the benchmark portfolio.

	Bench	BO	VC	RE	BVR
Panel A: Risk-Return Characteristics					
Return	6.58%	7.95%	8.62%	7.13%	8.15%
Standard Deviation (adjusted)	10.17%	9.16%	12.66%	8.34%	9.83%
Semi-Deviation (adjusted)	12.10%	11.16%	13.13%	10.74%	11.57%
Sharpe Ratio (adjusted)	0.597	0.646	0.521	0.612	0.624
Panel B: Average Allocations					
Fixed Income	-	39.7%	39.7%	39.8%	39.7%
Public Equity	-	39.4%	37.9%	40.0%	39.3%
Private Fund	-	20.9%	22.4%	20.2%	21.0%
Panel C: Deviation from Target Allocation					
Average	-	0.9%	2.4%	0.2%	1.0%
Standard Deviation	-	2.0%	6.9%	3.2%	3.2%
Panel D: Outperformance of Benchmark					
Return	-	100.0%	99.1%	100.0%	100.0%
Standard Deviation (adjusted)	-	100.0%	0.9%	100.0%	75.9%
Sharpe Ratio (adjusted)	-	98.9%	0.4%	80.4%	82.3%

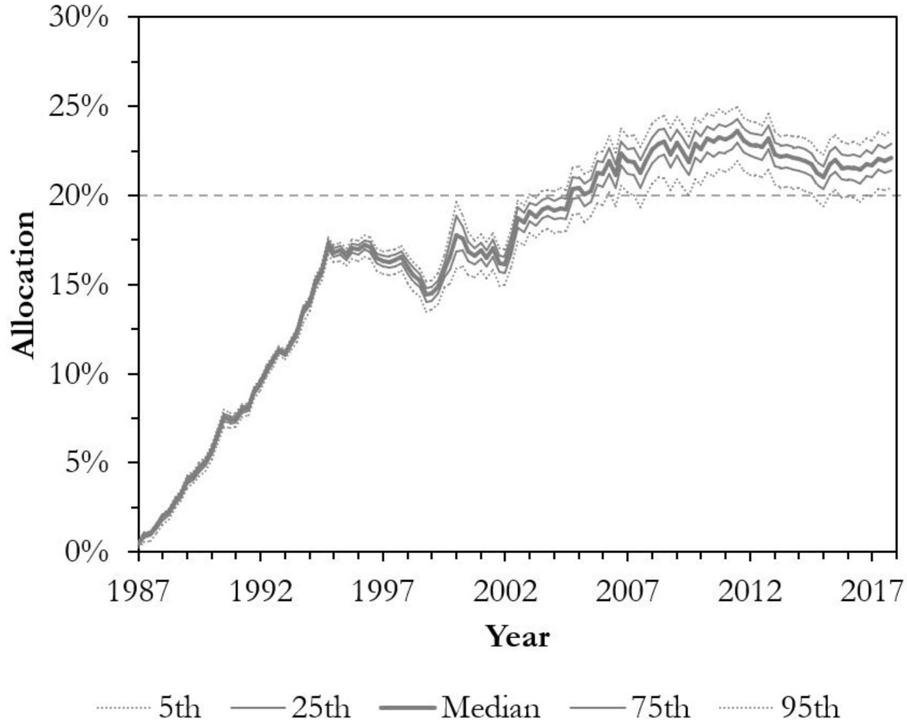
Table 5 Constant Beta Portfolio

This table shows the portfolio characteristics for a diversified portfolio if an investor follows a dynamic commitment strategy as described in Section 4, and if he targets a portfolio $\beta_{Portfolio} = 0.6$ (i.e. the long-run beta of the benchmark portfolio). We assume that $\beta_{PuE} = 1$, $\beta_{FI} = 0$, $\beta_{BO} = 1.3$, $\beta_{VC} = 1.6$, and $\beta_{RE} = 0.6$. Weighting follows a geometric weighting scheme that uses cash flow and net asset values of the last five years that, on average, the average portfolio allocation to private funds is 20%. Adjusted values follow the unsmoothing procedure outlined by Getmansky et al. (2004). We run 1,000 simulations and report average simulation outcomes and report average simulation outcomes for the steady state (1995-2018). In panel A, we report the first four moments of the return distribution of simulated portfolios as well as additional performance characteristics. In panel B, we report the average portfolio allocations and in panel C the first two moments of deviation from target. Panel D reports the percentages in which the diversified portfolio outperformed the benchmark portfolio.

