Does Private Equity Over-Lever Portfolio Companies? *

Sharjil M. Haque †

University of North Carolina at Chapel Hill

Job Market Paper

This Version: November 15, 2021. Latest version available here

Abstract

Detractors have warned that Private Equity (PE) funds tend to over-lever their portfolio companies because of an option-like payoff, building up debt overhang and widespread bankruptcy risks. Drawing on standard trade-off theory, this paper argues PE-ownership leads to higher levels of *optimal* (value-maximizing) leverage. I develop a dynamic trade-off model where a firm's capital structure and default decisions are made by the PE fund manager, whose payoff captures the PE institutional fee structure. PE-ownership can endogenously change tax benefits of debt and expected cost of financial distress though differences in (i) asset volatility (ii) expected future return and (iii) deadweight bankruptcy costs. Key model parameters are estimated using balance sheet data from a large sample of PE-sponsored leveraged buyouts (LBO). I find the estimated model is able to explain both the level and change in leverage ratios documented empirically following LBOs, driven primarily by changes in the portfolio company hypothesized above. Counterfactual analysis reveals significant loss in firm value if PE sub-optimally chose lower leverage. Post-LBO, I estimate Distance-to-Default increases by 28 percent and credit spreads narrow by 3.6 percentage points for the median firm, suggesting concerns about default risks may have been overstated. Additional tests show PE-backed firms receive equity injections from sponsoring funds if they fall into financial distress, corroborating the lower distress cost channel. My results suggest new policies regulating leveraged loans should focus on the borrower's optimal capital structure.

Keywords: Private Equity Funds; Non-Bank Financial Intermediaries; Capital Structure; Default Risk; Financial Fragility; Leveraged Loans; Structural Estimation
JEL Codes: D21, D22, G13, G32, G33

*I am indebted to Gregory Brown and Anusha Chari for their outstanding support and guidance. Many helpful comments and suggestions were received from Christian Lundblad, Elena Simintzi, Andri Babii, Lutz Hendricks, Stanislav Rabinovich, Siddhartha Biswas, Yunzhi Hu, Richard Varghese, Yannick Timmer, Anil K. Jain and seminar participants at Princeton's Young Economist Symposium, 2021 FMA Annual Meeting, Annual PE Research Conference, the University of Aix/Marseille and UNC Chapel Hill.

†Correspondence: `sharjilh@live.unc.edu`, Department of Economics, University of North Carolina at Chapel Hill

1 Introduction

Private Equity (PE) funds have become an important part of the non-bank financial system. At the end of 2020, global assets under management mushroomed to USD 2.6 trillion, with each dollar of equity typically leveraged with almost two dollars of debt.[1] The substantial increase in a portfolio company's leverage following a PE-sponsored leveraged buyout (LBO) has generated considerable controversy.[2] Critics argue PE fund managers over-leverage their portfolio companies, leading to socially inefficient outcomes such as debt-overhang and higher default rates (Andrade and Kaplan (1998), Hotchkiss et al. (2021)). They contend over-leveraging arises from the fund manager's option-like payoff that captures much of the gains from any profits on their investments but largely insulates them from any losses, leading to excessive risk-taking incentives (Axelson et al. (2009), Magnuson (2018)). Policy-makers such as senator Elizabeth Warren has demanded that PE funds should be "put on the hook for the debts of the companies they buy".[3]

In this paper, I ask "Do private equity funds systematically over-lever portfolio companies?". Since debt-holders are not naive investors, I argue PE-ownership leads to higher levels of *optimal* (value-maximizing) leverage because the expected cost of financial distress is lower and benefits of debt is higher, consistent with the canonical trade-off theory.[4] Studying this mechanism is challenging since we do not readily observe *optimal* leverage. Existing papers empirically examining leverage in PE rely on standard regressions of leverage on a number of factors that proxy costs and benefits of debt.[5] Given adequate choice of determinants, this approach is expected to explain most of the variation in leverage. However, the regression approach cannot detect if firms have too much debt or too little debt on average (Korteweg, 2010), which is central to this paper. Moreover, it cannot incorporate the endogeneity of the bankruptcy decision, which affects expected distress costs and leverage choice.

Consequently, I take a value-maximization approach. I introduce key features of private equity into a dynamic trade-off model of capital structure and examine evidence of over-leveraging by comparing estimated optimal leverage with actual data from a large and

[1] This figure refers to PE buyout funds only, which execute levered buyouts. Source: Preqin.

[2] In a traditional LBO, a PE fund buys majority control of an existing firm using relatively small share of equity and large portion of debt, assumed by the portfolio company (Kaplan and Stromberg, 2009). Brown (2021) and Axelson et al. (2013) document leverage nearly doubles in firms following LBOs.

[3] 2019, End Wall Street's Stranglehold on Our Economy, July 18th

[4] The trade-off theory suggests a firm's leverage choice optimally balances the benefits of debt such as tax shields and agency benefits from lower free cash flow against the direct cost of bankruptcy (Warner, 1977) and indirect costs such as debt overhang (Myers, 1977) and risk-shifting (Jensen and Meckling, 1976).

[5] See for example Axelson et al. (2013); De Maeseneire and Brinkhuis (2012); Guo et al. (2011); Kaplan and Stein (1993).

representative sample of levered buyouts. Through the lens of the model, I identify key factors that could explain a different level of optimal leverage and quantify the endogenous change in tax benefits and expected costs of debt following PE intervention. Given this unique setting, I also comprehensively examine PE investors' role in bankruptcy risk and financial fragility. To the best of my knowledge, this is the first paper to structurally estimate optimal leverage and default risk in PE-owned companies.

The salient feature of the model is that capital structure and any subsequent default-related decision is made by the PE fund manager, or General Partner (GP). The fund manager receives a unique portfolio company management fee (performance-invariant compensation) and an incentive fee that is contingent on the value of the portfolio company. The fund manager is protected by limited liability, which acts as a source of financial friction. The fund manager chooses how much defaultable debt to issue by optimally balancing tax and agency benefits with expected distress costs. Once debt is issued, the manager chooses an optimal default boundary to maximize equity value, which maximizes their dynamic incentive fees. In the model, expected distress costs depend on factors such as the portfolio company's future expected growth rate, asset risk and deadweight bankruptcy costs. Fund managers can improve efficiency and profitability (Kaplan and Stromberg, 2009), but may also over-invest due to limited liability and the call option-like payoff (Axelson et al., 2013). To capture these opposing forces, I introduce a PE-specific excess return on the firm's expected growth rate. Second, PE can lower deadweight bankruptcy costs by reducing asymmetric information between lenders and portfolio firms (Ivashina and Kovner (2011)). Finally, PE-ownership can lead to different levels of asset volatility (risk), capturing changes in agency costs resulting from asset substitution. In addition to leverage and the default boundary, credit spread is also determined endogenously.

I solve and structurally estimate the model using bayesian estimation methods that account for parameter uncertainty. Specifically, drawing on pre-LBO and post-LBO balance sheet data of PE-owned firms, I jointly estimate key parameters such as asset volatility and excess return which are central to optimal leverage.

I find the estimated model can explain both the level and change in leverage that we see in the data following an LBO. Decomposing the relative importance of key model primitives suggests lower asset volatility, followed closely by greater expected future return, are primarily responsible for the endogenous reduction in expected distress costs, leading to at least a 60 percent increase in optimal leverage post-LBO.[6] Reduction in deadweight

[6]The reduction in asset volatility is consistent with previous findings that show firms expand both their product basket and geographic product market following PE-takeover, thus diversifying risk. For details

costs of bankruptcy also raise optimal leverage, although the impact is relatively smaller. In other words, since bankruptcy is both less likely and less costly, higher leverage is optimal. To mitigate selection concerns that fund managers may be targeting companies that have low risk or high expected growth rate, I also examine a set of matched control firms and show these non-PE firms do not experience the same effects as PE-backed firms do. Furthermore, I also verify that my results are not confounded by variations in economy-wide credit conditions (Axelson et al., 2013, Malenko and Malenko, 2015).

My analysis proceeds in two steps. First, I develop a quantitative model of optimal capital structure and credit risk. To this end, I extend the classic Leland (1994) model of firm value and endogenous default where the agent (i.e. the GP) maximizes their expected payoffs. To remain consistent with standard features of an LBO, I also introduce a simple mechanism that allows a PE investment to be liquidated within a finite horizon. Consistent with previous studies, by maximizing firm value the equityholder internalizes debtholders' value at initial date 0. However, in choosing default times they ignore the considerations of debtholders, creating the usual conflict of interest between equity and debtholders (Bhamra et al. (2009), Strebulaev and Whited (2012)). Second, to estimate the model, I construct a large, international dataset of PE-backed firms, for which I observe panel information at the company-year level pre and post-LBO. My data matches median and inter-quartile leverage ratio in the Brown (2021) sample, a benchmark paper in the literature considered to have high-quality proprietary data of PE portfolio companies.

Given the model is quantitatively consistent with the data, I study the following counterfactual: how large is the cost if GPs sub-optimally chose lower leverage ratios? I find the median firm stands to lose as much as 5.8 percent of firm value from choosing a leverage ratio equivalent to half of its model-estimated optimal leverage. When I re-estimate this counterfactual in a simulated low-risk economy, this loss in firm value rises to 9.2 percent. In other words, the cost of choosing sub-optimally low leverage is much higher when aggregate uncertainty is low. This policy experiment indicates existing financial regulation, especially micro-prudential supervision, can be further strengthened by incorporating measures of *optimal* capital structure in the case of PE-sponsored leveraged loans.[7]

My second main contribution is to quantify PE's role in raising default risk and financial fragility. The European Central Bank has raised concerns related to financial stability from highly-leveraged PE transactions.[8] To this end, I estimate Distance-To-Default (DTD) and credit spreads using structural parameters from the model. I estimate both at an annual

see Fracassi et al. (2020). Similarly, Edgerton (2012) documents lower agency costs following LBOs.

[7]See Acharya (2013) for a discussion on micro-prudential supervision.

[8]See Giuzio et al. (2020) for a discussion on financial stability concerns due to private equity.

frequency for PE-backed as well as *matched* public non-PE companies. Following the systemic risk literature (e.g. Saldias (2012)), I also introduce a firm-size weighted measure of Distance-To-Default aggregated each year relative to a buyout since large firms are more likely to create risks related to financial stability. I find that median DTD rises in both the cross-section and the time-series, implying a *reduction* in the probability of default. The firm-size weighted DTD measure reveals even larger reductions in default probability. The median PE-backed firm experiences a narrowing of spreads from 6.28 percent to 2.7 percent which is also consistent with post-LBO data. On the other hand, matched control firms experience very little change in Distance-To-Default and credit spreads at the median. Thus, consistent with the finding that observed higher leverage is optimal, this analysis suggests previous concerns in the literature about PE's role in raising default risk and financial fragility may be overstated.

To corroborate findings from my model, I also provide reduced-form empirical evidence that supports the view that PE lowers expected distress costs and leads to higher tax benefits of debt. I use a difference-in-differences specification with firm and time fixed effects to show PE-owned companies generates higher returns relative to matched controls, which directly translates to higher profitability and greater tax benefits following the canonical predictions of Jensen (1986).[9] Second, I show that PE-backed firms receive greater equity injections when they fall into financial distress relative to matched control firms. This finding supports the view that PE-funds have "deep-pockets." Since funds are raised from institutional investors in the form of commitments that are invested over a series of years, GPs can make equity injections in their portfolio companies when accessing other sources of capital are difficult; this finding is novel.[10] Equity injection during financial distress reduces debt overhang and expected cost of distress as suggested by Hotchkiss et al. (2021).

In a final extension to the benchmark model, I show that only when a PE-backed company does not have access to equity capital from the fund (or equivalently cost of equity issuance is very high) to meet liquidity shortfalls, optimal leverage is lower. In this case, the GP finances the liquidity shortfall by raising additional debt which leads to higher debt-servicing costs. This mechanism drives ex ante expectations and result in more conservative capital structure choices. An implication is that for relatively more

[9]Specifically, if PE makes firms more profitable, greater debt will lower problems associated with greater free cash flow whereby managers invest available funds in low-return projects instead of paying out security holders.

[10]While Bernstein et al. (2019) find systematic evidence consistent with the equity-injection view only during the Great Financial Crisis, this paper is the first to document this mechanism at a more general level (i.e. whenever a portfolio company is in distress, regardless of aggregate macroeconomic conditions).

capital-constrained funds (e.g. new funds set up by first-time GPs), optimal leverage in portfolio companies may be lower and these funds are more likely to over-lever their investments.

Related Literature: This paper builds on several strands of literature: (i) Debt and Leverage in PE-sponsored buyouts, (ii) Structural models of capital structure, (iii) Default Risk and Financial Fragility, and (iv) Incentives of Non-Bank Financial Intermediaries.

First, my paper contributes to the large literature on debt and leverage private equity-sponsored leveraged buyouts. To the best of my knowledge, this is the first paper that structurally estimates the level of leverage in PE-backed companies that maximizes expected value and compares with data from a large and representative sample of buyouts. To date, papers investigating capital structure in PE rely on regression-based approaches. For example, Brown (2021) and Axelson et al. (2013) assess determinants of capital structure using rich multi-country sample of LBOs. De Maeseneire and Brinkhuis, Boucly et al. (2011) and Shive and Forster (2021) take similar approaches and rely on difference-in-differences regressions of leverage on a set of variables that proxy for the costs and benefits of debt. Guo et al. (2011) calculate the difference between actual interest deductions from higher post-buyout leverage from hypothetical tax payments if a firm had not been taken over by PE. Without identifying if a firm was under-levered or over-levered, tax benefits from increased leverage maybe systematically biased. I build on this literature by estimating leverage through a structural model that captures both the endogeneity of bankruptcy and the possibility that a firm may not be at the optimal level of leverage in a given instant in time. Additionally, little is known about how much value is destroyed if PE fund managers sub-optimally chose lower leverage ratios. I can use my model to answer the effects of such counterfactual leverage policies.

Second, my paper contributes to the theoretical capital structure literature, particularly Leland (1994), Leland and Toft (1996), Leland (1998) and Goldstein et al. (2001). Traditional capital structure models cannot be directly applied to portfolio companies in PE for two key reasons. First, while the fund manager has an equity claim on the firm's cash flows, it also receives additional fees unlike a standard CEO. Second, buyout investments have finite durations after which a deal is liquidated whereas in a standard firm value model, the CEO controls the firm forever. Without accounting for these differences, it is difficult to argue we are capturing the PE fund manager as the manager of the firm rather than a traditional CEO. To the best of my knowledge, this is the first paper that offers an analytical framework that embeds a PE fund manager's payoff, finite maturity horizon of an LBO and alpha-generation ability into a traditional capital structure model. To the best of my knowledge, the only two papers that offers theoretical understanding of leverage in PE and are related to mine is Axelson et al. (2007) and Malenko and Malenko (2015). However, neither paper structurally estimates optimal leverage, which may in part be due to the fact that their models do not lead to analytically tractable closed-form solutions for debt and equity payoffs. Finally, neither paper considers the role of PE-specific effect on the firm's cash flow growth rate, which is an element I also introduce similar to Lan et al. (2011).

Third, my paper also contributes to the literature on default risk and financial fragility. While a large literature exists on the role of hedge funds and exchange-traded funds in financial stability, very few papers examine financial stability implications from private equity funds using structural models. By extension, there is no clear understanding of PE's contribution to default risk which is of particular concern to policy-makers. The need for structural models of credit risk to understand private equity's contribution to financial fragility is vital since bankruptcy is endogenous and fund manager's capital structure choice is potentially forward-looking. I contribute to the structural credit risk literature of Bharath and Shumway (2008), Merton (1974), Leland (1994) and Leland and Toft (1996) as well as the private equity literature by estimating default probability in the PE industry. To the best of my knowledge, there is no structural model-based measure of default probability in private equity, a key gap in the literature this paper aims to fill. I provide a comprehensive analysis of default risk both in the cross-section and in the time-series taking into account larger firms may have greater implications on aggregate financial sector stability. Moreover, I use a unique firm-size weighted measure of default risk to examine evidence of systemic risk rising from LBOs.

Finally, I also shed light on incentive schemes governing private equity buyouts. Fund managers receive both a portfolio company management fee (Phalippou et al. (2017)) and an incentive fee which is directly contingent on the underlying asset value. Sorenson et al. (2014) is the closest paper in this aspect. However, in their model leverage is exogenous, while I develop a framework that embeds GP fees with endogenous leverage and credit spreads.

Paper Roadmap: The rest of the paper is organized as follows. Section 2 develops a structural PE model of firm value and optimal leverage. Section 3 describes the structural estimation strategy. Section 4 presents the data, construction of control group and summary statistics. Section 5 presents the benchmark results of the model. Section 6 details additional model results on default risk and financial fragility. Section 7 presents reduced-form tests related to difference-in-differences estimates corroborating key model mechanisms. It also presents the extension of the model where PE-backed firms no longer have access to a GP's "deep-pockets". Section 8 concludes.

2 A Model of Optimal Capital Structure in PE-owned Firms

The starting point of this paper is the observation that higher observed leverage in PE-backed companies can reflect over-leveraging or higher levels of optimal leverage, where over-leveraging is the difference between actual leverage and the level of leverage that maximizes a firm's expected value.[11] In this section, I develop an analytically tractable model of optimal leverage in PE. The economic intuition behind potential effect of PE on leverage can be illustrated using a simplified, static version of the trade-off model. As shown in Figure A1 in Appendix B, the level of leverage that maximizes expected value depends on the flow of benefits from tax shields and potential default costs. It is straightforward to see a reduction in distress costs (or rise in tax benefits) will shift the value-maximising level of leverage further to the right.

My objective is to extend the canonical model of firm value and endogenous default of Leland (1994) in order to explain the large difference in leverage ratios following PE-takeover. My model differs from the canonical framework in terms of capturing the GP's payoff (portfolio company management fee plus incentive fee), excess return which can be characterized as effort, and an exogenous deal liquidation probability that gives the LBO an implied average finite maturity. The GP acts as the manager (CEO) of the portfolio company. Three key factors that can potentially change optimal leverage for PE-specific firms are: (i) risk (standard deviation) of the asset return, (ii) expected growth rate of after-tax cash flows and (iii) deadweight costs of bankruptcy.[12] These "deep" parameters can affect the probability and cost of bankruptcy and lead to different levels of expected distress cost, thus changing optimal leverage.

2.1 Environment

Consider a continuous-time infinite-horizon model. Markets are complete, and I assume there exists a risk-neutral measure with risk-free rate r. Everything is observable implying there is no private information. Following Leland (1994), the value of corporate securities depend on underlying firm value but are otherwise time-independent.

All agents are risk-neutral. There is an infinitely-lived agent, called the GP, who can act as owner-manager in companies. At time t_0, an investor called a Limited Partner (LP) commits capital worth I_0 to the GP, where $m \in (0, 1)$. mI_0 captures an exogenous portfolio

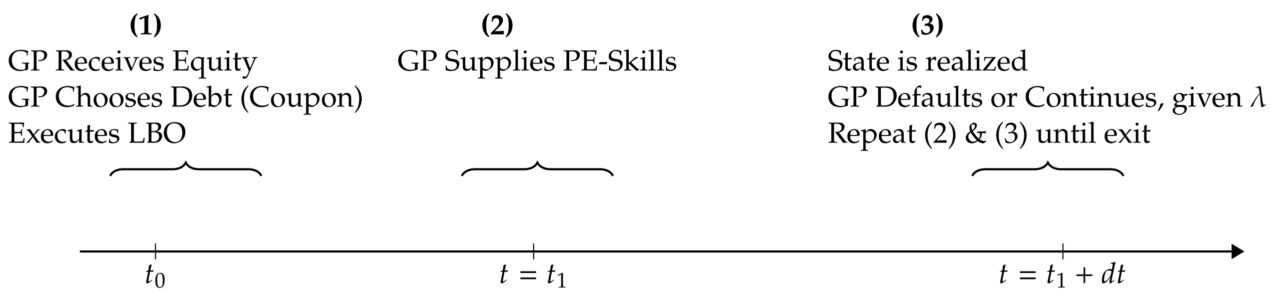
[11] It can reflect some combination of both, but we are interested in the dominant effect if there is one.

[12] While tax rates can also have significant effect on leverage, I do not focus on tax rates since both public and PE-owned companies face the same tax structure. Thus, tax rates are unlikely to explain any mechanism that leads to higher optimal leverage in PE.

company management fee and is discussed below when I outline the GP's compensation. The GP is left with I_0 initial investment. The GP leverages this amount with a perpetual bond by choosing C_0 coupon amount, in order to receive debt with market value D_0. The GP acquires a portfolio company worth $A_0 = D_0 + I_0$. In practice, a GP manages a PE fund and acquires 10-20 portfolio companies, but for tractability I develop my model at the deal-level.[13]

Stochastic Deal Exit Probability: I introduce an exogenous deal liquidation probability, λ, per unit of time. Let T denote the stochastic moment at which deal liquidation, or deal exit occurs. This assumption implies that the PE deal has a finite average duration and also keeps the model stationary and analytically tractable. λ is set such that the implied average duration of a deal is 10 years. Upon deal exit, the investors receive their committed capital. Figure 1 below outlines timing assumptions of a standard LBO:

Figure 1: Timeline of a Representative Deal



Following an acquisition, the GP supplies unique "PE-skills" that delivers an excess return. On the other hand, GPs may execute value-decreasing investments. These opposing forces will be captures as described below. Upon realizing the state, the GP chooses to default or continue operations based on the optimal default trigger conditional on the deal not being stochastically liquidated. The average life of the deal is determined by the stochastic exit probability, λ, outlined above.

2.1.1 Firm's Production Function

Without leverage, the portfolio company's asset-in-place generates value which evolves according to a return process represented by standard Geometric Brownian Motion (GBM) with expected return μ_a, volatility σ_a or the standard deviation of asset return and cash

[13]Consistent with Axelson et al. (2009), an implicit assumption in the model is that the GP cannot earn positive economic profit by simply storing money at the risk-free rate or fairly priced publicly traded asset. In other words, the model does not allow for private savings where the GP makes money using passive investment strategies.

payout rate δ:

$$dA_t = A_t[\mu - \delta]dt + A_t \sigma dB_t \tag{1}$$

where μ, σ and $\delta > 0$.[14] Sorenson et al. (2014) assume fund managers can deliver a higher risk-adjusted expected excess return that captures improved efficiency, managerial expertise, specific business knowledge etc. On the other hand, their option-like compensation may also lead to over-investing (Axelson et al., 2013). To capture these opposing effects I introduce a PE-specific excess return called $\alpha \in \mathbb{R}$. α can be both positive or negative and will be calibrated from the data. One interpretation of Eq. (1) is that the firm produces one unit of good per unit of time with market price fluctuating according to GBM. Net cash outflows may occur due to dividend payout to shareholders and/or since after-tax interest expenses are being paid without the canonical Leland (1994) assumption of fully offsetting equity financing. For ease of notation, I will supress the asset-to-cash conversion rate δ since it is constant. We then define the unlevered value of assets as the expected value of future discounted cash flows that these assets will produce:

$$E_U(A) = \mathbb{E}\left(\int_t^{\infty} e^{-r(s-t)} \delta A_s ds\right) = \frac{\delta A}{r - \mu} \tag{2}$$

Eq. (2) essentially captures so-called Earnings before Interest and Taxes. Figure A1 in Appendix B summarizes key economic activities representing a PE investment while distinguishing between activities at the deal-level and at the fund-level. PE-skills are flows from the GP to the company and results in an excess return called α. Net cashflows are cash flows net of interest payments. Observe that bond-holders are being repaid by the portfolio company while the debt is initially raised by the GP, reflecting the possibility of over-leveraging. A key point is that the GP has to improve company performance in order to earn profits. With the existing setup, the model incorporates these opposing forces on the GP's incentives.

2.2 Payoff to Debt and Equity Holder

2.2.1 Debt Value

Using standard pricing equations, we can then obtain explicit closed-form solutions for payoffs to each type of claimant (equity and. debt holder). Over each time interval $[t, t+dt]$ the firm is servicing its debt holders by paying coupon C. If $A_t < C_t$, the company is

[14] Recall B_t is a standard brownian motion (wiener process) where increments in each time period t is independent of past values, and the increments are normally distributed with mean 0 and variance u. In other words, it represents random shocks to the firm's fundamental. Initial value is typically set at 0. For further intuition in the context of capital structure see He (2012).

in distress, and receives equity injection from the PE fund to finance liquidity shortfall; if the company cannot get external financing or chooses not to, it defaults. Debt allows companies to exploit tax-shields, but it also comes with non-negligible costs. Higher leverage increases the expected cost of distress, and makes further debt issuance costlier. When choosing the optimal debt policies, companies trade off these costs and benefits.

I define ρ as the fraction of asset value A_B which is lost in the event of bankruptcy. Details of the derivation expressing debt-holder's payoff is shown in Appendix B1. Below, I outline the HJB equation for debt-holders:

$$rD(A) = c - \frac{\partial D(A)}{\partial t} + (r + \alpha - \delta)A_t \frac{\partial D(A)}{\partial A} + \frac{1}{2}\sigma^2 A_t^2 \frac{\partial^2 D(A)}{\partial A^2} \tag{3}$$

The first term on the right hand side express flow payoff to debt holders. Since the fund manager issues a perpetual bond resulting in time-independence of debt value, the second term is equal to 0. The last two terms on the right-hand side express the expected change in the value of debt as the underlying asset value fluctuates. Note the debt-holder internalizes the fact that the fund manager can change growth through $\alpha \neq 0$.

2.2.2 Equity Value and Endogenous Default Boundary

Now I outline the value process for the GP's payoffs, who is the owner-manager of the firm. In traditional capital structure models the equity-holder's payoff resemble a plain vanilla European call option in the sense that claimants cannot have negative equity. While the GP's payoffs are similar, the GP receives a portfolio company management fee which is senior in nature and is invariant to default probability. Additionally, the GP receives a share of profits as incentive fee. In practice, profit-sharing involves both a catch-up provision and a carried interest share percentage.[15] I ignore this distinction since both are essentially profit-sharing mechanisms and does not differentially impact the GP's optimization problem, which as will be described below, involves maximizing equity value with respect to an endogenous default boundary once debt is issued.

The incentive fee gives the GP upside potential. To keep the framework tractable, I will assume the GP receives a share of firm profits in each time interval (apart from t_0) as the incentive fee. Note that incentive fees are always non-negative. Hence, the GP's compensation can be outlined as follows:

$$F(A; A_B; C) = \underbrace{M(V; m)}_{Management\ Fee} + \underbrace{P(A_t, C, A_B)}_{Incentive\ Fee} \tag{4}$$

[15] See for example Sorenson, Wang and Yang (2014) for details.

Present value of management and incentive fees are, respectively:

$$M(I_t, m) = \mathbb{E}[\int_t^T e^{-\beta(s-t)} m I_0 \, ds)] \tag{5}$$

$$P(c, A_B, A_t) = \mathbb{E}\left[\int_t^T e^{-\beta(s-t)} \, max\{k[A_t - (1-\tau)C \ , 0]ds\}\right] \tag{6}$$

Recall management fee is an exogenous annual rate that pays a constant fraction of capital received from investors, mI_0. k is a fraction of profits the GP will receive net of debt payments. As I will show in deriving the payoff function, the share of profits the GP receives will not affect the optimal default-triggering asset level, and by extension, will not affect the choice of leverage. Without loss of generality, I assume $k = 1$, essentially capturing the spirit of a catchup provision.

Limited liability is the underlying financial friction. The embedded option in this payoff structure has two opposing implications: (i) Lan et al. (2013) suggest the risk-neutral fund manager has incentives to maximize the portfolio company's going concern so as to collect performance fees; (ii) Alternatively, Axelson et al. (2013) argue funds may be tempted to take excessive risk when they are compensated via incentive fees and protected by limited liability. Risk-shifting and related agency costs will be captured through the estimated asset volatility parameter. By construction, σ will capture which of the two effects outlined above will dominate.