	Bench	BO	VC	RE	BVR
Panel A: Risk-Return Characteristics					
Return	6.58%	7.81%	8.28%	6.94%	7.71%
Standard Deviation (adjusted)	10.17%	8.07%	10.41%	9.11%	8.59%
Semi-Deviation (adjusted)	12.10%	9.79%	8.78%	11.69%	9.92%
Sharpe Ratio (adjusted)	0.597	0.718	0.603	0.707	0.688
Panel B: Average Allocations					
Average Allocation Fixed Income	-	45.9%	57.4%	36.2%	46.7%
Average Allocation Public Equity	-	33.3%	20.2%	43.5%	32.6%
Average Allocation Alternative	-	20.8%	22.4%	20.4%	20.7%
Panel C: Deviation from Target Allocation					
Average	-	0.8%	2.4%	0.4%	0.7%
Standard Deviation	-	2.0%	6.8%	3.1%	3.3%
Panel D: Outperformance of Benchmark					
Return	-	100.0%	100.0%	100.0%	99.3%
Standard Deviation (adjusted)	-	100.0%	47.1%	100.0%	97.6%
Sharpe Ratio (adjusted)	-	100.0%	55.6%	100.0%	99.3%

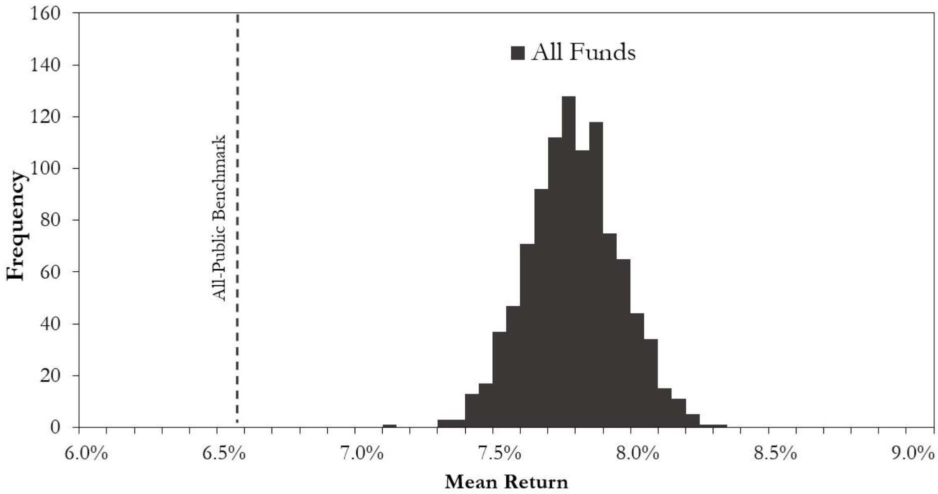


(a) Average allocation by asset class

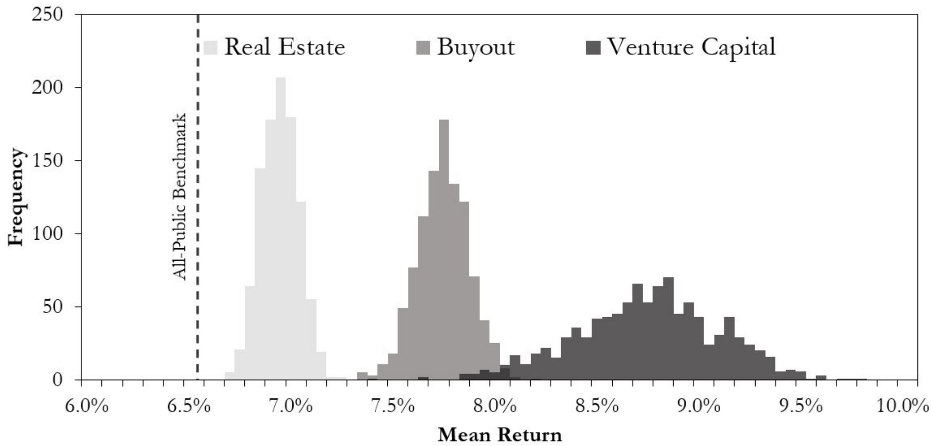


(b) Distribution in allocation to private equity funds

Figure 1: This figure shows time-series allocation for diversified portfolios following a fixed commitment strategy. The targeted allocation to buyout funds is 20%. Panel (a) shows the time-series of average allocations across asset classes. In panel (b), the distribution of allocations to private equity funds is highlighted.