Equity value can be computed through the following ordinary differential equation that equates the required rate of return for the GP with the expected rate of return on equity, which is the sum of the terms on the right hand side.

$$(r+\lambda)E(a) = mI_0 + k\big[A_t - (1-\tau)C + (r+\alpha-\delta)A\frac{\partial E}{\partial A} + \frac{1}{2}\sigma^2 A^2 \frac{\partial^2 E(A,t)}{\partial A^2}\big] \tag{7}$$

The discount rate on the left-hand side is elevated by the exogenous deal exit probability. The first term on the right-hand side captures a portfolio company management fee (Phalippou et al., 2017) and the second term captures profit-sharing. The third and fourth term capture the expected change in equity value caused by a fluctuation in the firm's asset value.

Details of the derivation of closed-form expressions for the GP's payoffs and endogenous default boundary is shown in Appendix B2. Using closed-form solution to the payoff function, we can obtain a standard expression for the endogenous default barrier, outlined in Eq. (8) below.

$$A_B = \big((1-\tau)C + mI_0\big)\frac{r+\lambda-\mu}{r+\lambda}\frac{\gamma}{1+\gamma} \tag{8}$$

The bankruptcy-triggering cash flow level differs from traditional capital structure models through the term $r+\lambda$, which is elevated through the stochastic deal exit probability. Additionally the risk-free asset management fee appears in the expression, mechanically raising the optimal default barrier. However, it is straight forward to see firm leverage (coupon) does not depend on the carried interest share percentage but depends on the fact that the GP can take away strictly positive payoff regardless of the performance of the company. This result helps explain invariance in the carried interest share percentage (almost universally set at 20.0 percent in the PE industry).

2.3 PE Fund Manager's Decision Problem

In standard corporate finance models, firms make their leverage and default decisions by balancing benefits of debt against the cost of default, with the objective of maximizing equity value. Consistent with the literature, the PE fund manager (GP) maximizes lifetime fees by maximizing equity value of the underlying portfolio company, since incentive fee is an equity claim on the underlying asset. The optimization problem assumes that incentive fees are non-decreasing in A_t. The PE-model in this paper differs along the dimension of the risk-free asset management fee, liquidation probability, PE-specific return and default boundary.

The GP's problem can be divided into two steps. At date 0, the GP chooses debt (coupon in the case of perpetual bonds) that optimally weighs the tax benefits and distress costs of debt. This is equivalent to maximizing value of the levered firm as shown in Eq. (9). Once debt is issued, it becomes a sunk benefit and thus, in the second step, the GP only maximizes equity value in choosing an optimal liquidation time as shown in Eq. (10). [16]

$$\underset{C_0}{max}\ \big(D(A_0, c_0) + E(A_0, c_0)\big) \tag{9}$$

$$E^L(A) = \underset{T_L \in F^{A_B}}{sup}\ \mathbb{E}\Big[\int_0^{T_L} e^{-rt}(mI_0 + (1-\tau)(A_t - c))dt\Big] \tag{10}$$

subject to the the return process of the fundamental value of the firm in Eq. (1). T_L is a stopping time with respect to the filtration F^A generated by A_t. It is standard to show

[16] See Strebulaev and Whited (2012) for further discussions on how this creates the standard ex ante vs. ex post conflict of interest in initial capital structure.

that $T_L \equiv inf[t : A_t \leq A_B]$. Eq. (10) is essentially equivalent to maximizing Eq. (7) with respect to the bankruptcy boundary A_B. For notational convenience I have suppressed λ in Eq. (10). We then compute the leverage ratio L_i that maximizes expected value for firm i as:

$$L_i = \frac{D(A, A_B, C_0^*)}{D(A, A_B, C_0^*) + E(A, A_B, C_0^*)} \tag{11}$$

Definition of Equilibrium. *A "Rational Expectations Equilibrium" is a collection of processes* $\{A_t, \mu_t, \sigma_t, r_t, \delta_t, \rho\}$ *that satisfy Eq. (1), Eq. (8),* $A_0 > 0$ *and debt is priced competitively at all dates* $t \geq 0$.

2.4 Comparative Statics and Empirical Predictions

With the model I have presented, we can obtain empirically testable predictions about the effects of PE-ownership on optimal leverage. In the baseline model, I focus on the impact of PE-ownership on three key factors that can potentially lead to different levels of optimal leverage: (i) asset risk, (ii) expected future return and (iii) deadweight costs of bankruptcy.

Prediction 1. (Lower Asset Risk and Distance to Default) *PE-owned companies are characterized by lower asset risk,* σ*, which increases distance to default* $\frac{A_t}{A_B}$ *leading to higher levels of optimal leverage, ceteris paribus.*

Lower risk captures aspects such as decrease in agency costs or related asset substitution that are prevalent in comparable public companies that suffer from governance problems. Alternatively PE may identify firms with lower risk or bring operational efficiencies that decrease risk. A negative relationship between leverage and riskiness of the firm is standard in the capital structure literature. Leland (1994) documents a negative relationship for reasonable values of risk. A negative relation between buyout leverage and asset risk is also consistent with the recent findings of Brown (2021).

Prediction 2. (Higher Expected Return and Distance to Default) *Consider any benchmark growth for all firms denoted* ϕ*. Without loss of generality set* $\phi = 0$*. If* $\alpha > 0$*, PE-ownership leads to a higher risk-adjusted expected growth rate,* $\phi + r$*, raising* $\frac{A_t}{A_B}$ *which raises optimal leverage.*

In the theoretical framework, I allow α to take any value to capture both benefits and risks of PE-ownership, and estimate α from the data. Standard models of PE rest upon the assumption that fund managers can bring operational efficiencies or managerial skill that leads to a unique excess return; a notable example is Sorenson et al. (2014). This can be justified by higher tax benefits of debt or through a lower default barrier. By extension, profitable firms face lower expected costs of financial distress and find interest tax shields more valuable. Thus, both the tax and the bankruptcy costs perspective predicts that profitable firms can optimally use more debt. In addition, the agency costs perspective predicts that the discipline provided by debt is more valuable for profitable firms (Frank and Goyal, 2009).

Prediction 3. (Lower Bankruptcy Cost) *Bankruptcy costs include costs involved in selling the firm's real assets to second-best customers during bankruptcy, loss of customers due to expectation of bankruptcy, lost managerial time in administrating a bankruptcy and legal fees. By lowering information asymmetry, PE-ownership lowers deadweight loss in bankruptcy, raising the optimal leverage ratio.*

Leland (1994), Leland (1998) and Uhrig-Homburg (2004) among others show that leverage is decreasing in bankruptcy costs. During bankruptcy, assets are sold at fire-sale prices which represent a deadweight cost. Andrade and Kaplan (1998) show that direct costs of bankruptcy is lower in PE-owned firms relative to comparable controls.

An implication of predictions 1-3 is that PE-backed firms should have similar (or lower) probability of default relative to comparable public companies. By raising firm value, higher optimal leverage ensures a firm's asset value is at least as far from its default barrier before the leverage "shock" from the LBO. Thus PE-ownership ensures there is a sustained ability to service the debt ensuring default likelihood does not spike. Alternatively, if higher observed leverage in PE-backed firms is sub-optimal, we should observe higher default probability relative to comparable controls on average.

3 Structural Estimation

3.1 Estimation Strategy and Empirical Specification

To facilitate estimation, I discretize the state space. Estimation follows two steps. In the first step, parameters that do not require the model structure such as tax rate are calibrated outside of the model or borrowed from the literature. For the corporate income tax rate, τ, I use 35 percent consistent with Leland (1994). I assume deadweight costs of bankruptcy is 0.35 or 35 percent pre-LBO, which is consistent with papers in capital structure theory. I reduce this parameter to 0.2 or 20 percent following PE-acquisition. For robustness, I repeat the analysis with various other calibrated values of rho randing from 0.25 to 0.5. The reduction in deadweight default cost is consistent with the literature that documents PE fund managers have closer relationships with debt-holders (e.g. Ivashina and Kovner (2011)).

In the second step of the estimation procedure, I estimate asset volatility (σ), excess

Table 1: Summary of Externally and Independently Estimated Parameter

Parameter	Value	Source
τ	0.35	Standard
r	-	Domestic Short-Term Treasury Rate, IMF
A_0	100	Standard
I_0	50	-
m	0.06	Phalippou et al. (2017)
ρ_{pre}	0.35	Standard
ρ_{post}	0.20	Andrade and Kaplan (1998)

return (α) and cash payout rate (δ).[17] Return R_{it} is estimated as a function of observable firm variables (size and leverage) that can affect drift, and an indicator function $1 \times [PE_{it}]$ that captures unique PE-effect. The indicator function will capture α. To capture selection effects, the minimization function below in Eq. (12) will be estimated jointly with matched non-PE owned companies. I measure R_{it} using Return on Capital, defined in Appendix table A1.

I follow Bartram et al. (2015) and estimate asset volatility by minimizing the squared deviation of predicted equity volatility (σ_e) from actual volatility. Equity volatility is defined as the standard deviation of year-on-year equity returns. I define equity volatility as shown in Eq. (13).

$$R_a = f\left(\mathbb{1}_{\{PE\}}, \mathbf{X}\right) \tag{12}$$

$$\sigma_e^2 = \Big(\frac{A}{E} \times \underbrace{N\big(\big(ln(\frac{A}{D}) + \sqrt{r + 0.5\sigma T}\big)/\sigma\sqrt{T}\big)}_{\frac{\partial E}{\partial A}}\Big)^2 \times \sigma_a^2 \tag{13}$$

The term inside the second parenthesis captures the riskiness of debt and that equity and debt values are correlated. Without this term, the estimates of asset volatility will suffer from a downward bias which becomes more severe as leverage increases. I parameterize asset volatility as a function of observable firm-level characteristics:

$$\sigma_a = \beta_0 + \sum^{n} \beta_i X_i \tag{14}$$

[17]In previous versions of this paper, I had calibrated the excess return, α, based on empirical findings. Sorenson et al. (2014) illustrates the effect of α rising from 0.0 percent to 2.0 perent on the Limited Partner's investment return. In their baseline calibration, they set α to 1.01. For my baseline estimation, I had set α to 1.5 percent. Overall, both my calibration and direct estimation led to relatively similar values for α.

where X_i is a set of covariates including firm size, tangibility, return on capital, debt, and profit volatility; β_i are the estimated coefficients. Next, debt maturity is parameterized between 1 and 10 years using the following expression:

$$T = 1 + 9\frac{Long\ term\ Debt}{Total\ Debt} \tag{15}$$

The face value of all outstanding debt is (P) is calculated as net book leverage. The payout rate δ is estimated as $Cashflow/Assets$. These observable variables allow for the calculation of all other variables in our optimization problem described in detail in Section B of the Appendix. As summarized in previous studies, this estimation procedure calibrates the model in a way that allows for the endogenous nature of the bankruptcy decision. Finally, as is standard in the literature, I set the initial asset value to 100.

I estimate the model in the pre-LBO and post-LBO samples for both PE-owned and matched control firms. Since fund managers do not randomly target companies, the control group will alleviate concerns related to selection. I estimate the parameters of the model using Markov Chain Monte Carlo simulation relying on a Gibbs sampling algorithm following Korteweg and Sorenson (2010). This estimation methodology is explained in detail in Robert and Casella (1999) and Johannes and Polson (2004), and in particular for structural models of the firm in Korteweg and Polson (2009). As Korteweg (2010) argues, MCMC provides a way of sampling the posterior distribution of the model's parameters and unobserved variables (the betas and unlevered asset values), given the observed values of debt and equity. The advantage of this method is that the unobserved variables may be simulated alongside the model parameters from their full posterior distribution. As we can replace the unobserved variables by simulated variables, we do not have to evaluate the (unconditional) likelihood function of the model. Once this sample is obtained, the unobserved variables are numerically integrated out, leaving the distribution of the parameters, conditional on the observed data. This integration step needs to be done only once.

By extension, missing observations are handled in exactly the same way as unobserved variables which is a key advantage given that my dataset is an unbalanced panel. This is particularly useful advantage since missing data is standard in corporate finance. Finally, MCMC provides a convenient way of dealing with unobserved heterogeneity when using estimation results for decision-making. For example the choice of risk undertaken by a firm will depend on firm fundamentals characterized by the vector X_i above. The algorithm deploys 12,500 iterations with a burn-in sample of 2,500. All acceptance rates are above 30 percent.

4 Data

4.1 Data Source

The data collection process is divided into three parts. First, I collect private equity deal-level data from Bureau Van Dijk's (BvD) Zephyr Merger and Acquisitions database. Zephyr has been increasingly utilized among PE researchers (Bernstein et al. (2019); Bansraj et al. (2019); Tykvova and Borell, 2012) and has been verified as a comprehensive and representative sample of PE transactions compared with other PE databases such as Standard and Poor's Capital IQ (Jenkinson and Stucke, 2011). I retrieve all Private Equity transactions labelled "Institutional Buyout" [18]. I select all deals from 2000 to 2020.

Second, I match target firms with their company-level accounting data from Orbis, using BvD identifiers. I follow Kalemli-Ozcan et. al (2015) to download and clean financial data for portfolio companies in order to reduce the survivorship bias present in online Orbis download through Wharton Research Data Services. Using this process and removing observations without the required data for the sample period yields a total of around 30,000 firm-year PE observations for 2820 verified and unique LBO deals; LBO verification is discussed in section 2.1.2. This is my starting sample for the treatment group that will be matched to a comparable control group.

Third, I construct a control group by retrieving company-level data of all publicly traded non-PE firms from Orbis for the same sample period. Jensen (1986) and Axelson et al. (2008) argue that PE-backed firms have superior governance to publicly traded firms. By extension, PE-backed firms have different agency costs that can affect capital structure, making publicly traded firms the natural comparison group for the analysis[19]. I require the relevant financial of control companies to be available in Orbis in at least one pre-deal year, where the deal year refers to the year a target was acquired by a private equity firm. However, my identification strategy faces challenges that are typical in empirical corporate finance due to the endogeneity of acquisition decisions. To alleviate selection issue, I match individual companies that are acquired by PE firms with non-acquired companies in the same country, sector and year to control for the common trends in the fundamentals. Details are outlined in Section 3.4.

[18] Zephyr defines this as "an acquisition where a Private Equity firm has taken a 50 percent stake or more in the Target company, or is the parent of the Acquiror. The acquisition often takes place through a 'new company' (newco) or an acquisition vehicle."

[19] Axelson et al. (2013) also use publicly traded firms as the control group.

4.2 Empirical Facts

Table 2 provides descriptive information for the sample that I use in analysis. A cursory glance shows there is large dispersion in the data. For example, the median value of total assets for buyout firms is $ 78 million while the mean is $ 431. The mean value is also similar to Cohn et al. (2014) who use tax return data in U.S. private firms and is significantly higher than the median suggesting there are a large number of small and mid-sized transactions and a few much larger deals. Comparison with other papers using reliable datasets such as Brown (2021) also confirm that the sample is representative of the traditional leveraged buyout universe. The median (mean) buyout firm has Shareholder Equity of $ 42 (168) million respectively.

Table 2: Descriptive Statistics of PE-backed Companies

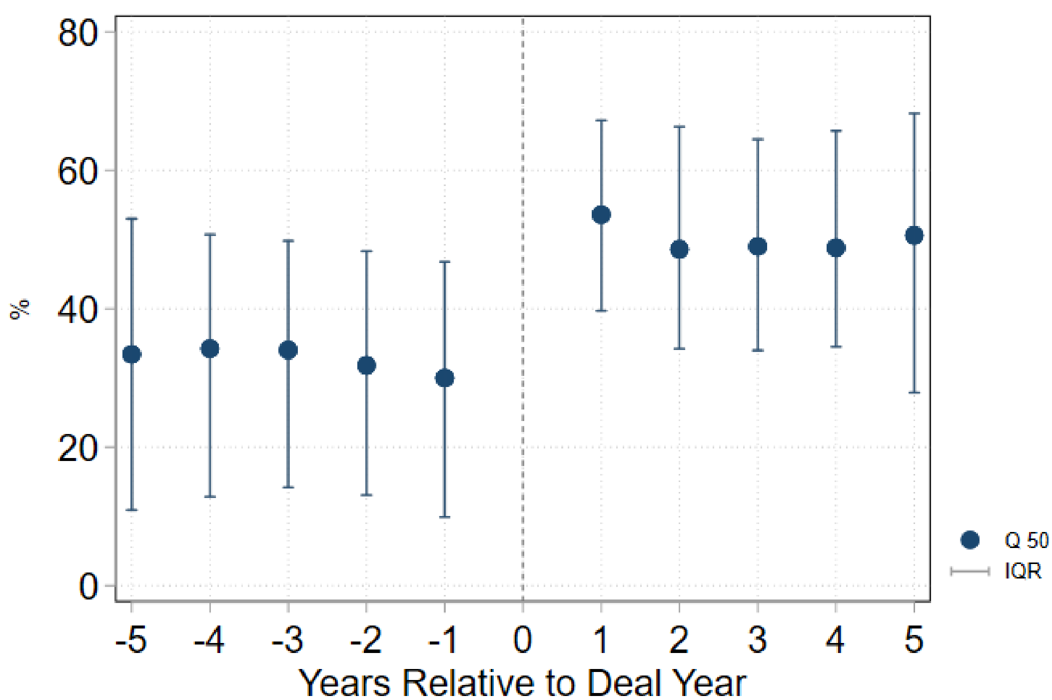
Panel A: Full Sample	N	Mean	Median
Total Assets ($ Mn)	20973	431	78
Net Debt/Asset (%)	19777	35	37
EBITDA Margin (%)	19594	7.08	6.55
ROA (%)	20566	5.6	5.6
Return on Capital (%)	15349	14.7	11.7
Growth in Sales (%)	17218	114	0.1
Panel B: Deal Entry Year			
Total Assets ($ Mn)	2022	306	63
Net Debt/Asset (%)	1677	48.8	49.5
EBITDA Margin (%)	1640	7.62	7.15
ROA (%)	1921	5.5	5.2
Return on Capital (%)	1570	15.7	11.6
Growth in Sales (%)	1084	136	1.4

Notes: This table reports summary statistics of key variables used in the analysis over the entire sample 1997-2020 calculated from BvD data. The observations are at the firm-year level. Panel A reports statistics for the full sample and does not distinguish between pre-LBO and post-LBO samples. Panel B reports statistics only for the deal year. Net Debt is Debt net of Cash and Cash Equivalents. Return on Assets is Net Profit over Assets. Return on Capital is defined as Net Profit over the sum of Equity and Non-Current Liabilities. All variables are winsorized at the 1 percent and 99 percent level.

I define Sales Growth as the one-year percentage growth in Sales. Buyout firms have median (mean) sales growth of close to 0 percent which is likely masking the difference between the pre-LBO and post-PE investment patterns. We also note that a small subset of firms have very high sales growth which has skewed the sample mean. They are reasonably profitable with EBITDA margin of around 7 percent. Next, we see that long-term debt constitutes a fairly low share of capital structure for the full sample. However, this likely masks the change in the level of long-term debt post-LBO. Mean (median) long-term debt value is reported at $ 10.0 (3.25) Mn. There is wide variability in the interest coverage ratio. As can be seen, some firms have very high interest coverage ratios suggesting a combination of high earnings relative to debt servicing requirement while some are much closer to vulnerable levels. Median interest cover stands at 4.3 indicating the firms on average have reasonable debt servicing capacity. Regarding the leverage ratio, since this table masks the change in leverage post-LBO. Median debt ratio is 35 percent which is affected by lower pre-LBO leverage ratios. Mean leverage ratio is quite similar based on the matched sample.

To shed light on the change in capital structure, Figure 2 plots the trend in $\frac{Net\ Debt}{Asset}$ around the buyout event. First, the plot reveals that leverage varies significantly across companies. Second, it reveals sizeable increase in debt following buyout. Median debt ratio rises sharply from 32% in $t = -1$ to 53% in $t = 0$, where $t = 0$ is the buyout year. We note that the debt ratio stays elevated within the 50-55% range for several years following the buyout. The inter-quartile range of entry debt ratio is 37% to 65%. Levels and trends

Figure 2: Key Motivating Fact - Leverage Ratio following Buyouts



Notes: This chart plots median and interquartile range (IQR) of the trend in Leverage Ratio (%) Relative to Deal Year. Leverage Ratio is defined as Net Debt/Asset. Net Debt is Debt net of Cash.

in my main outcomes are consistent with Brown (2021) who use deal-level debt data from StepStone and an anonymous global international bank.

4.3 Matching Procedure and Univariate Tests

Table 3: Covariate Balancing Test and Univariate Comparisons Post-LBO

		Treated	*Control*	*Diff*	*%Bias*	*%Decrease*
Panel A: Balancing Test						
Sales Growth	U	2.80	1.54	(***)	11.60	
	M	2.80	2.87		-0.60	94.8
ROA	U	7.55	4.32	(**)	-6.00	
	M	7.55	7.33		2.70	54.9
Log (Assets)	U	18.50	18.62	(***)	-8.30	
	M	18.50	18.45		2.70	67.9
Log $(\text{Assets})^2$	U	345.08	351.04	(***)	21.00	
	M	345.08	343.17		1.40	93.2
Panel B: Post-LBO Outcomes						
Net Debt/Asset (%)	M	49.8	23.3	(***)		
Interest Coverage Ratio	M	1.96	3.8	(***)		
Return on Capital Employed	M	9.10	3.83	(***)		
ROA	M	3.4	0.2	(***)		
Enterprise Value ($ Mn)	M	679	854	(***)		
Enterprise Value/Asset	M	1.04	1.32	(***)		

Notes: This table reports tests effectiveness of the propensity-score matching algorithm to identify comparable public control firms as well as Post-LBO univariate comparisons between the treatment and control sample. Matching with Propensity Scores is undertaken within every country-sector-year combination using Pre-LBO data. If a match cannot be found in the same year, I expand the search horizon to $[t-3, t+3]$, where t refers to the year an LBO was executed. Panel A reports sample means for treated and control samples before and after matching. U refers to the unmatched sample, M refers to the matched sample. T-tests for differences in means are reported in column (5) and the reduction in bias in column (7) titled "% Decrease". In panel B, I report key firm-level outcomes after a firm is acquired by a PE fund and the corresponding outcome for the matched control sample.

My matching criterion is a more conservative version of those in Boucly et al. (2011) and Bernstein et al. (2019). This is because my matching on observable characteristics is based on a propensity score model. The nature of traditional venture capital and leveraged buyout targets guides my choice of matching variables. Following previous studies, I match on log of total assets, square of the log of total assets, growth in sales and return on assets.

Specifically, I use a logit model to generate the conditional treatment probability (or propensity) of receiving an LBO investment based on observable firm characteristics mentioned above. Next, I match each treated company to control firms within each country-sector-year combination. If I cannot find an acceptable match in the same year I expand my search horizon to $[t+3, t-3]$ where t refers to the deal year. In order to ensure a large control group, I retain the 5 closest neighbors if available similar to Bansraj et al. (2019). This approach allows me to successfully find identical control firms for 1,697 LBO deals (PE-owned companies) in my sample. For this matched treatment sample, I find a total of 5,752 control firms that are in principle similar to PE-owned firms.

Table 3 presents effectiveness of the matching algorithm. As can be seen, the matching algorithm successfully eliminates statistical difference between treated and control samples along the observable covariates. T-tests for differences in means reveal there is no statistically significant difference between treated and control group after matching on size, profitablity and growth which are key attributes PE fund managers consider when selecting a target.[20].

5 Estimation Results

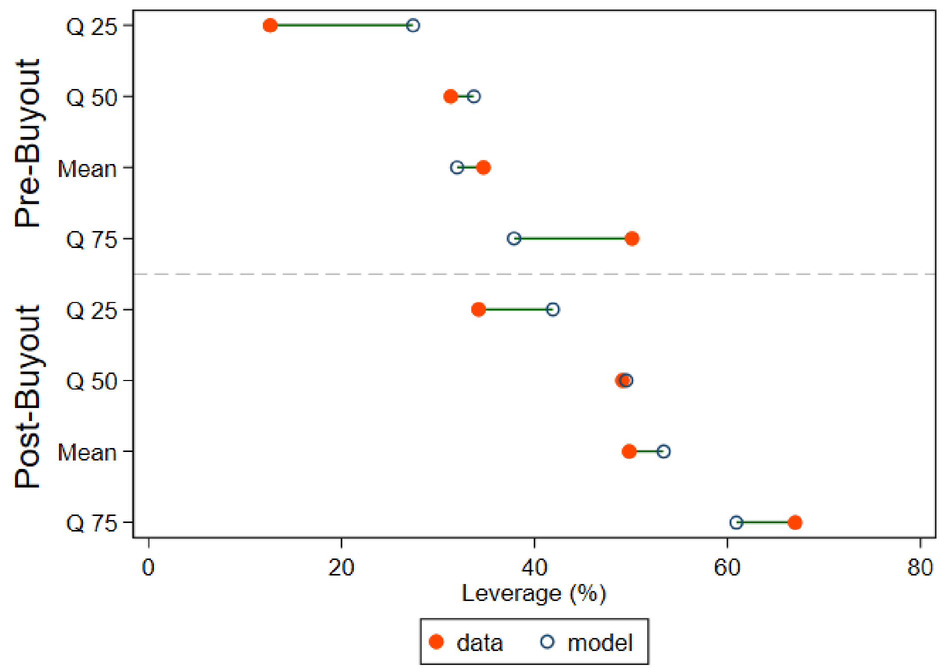
5.1 Structural Estimates of Optimal Leverage

This section presents the quantitative results of the structural model. Previous studies typically calibrate values for key model parameters. However, optimal leverage is particularly sensitive to parameters such as risk and excess return and thus structural estimation will allow for a more accurate representation of capital structure of PE-backed firms which are systematically different than standard public companies. Parameter estimates from Eq. (12) and Eq. (13) are reported in Appendix D Table A2, and in this section I primarily focus on the main outcome of interest, leverage.

First, Figure 3 compares key moments generated from the model with the actual data. As can be seen, the model is quantitatively consistent with the data both before and after PE-takeover. For a more thorough analysis, Table 4 reports structural estimates of leverage in more detail using the estimated structural parameters. I begin by comparing the estimates from the baseline model with actual values. Both the mean and particularly median values are quite close to what we observe in the data. For example, median leverage post-buyout is 49.4, while model-implied optimal leverage is 49.5. The mean is

[20] To ensure that my results are not sensitive to alternate matching criterion or loss of data due to strict matches, I carry out two additional propensity-score matching exercises. My results are not affected by alternate matching criterion. These results are available upon request.

Figure 3: Leverage Estimates



Notes: This chart shows estimates of optimal leverage generated from the structural model outlined in Section 2, and compares with actual sample moments. Estimation procedure utilizes a Markov Chain Monte Carlo simulation with Gibbs sampling using diffuse priors. I use a total of 12500 iterations with a burn-in sample of 2500. Robustness checks have been carried out to ensure results are not sensitive to alternate specifications for asset risk as well as a variety of plausible priors for the simulation.

only higher from the actual data by 3.57 percentage points. I also note the baseline model comes reasonably close to matching actual leverage ratios in the $75th$ percentiles. Taken together, these estimates strongly suggest higher observed leverage in the data is optimal rather than reflections of over-leveraging.

Next, we note that pre-LBO, model-implied leverage is around 33.71 percent and very close to actual leverage pre-LBO. Simiarly the model does a good job of matching the median leverage ratio pre-LBO. It is easy to see the change in leverage that occurs after an LBO is quite closely captured by the model. Specifically, back-of-the-envelope calculations show the model predicts an increase of 19.66 percentage points in mean leverage, compared to 18.50 percentage points in the actual data. Therefore, the PE-shock nearly doubles optimal leverage. When we turn to matched public comparable companies, we note that the control group has leverage ratios much lower than what the model estimates is optimal for these companies. This is particularly pronounced at the mean. The data however matches the third-quartile quite well (46.04 compared to 46.70).