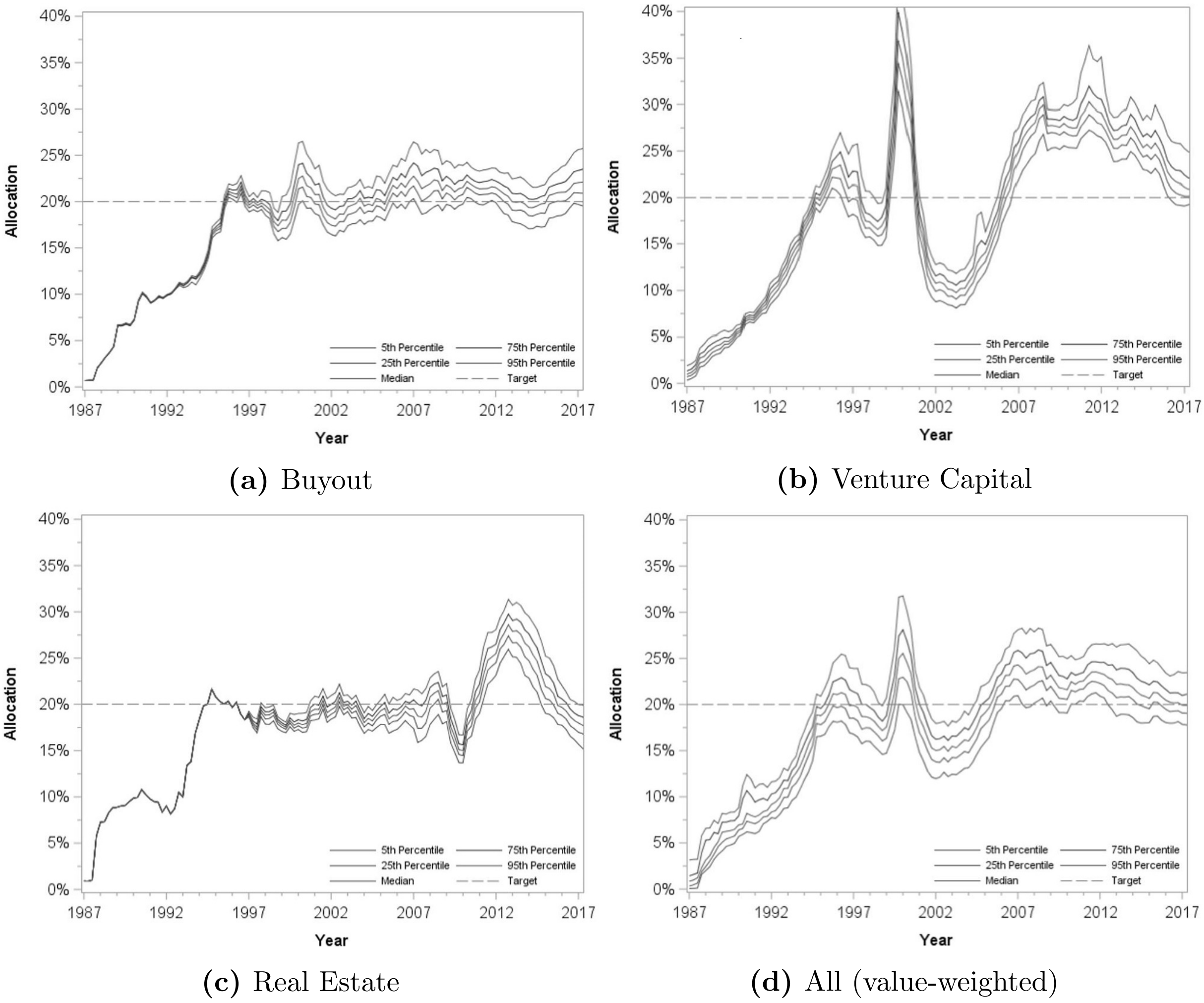


(a) Return distribution for portfolios diversified across all funds



(b) Return distribution by fund strategy

Figure 2: This figure shows the return distributions of diversified portfolios following a fixed over-allocating strategy with 20% target allocation to private market funds.