Table 4: Structural Estimates of Optimal Leverage

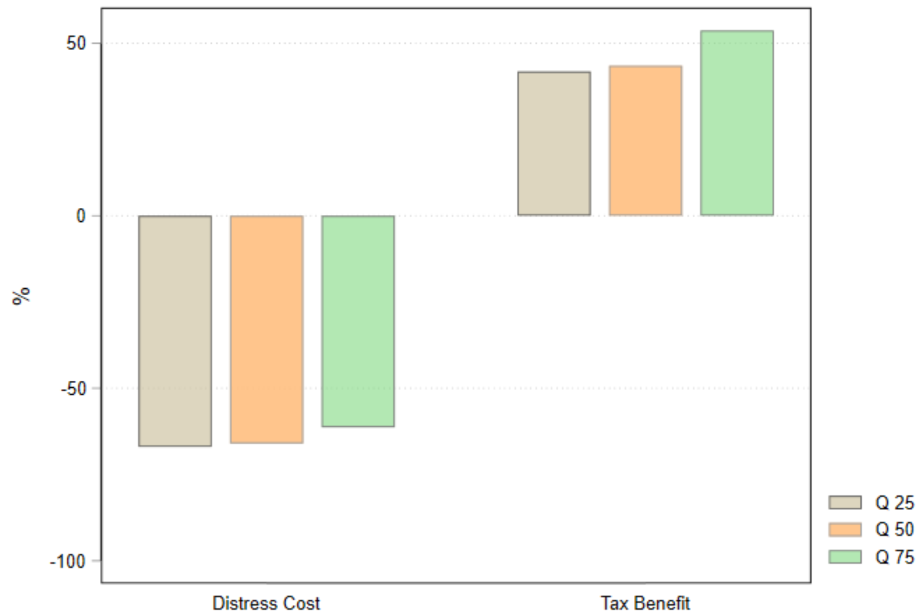
	Mean	p50	p75	SD
Panel A: Leverage (%)				
PE Firm Post-Buyout				
Actual Leverage	49.80	49.40	67.04	37.02
Benchmark Model	53.37	49.46	60.91	15.61
PE Firm Pre-Buyout				
Actual Leverage	31.30	34.70	50.10	32.09
Benchmark Model	33.71	31.96	37.87	8.29
Matched Control				
Actual Leverage	23.30	28.00	46.70	31.56
Benchmark Model	36.46	38.87	46.04	12.55
Panel B: Decomposition of Key Factors				
PE Firm Post-Buyout Leverage				
Estimated $\sigma, \alpha = 0, \rho = 0.5$	43.42	36.33	54.47	21.05
Estimated $\sigma, \alpha = .019, \rho = 0.35$	52.68	48.60	60.63	16.07

Notes: This table reports model-estimated values of optimal leverage. Optimal Leverage is calculated as outlined in Section 2. The model is estimated separately for the pre-LBO and post-LBO sample for PE-backed firms and in the full sample for matched control firms. Estimation procedure utilizes a Markov Chain Monte Carlo simulation with Gibbs sampling using diffuse priors. I use a total of 12500 iterations with a burn-in sample of 2500. Robustness checks have been carried out to ensure results are not sensitive to alternate specifications for asset risk as well as a variety of plausible priors for the simulation. Panel B reports differential impact on Leverage due to changes in model primitives.

One simple explanation consistent with much of the capital structure literature is that matched public companies are under-leveraged potentially reflecting agency problems that typically characterize these types of firms.

To get a deeper understanding on which key factors contributed to the rise in leverage, I decompose the contribution of the three factors that contribute to the approximate 60 percent increase in optimal leverage post-buyout in the baseline version of the model. These decomposition exercise is reported in Panel B. First, I set the risk-adjusted excess return to 0 and keep the bankruptcy costs unchanged in the post-buyout sample and estimate optimal leverage. All variation will now be driven by asset risk only. Approximately 50 percent of the increase in model-implied leverage post-buyout is driven by the lowering of asset risk as model-estimated leverage rises from 33.71 to 43.42 at the mean. In the last column, I re-introduce the estimated excess return but keep bankruptcy costs unchanged. It becomes clear most of the remaining variation is driven by higher expected future return, α. The direct deadweight costs of bankruptcy does not raise optimal leverage by any

Figure 4: Endogenous change in Tax Benefits and Distress Costs



Notes: This figure reports model-estimated values of the change in tax benefits and costs of debt from PE-ownership. The figure reports the percentage change in the tax benefits and expected distress costs following PE-ownership which are functions of key parameters of the model. Both tax benefits and expected distress costs are scaled by model-estimated levered firm value.

significant level relative to risk and excess return.

Finally, Table A2 reports estimated values of asset risk. We note that PE-ownership lowers asset risk consistent with a large literature that documents PE fund managers bring managerial oversight and improved governance that reduces volatility. We document a decrease of approximately 60 percent in asset risk. The size of the standard errors suggest the model is well-identified and each parameter of estimate significantly explains equity volatility.

I end the section by estimating the percentage change in the endogenous determinants of optimal leverage. Specifically, I compute the change in tax benefits and expected costs of debt from the estimated model as the difference between estimated values pre-LBO and post-LBO. Figure 4 reports these estimates. As can be seen, there is an increase in the tax benefits and a much larger decrease in expected costs of debt (at the median) consistent with the hypothesis in the paper. While the levels are not reported because we are primarily interested in understanding the change PE delivers, I estimate expected distress cost as a share of firm value is less than 1 percent for the median firm which is also consistent with Brown (2021). This is available upon request.

5.1.1 Model Validation

To validate the model, I take the following three steps. First, I show that a long time-series of actual leverage and model-estimated leverage post-LBO track each other closely. Second, I show the model estimates also matches non-targeted moments. Third, I run reduced-form regressions with firm fixed effects of leverage on firm value using both actual and model simulated data and show that estimated coefficients, standard errors and R^2 are very close to each other.

For brevity, I report figures and tables from this section in Appendix D. To begin, Figure A1 in Appendix plots the time-series of actual and model-estimated leverage post-LBO from 2001 to 2019. As can be seen, in general the two variables track each other closely. The model is not able to entirely replicate the cyclical nature of PE leverage, but overall the absolute values are quite close to each other and in general range between 50 to 60 percent.

Second, in Table A3 in the Appendix, I show the model approximately matches other moments related to leverage and firm value. Specifically, I show model-estimated credit spread is nearly identical to actual data. I also show estimates of cost of debt is reasonably close. First, I estimate credit spreads following Leland (1994).[21] The second objective of estimating spreads is to understand default risk, which will be discussed in the next section. For the purposes of this section, I compare median model-estimated credit spread with actual data. My sample does not have information on spreads. Thus, I rely on information from a canonical paper in the literature that has detailed debt pricing information on LBOs. Specifically, I use information from Axelson et al. (2013), who find that the typical LBO is characterized by term loans that have spreads over LIBOR of *306 basis points*. I find the median firm in my sample has estimated spread of *302* basis points in the first 5 years post-LBO, which is nearly identical to the data. I also document the median spread is *318* in the year after the buyout: this estimate is available upon request. Next, I compare the cost of debt. I compute cost of debt as the interest paid on short and long-term debt in my sample. I find the mean interest is 7.1 percent, while the 75th percentile is 4.6 percent, highlighting the skewed nature of the data. In the model, the mean is a bit lower at 5.5 percent while the 75th percentile is a bit higher at 6.1 percent. However both actual data is within the standard deviation of model estimated cost of debt.

Finally, I run a reduced form fixed-effects regression of leverage on value using both

[21] Leland (1994) estimates spreads as the difference between interest on risky debt and the risk-free rate. Interest on risky debt is given by: $R = r[1 - (C/V)^X k]^{-1}$, where a closed-form solution for all parameters are also provided.

actual and simulated data. Eq. (40) in Appendix D highlights this specification and results are reported in Table A3 in Appendix D. Using actual data, I find a negative relationship between value (which we can think of profitability in this case) and leverage. The estimated coefficient is -0.0297 and is highly significant. When I re-estimate this using simulated data, the coefficient is negative and significant and is reported at -0.0231. We can also observe that the standard errors are very close and the R-squares are nearly identical.

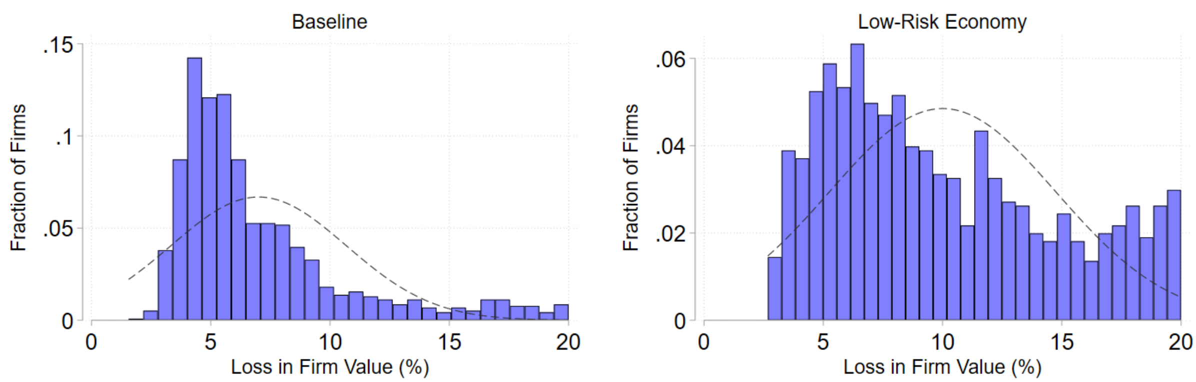
5.2 Counterfactual Experiment: What if PE did not lever up?

Since much of the criticism of PE centers around high leverage ratios, a natural counterfactual analysis centers around quantifying the loss in firm value from deviating from optimal leverage. In this section, I ask how much value is lost if PE-backed companies remained at leverage ratios similar to non-PE companies? Note, this question implicitly assumes all other characteristics of PE-backed companies still exist: higher excess return resulting from standard PE characteristics such as better corporate governance, lower dead-weight costs of bankruptcy, lower asset risk etc.

To answer this question, I run an algorithm such that each PE-backed firm now chooses leverage equivalent to *half* of their model-estimated optimal leverage. I estimate firm value $V_i^{'}$ corresponding to this sub-optimal leverage ratio. Letting, V^* denote firm value at PE's optimal leverage ratio reported in Table 4, I compute $Cost\ of\ Deviation_i = V^* - V^{'}$ for each PE-backed firm i. Next, I repeat the exercise for a simulated economy low aggregate uncertainty or risk. For ease of exposition, we can also label this a *boom* state. Since boom states are typically characterized by low uncertainty, firms are likely to lever up more implying the cost of a sub-optimal capital structure in such states of the world is likely to be higher relative to the baseline. Armed with my model, I can quantify the exact cost of choosing sub-optimal capital structure, which can be used to inform policy both at the PE-industry and overall financial-sector level.

The results are plotted in Figure 5. I find that cost of remaining at sub-optimally low leverage ratios is quite substantial. Beginning with the chart on the left which reports the baseline estimates, we see a wide range in terms of lost value from choosing a much lower leverage ratio. For a small share of firms, this cost can be as high as 20 percent of firm value. Simple back of the envelope calculations reveal median loss in firm value is 5.79 percent. This is consistent with Korteweg (2010) who estimated a median loss from not choosing the optimal leverage ratio of 5.5 percent. Looking at the left-tail, it is clear that there is a strictly non-zero cost of deviating from the optimal capital structure. Thus this initial counterfactual underscores the importance of a higher optimized leverage ratio.

Figure 5: Counterfactual Experiment: Cost of Choosing lower (sub-optimal) Leverage



Notes: The chart above reports results from a counterfactual analysis on the cost of deviating from optimal leverage for a PE-backed company. Both charts plots the difference in firm value at the optimal C^* and a sub-optimal C_{sub}, where $C_{sub} = 0.5 * C^*$. This particular formulation of sub-optimal capital structure was chosen to match leverage ratios of standard non-PE companies. The top chart is the baseline counterfactual. The bottom chart plots the same in a simulated low-risk aggregate economy. To simulate a low-risk economy, I introduce (add) a common shock of $\sigma = -0.1$ to the distribution of firm risk. Median loss in the baseline: 5.79 percent. Median loss in low-risk economy: 9.23 percent.

Next, I repeat the exercise in a boom-economy. To simulate *boom*-economy, I follow the literature on risk shocks (Christiano et al., 2014) and assume the variance of idiosyncratic firm cash flows decreases in the boom state, which ceteris paribus lowers the cost of distress. The magnitude of the risk shock is approximately 50 percent of the median PE-backed firm post-LBO in terms of asset risk (or just over 20 percent of the median PE-backed firm in the pre-LBO sample). The right panel in Figure 5 plots the distribution in the boom-state. It is apparent the cost of not levering up to the optimal level is substantially higher. A substantial fraction of firms now stand to lose over 10 percent of firm value. Again, while not reported, back-of-the-envelope calculations show the median firm stands to loose 9.23 percent of firm value in this low-risk economy.

6 Default Risk and Financial Fragility

6.1 Overview of Methodology

A standard concern is that PE-owned firms characterized by high levels of debt could make the financial system fragile considering the possibility of elevated default risk. This is particularly crucial since banks hold a lot of private equity-sponsored debt as shown in Ivashina and Kovner (2011). For example, with the publication of the leveraged lending guideline in March 2013, U.S. regulators have become worried about private equity firms with their high-risk deals (Grupp, 2015). Despite the attention from regulatory authorities to LBOs, the literature investigating the impact of PE on corporate distress and financial fragility using structural distance-to-default estimates is virtually non-existent.[22] In this section, I derive structural estimates of Distance-To-Default and use this measure to infer the contribution of PE-ownership to default risk. Consistent with the primary hypothesis proposed, if higher leverage is optimal then we should not observe deteriorating credit risk and acceleration of default risk and financial fragility due to PE. To alleviate concerns that PE-backed firms may have been on a particular trajectory in terms of default risk, I estimate the same for a set of matched control firms.

There are several methods to compute default risk and examine financial fragility, some of which are beyond the scope of this paper. In this paper, I follow Saldias (2012) and use aggregated Distance-To-Default estimates and infer the change in DTD following PE-investment (in the cross-section of the distribution) as a measure of default risk due to PE intervention. Next, I compute *firm-size weighted Distance-To-Default* to take into account the fact that larger firms are more likely to be influential in generating aggregate corporate and financial sector risks.[23]

Following Vassalou and Xing (2004), I compute DTD as outlined below:

$$DTD = \frac{ln(\frac{A_t}{A_B}) + (\mu_A - \delta + 0.5\sigma_A^2)T}{\sigma_A\sqrt{T}} \tag{16}$$

where A_t and A_B are firm asset value and default barrier respectively. All other variables in Eq. (13) are described in the model in Section 2 and estimated as described in Section 4.

Finally I follow Leland (1994) and estimate the interest raid paid on debt and corresponding yield spread over a risk free asset in the same manner as described in Section 5.1.1. Since credit spreads embody the inherent credit risk in a corporation, it serves as a check to our main default risk analysis which relies on the DTD measure. For both estimates, I restrict the sample window to five years surrounding a buyout.

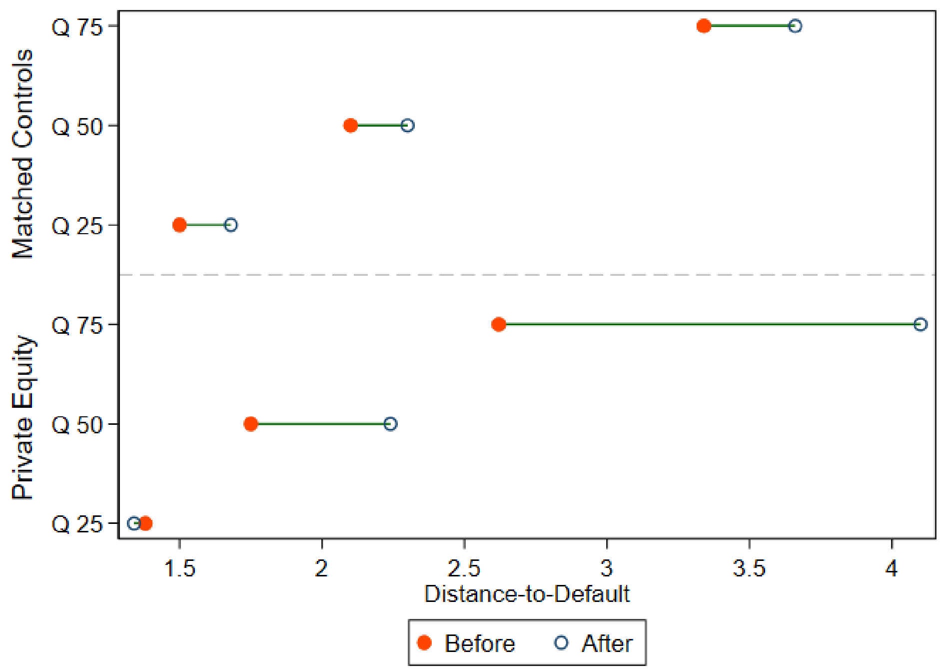
[22] Tykvova and Borell (2012) investigate distress risk in PE-backed firms using accounting-based measures such as Altman's Z-score. The disadvantage using this method is that it does not take into account the fact that default probability will inevitably depend on a firm's unobserved unlevered asset values, expected return, volatility, maturity of debt and default barrier. However, an advantage of the Z-score is that it is not subject to model misspecification. As a result, I will use the Altman score as part of a robustness check in Section 7, as will be described subsequently.

[23] One drawback of my analysis is that DTD measures implicitly assumes perfect correlation across assets and does not take into account risk interdependance and sector-wide tail risks. Thus, at this point we cannot explicitly infer an impact on systemic risk due to PE.

6.2 Distance-To-Default and Credit Spreads

I begin by first presenting results in the cross-section, and then in the time-series relative to years around an LBO event. Figure 6 reports the distribution of DTD. I interpret the change in DTD as a measure of PE's contribution to default risk. Recall higher values of DTD is equivalent to lower default probability.

Figure 6: Structural model Distance to Default Estimates



Notes: This figure reports model-estimated values of Distance-To-Default. The estimation window is restricted to $t = [-5, 5]$*. Distance-to-Default indicates the number of standard deviations at which the market value of assets is away from the default barrier. To estimate asset volatility, I deploy a Markov Chain Monte Carlo simulation with Gibbs sampling using diffuse priors. I use a total of 12500 iterations with a burn-in sample of 2500. Robustness checks have been carried out to ensure results are not sensitive to alternate specifications for asset risk as well as a variety of plausible priors for the simulation. Default barrier is defined as a firm's short-term debt plus half of long-term debt. Debt Maturity and drift is calibrated as outlined in Section 4.*

We begin by examining the cross-sectional variation before and after a buyout. Row (1) shows unchanged default risk following a buyout corresponding to the first quartile. However, column (2) shows the median firm experiences non-negligible increase in DTD in the first five years following a buyout. Specifically, the median firm's DTD rises from 1.75 to 2.24, which simple back of the envelope calculation shows is an approximate increase of 28 percent. Row (3) reveals a substantial increase at the third quartile as DTD rises from 2.62 to 4.1, which is an increase of 56.5 percent. Turning to matched controls, much smaller differences in the post-buyout window. Overall, these cross-sectional estimates suggest default risk there is little evidence of an increase in default risk since the median PE firm experiences substantial improvement in default probability. The matched control group shows that there is non-negligible PE-effect driving this change in DTD since non-PE firms experience much smaller effects.

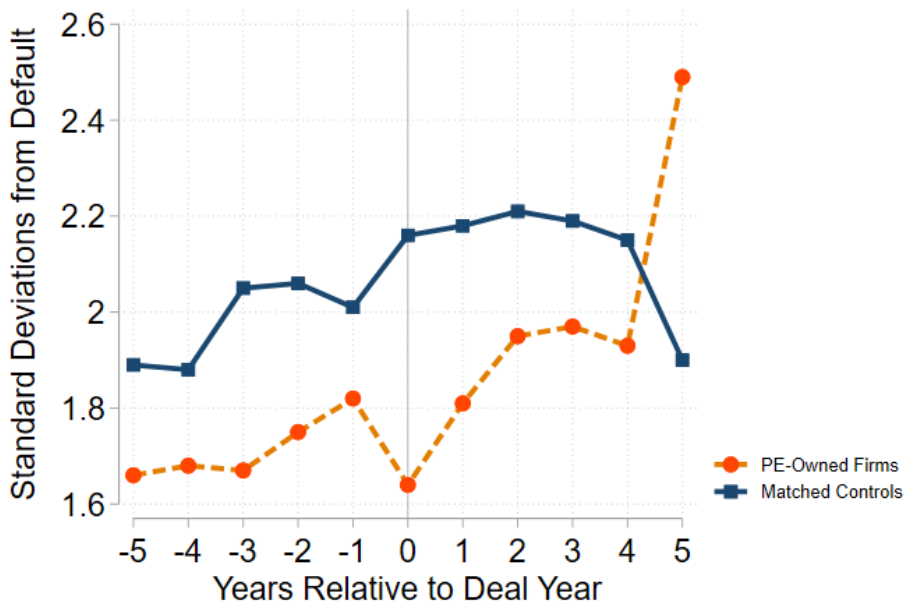
Next, credit spread estimates are reported in Figure A3 in Appendix D, for brevity. Across the board, we observe substantial narrowing of credit spreads. For example, the median firm experiences a reduction in spreads from 6.28 percentage points to 2.7 percentage points. On the other hand, matched controls experience very little narrowing of spreads following an LBO. Overall, the results qualitatively mirror the same patterns as those shown in the distance-to-default estimates. It is also worth noting since the model does not capture liquidity risks, the spread is a measure of default risks alone.

Turning to examining the time-series variation yields interesting patterns. Figure 7, Panel A reports (unweighted) median DTD estimates in each year relative to a buyout for PE-backed and matched controls. It is interesting that the median PE-backed firm actually had relatively low DTD value pre-LBO. Infact, we see DTD of the median firm was below 2.0. In contrast, matched controls were at a moderately higher level at 2.0 at year $t = -1$. Following PE-acquisition, we see DTD rises sharply in PE backed firms in the first two years, before tapering four a couple of years and ending at a much higher leve at $t = 5$. We note a much lower rate of increase among matched controls. After $t = 2$, we note opposing trends with PE-backed firms continuing to experience increase in DTD while matched controls essentially revert back to their pre-LBO mean. At $t = 5$, we note the median PE-backed firm's DTD surpasses that of the matched control.

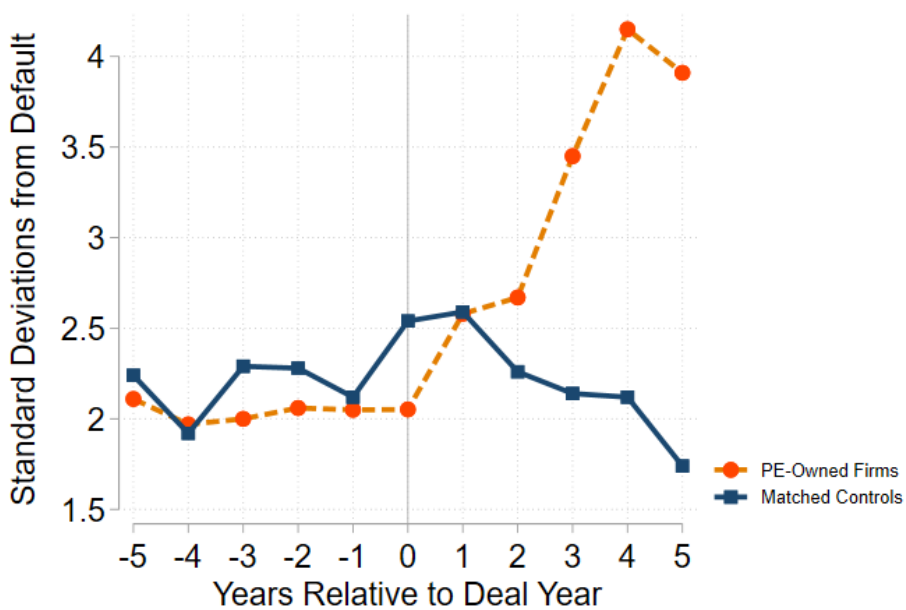
Panel B in Figure 7 paints a significantly different picture. Weighted by firm size, DTD among PE-backed firms rise much faster following acquisition, while that of matched controls moderately decline to around 2.0 at year $t = 5$. PE-backed firms surpasss matched controls in terms of DTD in $t = 2$. Since, larger firms are more likely to generate systemic risk, these charts strongly suggest concerns related to high levels of debt and risks posed to associated banks may be overstated. Overall, the analysis in this section reveals little evidence that PE-ownership increases corporate distress or financial fragility despite higher levels of debt financing.

Figure 7: Distance-To-Default: Time-Series Variation

Panel A: Median



Panel B: Firm-Size Weighted Estimates



Notes: This chart reports Distance-To-Default (DTD) estimates. The top chart reports the median value and the bottom chart aggregates by each year relative to a buyout using firm size as weights. Methodology for DTD estimation is described in Section 6.1.

7 Additional Tests and Model Extensions

7.1 Reduced-form Evidence

The key prediction of the structural model is the endogenous change in tax benefits and expected distress costs of debt due to PE ownership. To substantiate model predictions, in this section, I provide reduced-form empirical evidence consistent with lower expected distress costs and higher tax benefits of debt under PE-ownership. Finally, in section 7.2, I study an extension to the baseline model.

7.1.1 Equity Injection During Financial Distress

First, I show that PE funds inject additional equity into a company when it is in distress. This mechanism can explain why bankruptcy cost can be lower and particularly why expected distress cost is lower. Bernstein et al. (2019) find that PE-backed companies behaved differently than a matched control group during the financial crisis. They estimated net equity issuances over assets increased by two percentage points relative to their peers. Because PE groups raise funds that are drawn down and invested over multiple years—commitments that are very rarely abrogated—they may have "deep pockets" during downturns. These capital commitments may allow them to make equity investments in their firms when accessing other sources of equity, or financing in general, is challenging. The literature has not yet documented if this mechanism is at play any time a PE-backed firm is in distress. To test this hypothesis more generally, as opposed to only during an aggregate financial crisis, I define an indicator variable $Distress$ as follows:

$$Distress = \begin{cases} 1 & \text{if Altman Z-Score} < x \\ 0 & \text{otherwise} \end{cases}$$

where the Altman Z-score is computed following Altman (1968) at the *company-year* level and x is a positive constant.[24] Using this $Distress$ variable, I estimate Eq. (15) below where the dependant variable is Net Equity Contribution/Asset at the firm-year level. Equity Contribution is defined as the difference in Total Equity (shareholder value) over the past year, minus profit following Bernstein et al. (2019) [25]. I introduce a triple interaction between PE, $Post$ and $Distress$. A positive coefficient is indicative of PE-

[24]Since I do not observe data on Retained Earnings, I proxy with Cash flows which Orbis (2007) defines as Profit for the Period plus Depreciation.