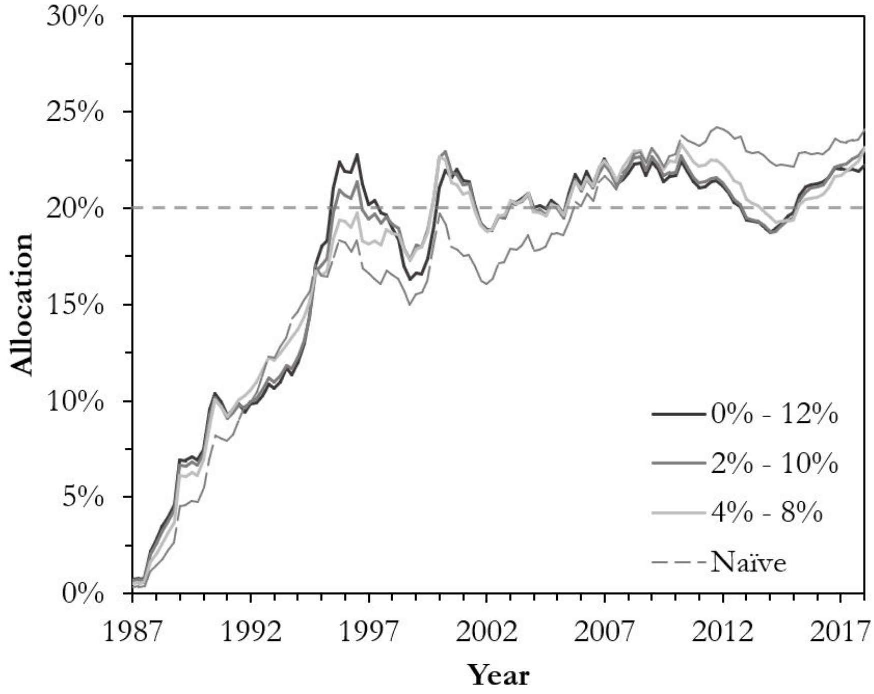
(a) Buyout

(b) Venture Capital

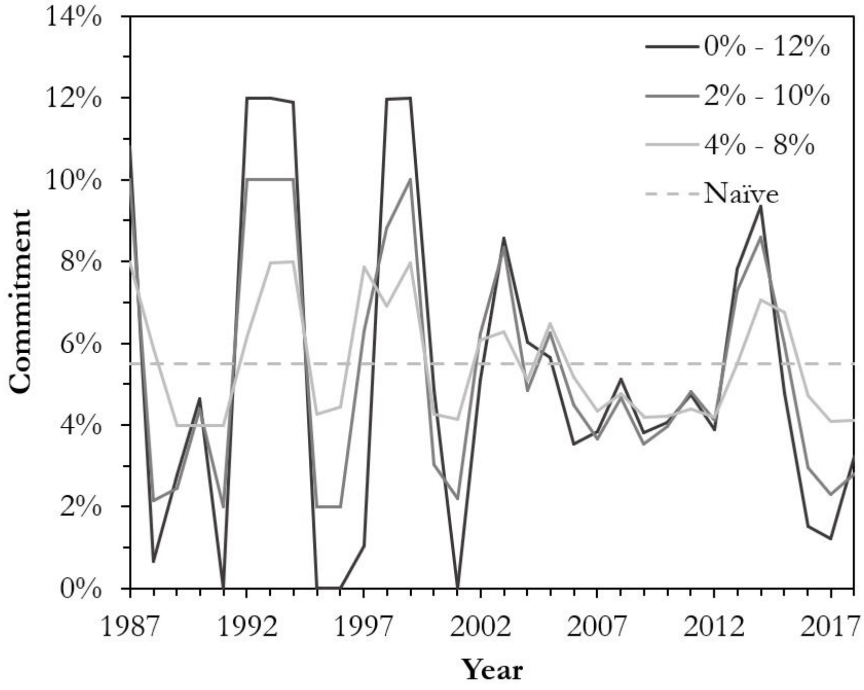
(c) Real Estate

(d) All (value-weighted)

Figure 3: This figure shows the time-varying allocation to private market funds in a diversified portfolio. Panel (a) to (c) display the allocations from a dynamic commitment strategy with 20% target allocation for buyout, VC, RE funds. In panel (d), the same strategy is employed but commitments are value-weighted across all three private asset classes.

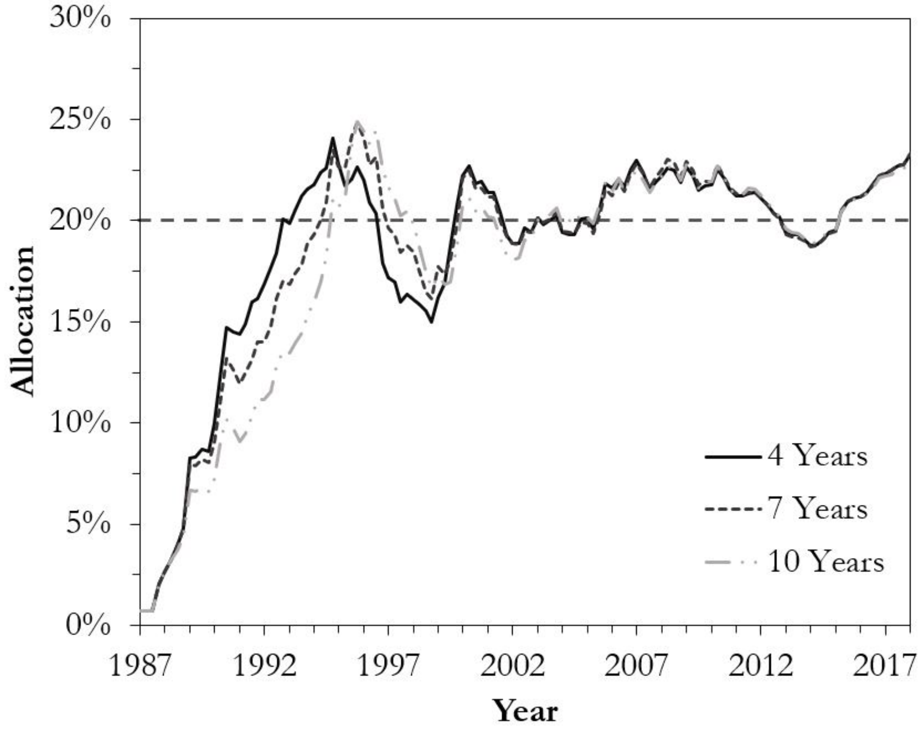


(a) Median allocation

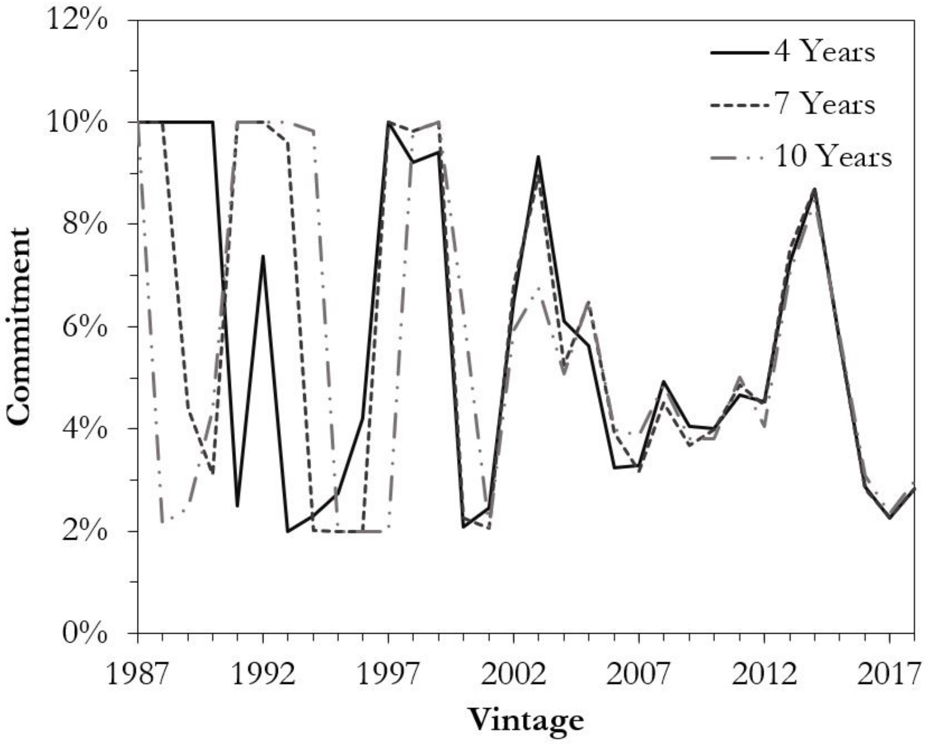


(b) Median annual commitment

Figure 4: Diversified portfolio following a dynamic commitment strategy with buyout funds. Annual commitments are restricted to ex-ante defined range (commitment as a percentage of total portfolio value). The naive benchmark corresponds to a simple over-commitment strategy of 5.6% annually that aims to reach target allocation of 20% after 5 years.



(a) Median allocation



(b) Median annual commitment

Figure 5: Diversified portfolio following a dynamic commitment strategy with buyout funds. The commitment strategies are designed to reach target allocation after ex-ante defined years of ramp-up. Annual commitment is restricted from 2% to 10% of total portfolio value. The naive benchmark corresponds to a simple over-commitment strategy of 5.6% annually that aims to reach target allocation of 20% after 5 years.