[25]For profit I proxy with Cash Flows in period t. I also verify that my results are not affected if I used other measures of Profit such as Profit Before Taxes. These are available upon request.

backed firms receiving additional equity contribution compared to a matched control group when they are pushed into distress. To summarize, I estimate variants of the following triple-interaction equation:

$$Y_{it} = \beta_1 Post_{it} + \beta_1 PE_i \times Post_{it} + \beta_2 PE_i \times Post_{it} \times Distress_{it} + \gamma' \mathbf{X}_{it} + \alpha_i + \delta_t + \epsilon_{it} \quad (17)$$

Table 5: Equity Injection During Financial Distress

Y_{jt} : *Net Equity Injection*	$x = 1.0$		$x = 1.8$	
	(1)	(2)	(3)	(4)
$Distress \times PE \times Post$	0.037***	0.040***	0.030**	0.032**
	(0.014)	(0.015)	(0.014)	(0.015)
$Distress$	0.126***	0.135***	0.128***	0.135***
	(0.006)	(0.007)	(0.011)	(0.012)
R-squared	0.086	0.065	0.093	0.071
Firm FE	Y	Y	Y	Y
Time FE	Y	Y	Y	Y
Controls	Y	N	Y	N
N	18291	18591	18291	18591

*Notes: * $p < .10$, ** $p < .05$, *** $p < .01$. This table reports results of matched difference-in-differences regressions of outcomes of PE-backed companies relative to public controls. Following Bernstein et al. (2019), Y_{jt} is Net Equity Contribution/Asset. Equity Contribution is measured as the difference in total equity (shareholder value) over the past year, minus profit. Specifications vary by fixed effects and controls. Matching procedure is outlined in Section 2.1.3. $Post_{it}$ takes value 1 in years after a buyout. $Distress_{it}$ takes value 1 if the computed Altman Z-score is less than 1 in a given company-year. In columns (3) and (4) I check robustness using a threshold of 1.8 which the literature also suggests is a potential bankruptcy threshold. The specification controls for firm and time fixed effects. I also control for confounding pairwise interactions if they are not absorbed by firm fixed effects. Standard errors are clustered by firm*

I present these results in Table 5 columns (1) to (4). Estimates in columns (1) to (2) that alternatively vary firm controls suggest PE-sponsored firms receive equity injections of about 3.7 to 4.0 percentage points higher compared to matched public controls when in financial distress. These estimates are robust in the presence of time-invariant firm-level fixed effects as well as time-varying controls which include size and leverage. It can be argued that the results are sensitive to the choice of threshold used to define $Distress$. In order to alleviate such concerns, I re-define $Distress$ using a different threshold of 1.8, which is a recognized threshold in the literature. My results remain robust and perhaps intuitively we find the point estimates are marginally smaller, capturing the fact that companies are relatively less in distress when their Altman score is at 1.8 relative to 1.0. By extension, they require lower equity injections. It is also worth mentioning that my equity injection estimates are quite comparable to Bernstein et al. (2019) mentioned above.

Recall that a key mechanism underlying higher optimal leverage is substantial reduction in expected distress costs. Standard trade-off thoery tells us expected costs depend on both the direct costs of bankruptcy as well as indirect costs. The latter includes asset substitution, asset fire-sales and debt overhang. While decomposing these components of the indirect costs is beyond the objective of this paper, the result on equity injection during financial distress is clear evidence of lower debt overhang when a company is backed by PE. This in turn implies lower expected cost of debt. Moreover, to the extent that the incentives to prevent value-transferring to debt-holders is reduced and equity holders avail profitable investment opportunities, the result on equity injection is also consistent with higher expected future return. These implications are consistent with Hotchkiss et al. (2021) and Malenko and Malenko (2015).

7.1.2 Higher Profitablity and Greater Tax Benefits

Second, for tax benefits, I show that PE makes firms more profitable which directly implies higher tax benefits of debt following Jensen (1986). I compare the average outcome of acquired (treated) companies with non-acquired (control) companies using a matched difference-in-differences specification outlined in Eq. (16) below.[26] Specifically, following the canonical specification in Boucly et al. (2011), I estimate:

$$Y_{it} = \alpha_i + \delta_t + Post_{it} + \beta Post_{it} \times PE_i + \gamma^{'} \mathbf{X}_{it} + \epsilon_{it} \tag{18}$$

For firm i at time (year) t, the dependant variable will alternatively be (i) Return on Assets, (ii) Return on Capital Employed, defined as Profit before Taxes plus Interest Paid divided by the sum of Equity and Non-Current Liabilities, (iii) Enterprise Value/Assets, which is a standard measure of firm valuation. PE_i, takes value 1 if a company is PE-sponsored. $Post_{it}$ takes a value of 1 in the years following the leveraged buyout. For all specifications, the vector X consists of firm size measured by the log of total assets and leverage. Since these variables are affected following the interventions, I report results with and without controls. I include firm α_j and year δ_y fixed effects. I cluster standard errors at the firm and year level. The coefficient of interest is β, the DiD estimate of the PE

[26]For a difference-in-differences setting with more than two time periods, Imbens and Woolridge (2007) suggest introducing a policy dummy that is simply defined to be unity for groups and time periods subject to the policy along with a full set of time-period dummies. However, since I do not distinguish between permanent and temporary effect of PE-ownership, the need for a composite indicator that turns "on" only during the years of PE-ownership is redundant, reducing the problem to the canonical DiD.

effect on firm-level outcomes[27]. [28]

Table 6 summarizes results on returns. Even numbered columns include firm-level controls. I begin by documenting the fact that Return on Assets rises significantly following PE acquisition. Specifically, ROA rises by 2.5 percent relative to comparable public controls. This value rises to 9.5 percentage points if we include firm controls. Next we see significant increase in Return on Capital Employed. This measure captures how efficiently a firm is generating profits and is particularly applicable to debt-heavy companies. These findings indicate PE-sponsored firms generate return on employed capital that is 3-6 percentage points greater than comparable public companies as shown in columns (3) and (4). Finally we estimate the impact on firm valuation, proxied by Enterprise Value scaled by Total Assets. I document substantial increase in the valuation of the company following PE-ownership. Both columns (5) and (6) indicate an increase in firm valuation of around 30 percentage points relative to comparable controls. Collectively these findings suggest PE-ownership adds value that is not replicable by non-PE managers highlighting the former's unique knowledge of business models and industry characteristics. Conceptually, this is analogous to an excess return that explained part of the increase in optimal levels of leverage post-LBO.

Table 6: Evidence of Excess Return from Matched DiD Estimates

	Return on Asset		*Return on Capital*		*Enterprise Value/Asset*	
	(1)	(2)	(3)	(4)	(5)	(6)
$PE \times Post$	0.025**	0.095***	0.032**	0.060***	0.314***	0.327**
	(0.010)	(0.034)	(0.013)	(0.014)	(0.116)	(0.143)
R^2	0.001	0.064	0.034	0.046	0.030	0.051
Firm FE	Y	Y	Y	Y	Y	Y
Time FE	Y	Y	Y	Y	Y	Y
Controls	N	Y	N	Y	N	Y
N	25513	24860	21511	21079	10126	10085

*Notes: * $p < .10$, ** $p < .05$, *** $p < .01$. This table reports results of within-firm difference-in-differences regressions of outcomes of PE-backed companies relative to matched controls. Matching procedure is outlined in Section 3. $Post_{it}$ takes value 1 in years after a buyout for both treated and corresponding matched controls. PE takes value 1 for PE-owned firms. Standard errors are clustered by firm and time.*

Finally, to validate the parallel trends assumption I run dynamic difference-in-differences

[27] In unreported regressions, I also cluster at the $firm \times Post$ level and $firm$ level and confirm results are not sensitive to the level of clustering. Additionally, since the controls may also be affected by PE-ownership, I repeat the regressions without controls. All results remain unchanged and are available on request.

[28] It is worth noting this exercise is analogous to the estimation of excess return in the model, however I did not include any form of fixed effects. Thus this exercise also serves as a robustness check on the model.

and plot estimated coefficients following Gupta et al. (2020). I use variants of the following specification:

$$Y_{it} = \alpha_i + \delta_t + \sum_{s \neq 0} \beta_s Deal\ Year_{i,s} + \gamma' X_{it} + \epsilon_{i,t} \tag{19}$$

Estimates from Eq. (15) are shown in Figure A2 in Appendix D2 below. Thus these estimates can be interpreted as plausibly causal, since the implicit parallel trends assumption is not violated.

7.2 Introducing Capital Constraints in PE-Owned Firms

One concern surrounding the model may be that the GP can always overcome liquidity problems. In practice, this is likely to be the case for investments made in the early stage in the life of a fund when GPs have access to uncalled capital that can be used to overcome liquidity shortfalls in portfolio companies, allowing bankruptcy to arise endogenously. However when all uncalled capital has been deployed, liquidity shortfalls cannot be met by injecting further equity. Past studies in capital structure have dealt with this issue in various ways. For example, Uhrig-Homburg (2004) introduces moderate equity issuance costs and finds optimal leverage in companies is much lower relative to the unconstrained case where equity can be issued costlessly.

In this section, I extend the baseline model of optimal leverage by introducing GP capital constraints. Specifically, PE's "deep-pockets" assumption is relaxed and I allow debt issuance when portfolio company cash flows fall below required coupon payments. This is equivalent to assuming very costly equity issuance, making debt issuance the more viable choice. The GP has two options: (i) default if it hits the default barrier, or (ii) issue further debt by promising a higher coupon payment. The analysis can capture PE deals made towards the end of the life of a fund or more generally any setting when a PE fund does not have access to capital from limited partners, net of initial investment and management fees (e.g. funds with limited resource or "dry-powder").

7.2.1 Equity-Holder's Payoff in a Liquidity Shortfall

Once again the PE-backed firm is characterized by its cash flows in each time-interval $A_t dt$, and needs to meet required coupon payment c from a perpetual consol bond. When cash flows are greater than required coupon payments, the HJB equation is the same as the baseline case as long as cash flows can cover required coupon. Now, consider the case where the firm's realized cash flow at the end of the period is less than the required coupon payment. Unlike the baseline, the company cannot rely on the GP to meet the liquidity shortfall. The firm can either default or choose to raise additional debt to meet liquidity shortfall depending on the outcome that maximizes firm value.

I assume there are no transaction costs of raising debt, and newly issued debt has the same seniority level as buyout leverage. Let $F(A,c)$ denote the value of the firm's debt and dF the value of newly issued debt. I also assume debt is not raised for new investments similar to the baseline case. The company will only raise an amount of debt equivalent to the liquidity shortfall shown below:

$$\big(c - (A + (c - A)\tau\big)dt = (c - A)(1 - \tau)dt = dF(\cdot) = dc\frac{\partial F(\cdot)}{\partial c} \tag{20}$$

The first equality captures a liquidity shortfall as the difference between required debt payment and realized cash flows plus tax return from the government. The last equality shows the change in the value of the debt is achieved by increasing current debt service by dc. Leland (1994) shows debt-holders will always resist increasing the total coupon payments through additional debt issuance, even though such sales may increase the value of equity and the firm by differentiating between current and new debt-holders and concavity assumption of F in c. I do not distinguish between current and new debt-holders. This is plausible under the assumption that PE groups have strong ties with the financial sector (Ivashina and Kovner, (2011)) and can approach their original lenders for further debt if optimal.

When $c - (A + (c - A) > 0$, the GP's HJB equation internalizes the fact that coupon changes:

$$rE(A) = \frac{c(1-\tau) - A}{\frac{\partial F(\cdot)}{\partial c}}E_c' + mI_0 + \mu A E_A' + \frac{\sigma^2 A^2}{2}E_{xx}'' \tag{21}$$

Eq. (19) does not have a closed-form solution, and needs to be solved numerically. I derive the value of $\frac{\partial F(\cdot)}{\partial c}$ using the closed form expression from Eq. (13). The value to the equity-holder at default remains the same as the baseline. Using standard smooth-pasting conditions at the critical point where $A = c$, we know the solution must satisfy the following conditions:

$$E_c' = \frac{1-\tau}{r} + (\frac{A}{A_B})^{-\gamma}\frac{1-\tau}{r} = \frac{1-\tau}{r}\big(1 + (\frac{A}{A_B})^{-\gamma}\big) \tag{22}$$

$$\frac{\partial F(\cdot)}{\partial c} = 0 \tag{23}$$

7.2.2 Debt-Holder's Payoff

When cash flows are sufficient to cover coupon, the debt-holder's payoff are given by the baseline model where the flow payoff per interval of time is cdt. When companies run into liquidity shortfalls and the GP cannot inject equity, the debt-holder lends additional capital in exchange for an increase in debt service payments. Thus, debt-holder's HJB equation should be the same in both intervals. When there is a liquidity shortfall, the debt-holder gets:

$$(A + (c - A)\tau) + dF(\cdot) = cdt \tag{24}$$

This makes intuitive sense since debt-holders still have absolutely priority in default, and the value of debt is not allowed to go to infinity implying boundary conditions are unchanged.

7.2.3 Numerical Analysis of Optimal Bankruptcy-Trigger and Leverage

In this section I analyze optimal bankruptcy point that maximises firm and equity value in details. I find my results are generally robust to a wide set of values after extensive sensitivity tests. As before I consider the same values for all the key parameters outlined in Table 4. There are two related objectives. First, determine an optimal leverage ratio. Second, determine if this is the global max for equity and firm value compared to the baseline case where the GP could meet liquidity shortfalls by injecting equity. Note that it is straight-forward to see both choices strictly dominate an exogenous-default case where the firm declares bankruptcy the first time it fails to meet coupon payments since the GP will always receive management fees, regardless of the state of the world.

Prediction 5. Optimal Leverage when GP is Capital Constrained. *When the GP does not have access to equity capital to meet a portfolio company's liquidity needs, optimal leverage is much lower since debt-financed coupon payments raise interest servicing costs.*

In order to examine optimal leverage I assume the same values for all key variables outlined in Table 4. I again set $A_0 = 100$ and assume an excess return of $\alpha = 1\%$. I begin with an initial value of $A_B = 15$ and a coupon rate of 2%. I repeat this procedure until optimal leverage and firm value converges to a maximum. I repeat the exercise for alternate values of A_B to find the global maximum. Results are reported in Table 9 below. I find optimal leverage is approximately 22.7 percentage points lower when the company cannot meet coupon payments by injecting additional equity. Optimal leverage is at 37.3 percent, which is quantitatively similar to that of a non-PE company. An implication is to the extent that leverage ratios are invariant to the life of a fund in practice, it is possible that deals made towards the end of the life of a fund or by a fund with limited access to dry powder or other sources of capital may be over-levered. Examination of buyout leverage for each portfolio company sorted by its order in the sequence of investments made by a fund since inception can confirm this prediction, and is beyond the objective of this paper. For now, caution is warranted on these interpretations since (i) I do not observe the sponsoring fund's experience or access to external capital and (ii) this is not a standard case.