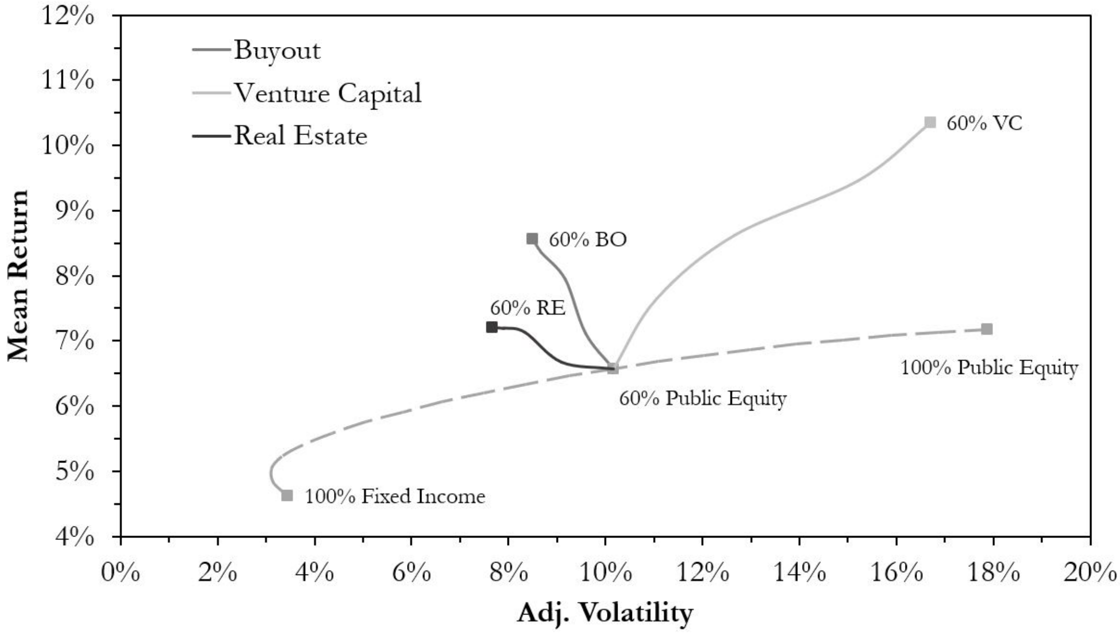


Figure 6: This figure shows the impact of private market funds on the efficient frontier. The dashed line represents the investable portfolios using public market funds only. Specifically, the graph shows the expansion of the frontier resulting from the substitution of 60% public equity with three different private market funds.

# of Funds	Mean	SD	Min	5%	25%	50%	75%	95%	Max
2	8.94%	0.29%	7.96%	8.47%	8.74%	8.94%	9.14%	9.41%	9.86%
5	8.94%	0.17%	8.41%	8.65%	8.84%	8.95%	9.06%	9.22%	9.51%
10	8.91%	0.10%	8.64%	8.74%	8.84%	8.91%	8.98%	9.08%	9.23%
20	8.95%	0.06%	8.75%	8.86%	8.91%	8.95%	8.99%	9.04%	9.15%

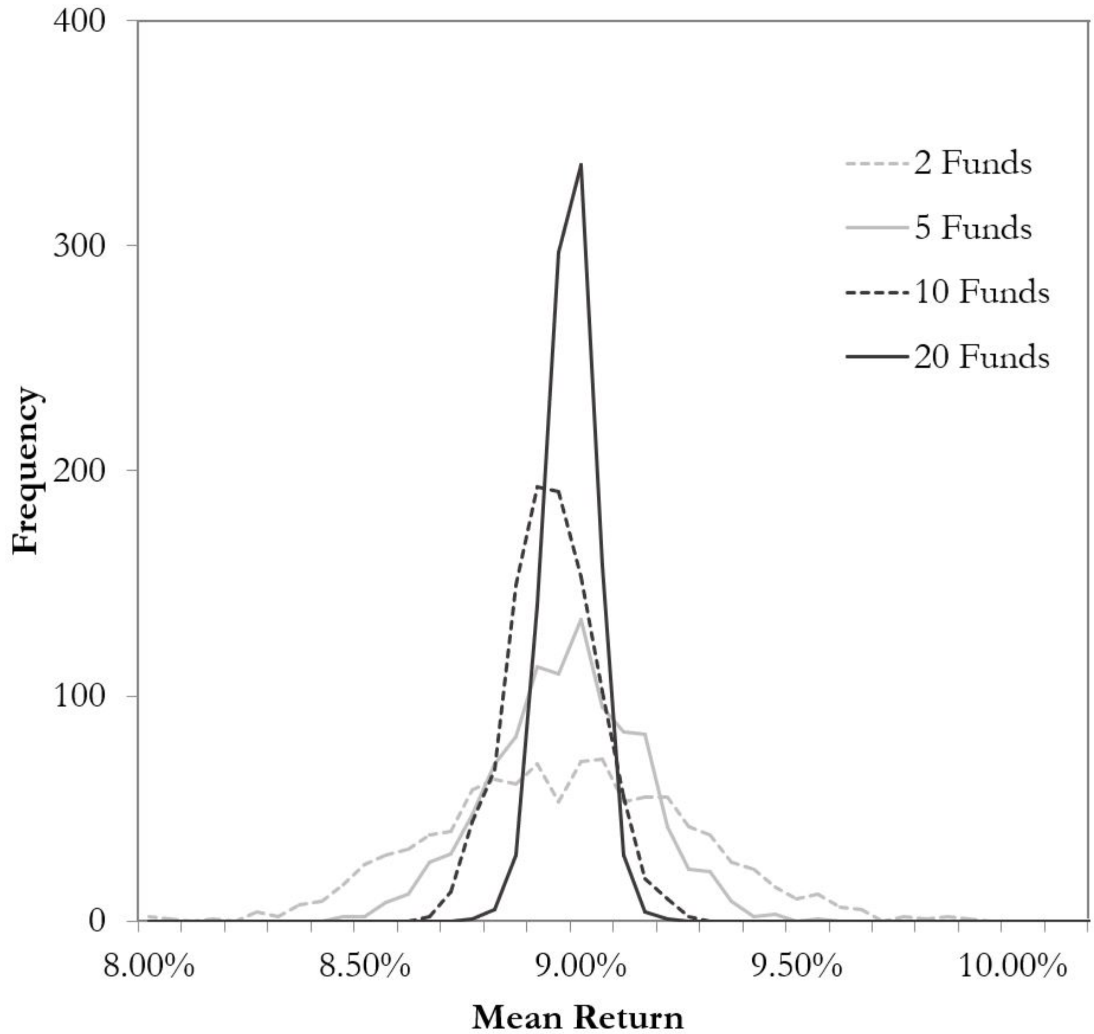
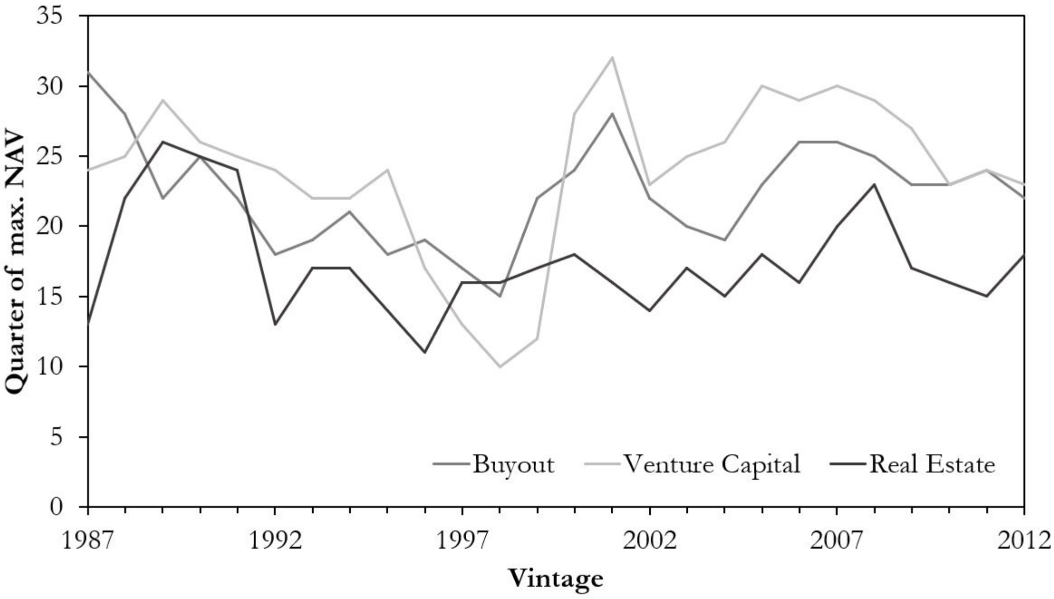


Figure 7: This figure shows the effect of within-vintage diversification. Each vintage, the investor commits to 2, 5, 10 or 20 buyout funds (or the maximum number of funds available). Distribution are shown for a total of 1,000 simulations.



(a) Median quarter of maximum NAV per vintage

Correlation Matrix			
	BO	VC	RE
BO	1.00	0.60	0.27
VC		1.00	0.35
RE			1.00

(b) Correlation in maximum NAV quarter

Figure 8: Panel (a) of this figure shows the cyclicality in maximum fund valuation over vintage years. Vintage years are defined by the date of the first investment by a fund. The last years of the sample are excluded as the maximum valuation can not clearly be identified early in a fund's life-cycle. Panel (b) shows the time-series correlation of maximum NAV for the among private market funds.