Table 7: Results

Variables	
Optimal Leverage (%)	37.3
Firm Value Relative to Baseline Case	152.29
Firm Value in Baseline Case	155.60

Notes: This table reports optimal leverage and firm value in the capital constrained case. Firm values are normalized to the non-PE company reported in the baseline case. Firm Value in the baseline case is retrieved from Table 5 and represents a PE-backed company with a GP that has access to PE's "deep-pockets" and can meet liquidity shortfalls by injecting additional equity.

8 Conclusion

Private Equity is widely criticised for putting on too much debt on their portfolio companies' balance sheets. Critiques often cite cases such as Toys R Us to support their claim. This standard criticism implicitly assumes what optimal leverage should be for a company. However, optimal leverage is extremely challenging to pin down without a structural model that endogenizes key benefits and costs of debt as well as the incentives governing the agent choosing capital structure, the GP. This paper, is the first to show using a structural model, that private equity ownership can lead to higher levels of optimal debt.

I begin by marrying two distinct literatures: private equity and capital structure. To the best of my knowledge, this paper is the first to provide a structural estimate of optimal leverage specific to companies owned by PE funds. Even though the model is essentially developed at the deal-level, it can make important fund-level predictions. The results indicate PE-owned firms are not systematically over-levered. Rather higher observed leverage is optimal as long as PE firms can generate an α, reduce asset risk and bankruptcy costs as well as choose the default-triggering asset level optimally. Consistent with higher optimal leverage, I show that PE's contribution to corporate distress and financial fragility is lower than previously assumed. Aggregated firm-size weighted Distance-to-Default estimates uncover little evidence that PE increases corporate distress or weakens the financial system.

Finally, I provide additional empirical evidence consistent with key factors that drive higher optimal leverage. Specifically, using a set of matched difference-in-differences regressions, I show that PE backed firms generate higher return on asset, return on capital, valuation and also receive greater equity injections when in financial distress relative to a set of comparable publicly traded companies. These results justify PE's unique excess return, and deadweight costs of bankruptcy. These findings are novel in both the PE literature and the broader capital structure literature. Overall, my results suggest new policies regulating PE funds should focus on optimal risk at the deal-level instead of capital gains tax or dividend payout policies.

References

[1] Acharya, Viral V., 2013. "Adapting Microprudential Regulation for Emerging Markets", in Otaviano Canuto and Swati R. Ghosh (eds.), Dealing with the Challenges of Macro Financial Linkages in Emerging Markets, World Bank, Washington, D.C. pp. 57-89

[2] Altman, E. (1968). Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy. The Journal of Finance, 23(4), p.589.

[3] Adrade, Gregor, and Steven N. Kaplan, How Costly is Financial (Not Economic) Distress? Evidence from Highly Leveraged Transactions that Became Distressed, 1998, The Journal of Finance, 53 (5)

[4] Asis, Gonzalo, Anusha Chari and Adam Haas, 2020, In Search of Distress Risk in Emerging Markets, NBER Working Paper 27213

[5] Axelson, U., Jenkinson, T., Stromberg, P., Weisbach, M. S. (2013). Borrow cheap, buy high? The determinants of leverage and pricing in buyouts. Journal of Finance, 68(6), 2223–2267.

[6] Ayash, Brian, Mahdi Rastad, (2018). Leveraged buyouts and financial distress, SSRN working paper 3423290.

[7] Bharath, Sreedhar and Tylar Shumway, Forecasting Default with the Merton Distance to Default Model, The Review of Financial Studies, Volume 21, Issue 3, May 2008, Pages 1339–1369,

[8] Bhamra, Harjoat S., Lars-Alexander Kuehn, Ilya A. Strebulaev , The Levered Equity Risk Premium and Credit Spreads: A Unified Framework, The Review of Financial Studies, Volume 23, Issue 2, February 2010, Pages 645–703,

[9] Brown, Greg, 2021, Debt and Leverage in Private Equity: A Survey of Existing Results and New Findings, Working Paper, Institute for Private Capital, Private Equity Research Consortium

[10] Brown, G., Harris R., Jenkinson T. and Robinson, D., (2020a). Private Equity: Accomplishments and Challenges, Journal of Applied Corporate Finance, 32 (3), 8-10.

[11] Brown, G., Carnelli Dompe and Kenyon, S., (2020b). Public or Private? Determining the optimal ownership structure, SSRN Working Paper, 3529421

[12] Brown, Greg and Steven N. Kaplan, Have Private Equity Returns Really Declined?, 2019, The Journal of Private Equity, 22 (4) 11-18.

[13] Bartram, Sohnke, Gregory W. Brown, William Waller, 2015, How Important is Financial Risk, Journal of Financial and Quantitative Analysis, 50, 804-524.

[14] Brown, Gregory, Robert Harris, Wendy Hu, Tim Jenkinson, Steven Kaplan, David Robinson, 2020, Private equity portfolio companies: A first look at Burgiss holdings data, Working Paper, Retrieved from Social Science Research Network.

[15] Bansraj, Smit, Volosovych, 2019, Can Private Equity Funds act as Stategic Buyers?. Working Paper. Retrieved from Social Science Research Network.

[16] Boucly, Quentin, David Sraer, David Thesmar, 2011, Growth LBOs, Journal of Financial Economics, 102, 432-453

[17] Black, F., and M. Scholes (1973): The Pricing of Options and Corporate Liabilities," Journal of Political Economy, 81(3), 637-654

[18] Christiano, Lawrence J., Roberto Motto, and Massimo Rostagno (2014), Risk Shocks, American Economic Review, 104 (1) 27-65

[19] Wouter De Maeseneire, Samantha Brinkhuis, 2012, What drives leverage in leveraged buyouts? An analysis of European leveraged buyouts' capital structure, Accounting and Finance, 52, 155-182

[20] Frank, Murray and Vidhan Goyal, 2009, Capital Structure Decisions: Which Factors Are Reliably Important?, Financial Management, 38, 1-37

[21] Edgerton J.,Agency Problems in Public Firms: Evidence from Corporate Jets in Leveraged Buyouts, 2012 The Journal of Finance, 67, 2187-2283

[22] Fracassi, Cesare and Previtero, Alessandro and Sheen, Albert, Barbarians at the Store? Private Equity, Products, and Consumers (June 16, 2021). Journal of Finance, Forthcoming

[23] Florian Szucs, 2013, MA and RD: Assymetric effects on Targets and Acquirers?, Working Paper. Retrieved from Social Science Research Network.

[24] He, Zhiguo and Wei Xiong, Rollover Risk and Credit Risk, 2012, The Journal of Finance

[25] Ivashina, Victoria and Kovner, Anna, The Private Equity Advantage: Leveraged Buyout Firms and Relationship Banking (April 5, 2010). EFA 2008 Athens Meetings Paper, Review of Financial Studies, 24 (7), 2462-2498, 2011 , Available at SSRN:

https://ssrn.com/abstract=1017857

[26] Kaplan, Steven and Jeremy Stein, 1993, The evolution of buyout pricing and financial structure in the 1980s, Quarterly Journal of Economics, 108, 313-357

[27] Jensen, 1989, Eclipse of the Public Corporation, Harvard Business Review

[28] Jensen, Michael, Agency Costs of Free Cash Flow, Corporate Finance and Takeover, 1986, American Economic Review, 76 (2)

[29] Jenkinson and Stucke, 2011, Who benefits from the leverage in LBOs? Working paper

[30] Grupp, Marcel, On the Impact of Leveraged Buyouts on Bank Systemic Risk, 2015, SAFE Working Paper No. 101, SAFE

[31] Goyal, Frank, Vidhan K. Goyal, 2007, Trade-off and Pecking Order Theories of Debt, Working Paper

[32] Giuzio, Margherita, Claudiu Moldovan and Danilo Vassallo, Financial stability implications of private equity, Financial Stability Review, May 2020, European Central Bank

[33] Faulkender, Michael, Mark J. Flannery, Kristine Watson Hankins, Jason Smith, 2011, Journal of Financial Economics, 103, 632-646

[34] Rajan, Raghuram, Luigi Zingales, 1995, What Do We Know About Capital Structure? Some Evidence from International Data, The Journal of Finance, 50, 1421 - 1460

[35] Kaplan, Steve and Per Stromberg, 2009, Leveraged Buyout and Private Equity, Journal of Economic Perspectives, 23, 121-143

[36] Korteweg, Arthur, 2010, The Net Benefits to Leverage, The Journal of Finance Vol. 65, No. 6 (DECEMBER 2010), pp. 2137-2170

[37] Lan, Yingcong and Wang, Neng and Yang, Jinqiang, The Economics of Hedge Funds, 2013, The Journal of Financial Economics, 110, 300-323

[38] Malenko, A., and Malenko, N. (2015). A theory of LBO activity based on repeated debt-equity conflicts. Journal of Financial Economics, 117(3), 607–627.

[39] Magnuson, William J., The Public Cost of Private Equity (April 19, 2017). Minnesota Law Review, Vol. 102, 2018, Forthcoming, Texas AM University School of Law Legal Studies Research Paper No. 17-28

[40] Axelson, Ulf, Tim Jenkinson, Per Stromberg, Michael Weisbach, 2013, Borrow Cheap, Buy High? The Determinants of Leverage and Pricing in Buyouts, The Journal of Finance

[41] Axelson, Ulf, Tim Jenkinson, Per Stromberg, Michael Weisbach, 2008, Leverage and Pricing in Buyouts: An Empirical Analysis, Working Paper

[42] Mark Flannery and Kasturi P. Rangan, (2006), Partial adjustment toward target capital structures, Journal of Financial Economics, 79, (3), 469-506

[43] Guo, Shuoron, Edith Hotchkiss, Weihong Song, Do Buyouts (Still) Create Value, The Journal of Finance, Vol. 66, No. 2 (April 2011), pp. 479-517

[44] He Huang, Ye, Rethinking capital structure decision and corporate social responsibility in response to COVID-19, 2021, 2021, Journal of Accounting and Finance

[45] Hotchkiss, Edith S. and Stromberg, Per and Smith, David Carl, Private Equity and the Resolution of Financial Distress (July 7, 2021). AFA 2012 Chicago Meetings Paper, ECGI - Finance Working Paper No. 331/2012, Available at SSRN: https://ssrn.com/abstract=1787446

[46] Imbens, Guido, Jeffrey Woolridge, 2007, Estimation of Average Treatment Effects Under Unconfoundedness, NBER

[47] Merton, Robert, 1974, On the Pricing of Corporate Debt: The Risk-Structure of Interest Rates, The Journal of Finance, 29, 449-470

[48] Kraus, Alan, Robert Litzenberger, 1973, A State Preference Model of Optimal Financial Leverage, The Journal of Finance, 28, pp. 911-922

[49] Korteweg, Arthur, The Net Benefits to Leverage, 2010, The Journal of Finance

[50] Jensen, Michael C., and William H. Meckling, 1976, Theory of the firm: Managerial behavior, agency costs, and ownership structure, Journal of Financial Economics 3, 305–360.

[51] Myers, Stewart C., 1977, Determinants of corporate borrowing, Journal of Financial Economics 5, 147–175.

[52] Johannes, Michael, and Nicholas Polson, 2004, MCMC methods for continuous-time asset pricin gmodels, in Lars Hansen, eds.: Handbook of Financial Econometrics (Elsevier, North Holland).

[53] Lerner, Josh, Morten Sorenson, Per Stromberg, 2011, Private Equity and Long-Run Investment: The Case of Innovation, The Journal of Finance, 66, pp. 445-477

[54] Leland, Hayne, 1994, Corporate Debt Value, Bond Covenants and Optimal Capital Structure. The Journal of Finance, 49, pp 1213-1252

[55] Leland, Hayne, Klaus Bjerre Toft, 1996, Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads. The Journal of Finance, 51, pp 987-1019

[56] Leland, Hayne, 1998, Agency Costs, Risk Management, and Capital Structure. The Journal of Finance, 53, pp 987-1019

[57] Goldstein, Robert, Nenjiu Ju and Hayne Leland, 2001, An EBIT-based Model of Dynamic Capital Structure, Journal of Business

[58] Uhrig-Homberg, Ulrig, 2004, Cash-flow shortage as an endogenous bankruptcy reason, Journal of Banking and Finance, 29, 1509-1534

[59] Merton, Robert C., Theory of Rational Option Pricing, 1973, The Bell Journal of Economics and Management Science, 4, 141 - 183

[60] Öztekin, Özde Flannery, Mark J., 2012. "Institutional determinants of capital structure adjustment speeds," Journal of Financial Economics, Elsevier, vol. 103(1), pages 88-112.

[61] Bobeldijk, Yolanda, Private Equity International (14 March 2013), Bank of England Warns of systemic risk from Private Equity, Private Equity International

[62] Bernstein, Shai, Josh Lerner and Filippo Mezanotti, 2019, Private Equity and Financial Fragility During the Crisis, The Review of Financial Studies, Volume 32, Issue 4, April 2019, pp 1309–1373

[63] Saldias, Martin, Systemic Risk Analysis Using Forward-Looking Distance-To-Default Series, 2012 Working Paper, Bank of Portugal

[64] Sorenson, Morten, Neng Wang, Jiangang Yang, 2014, Valuing Private Equity, The Review of Financial Studies, 27, pp. 1977-2021

[65] Strebulaev, Ilya A. and Whited, Toni M., Dynamic Models and Structural Estimation in Corporate Finance (December 1, 2012). Final pre-publication version, published in Foundations and Trends in Finance 6 (2012), 1-163., Available at SSRN: https://ssrn.com/abstract=2091854

[66] Tereza Tykvová and Mariela Borell, 2012, Do Private Equity Owners Increase Risk of Financial Distress and Bankruptcy, Center for European Economic Research, Discussion Paper No. 11-076

[67] Warner, Jerold B., 1977, Bankruptcy costs: Some evidence, Journal of Finance 32, 337–347.

[68] Weiss, Lawrence A., 1990, Bankruptcy resolution: Direct costs and violation of priority of claims, Journal of Financial Economics 27, 285–314.

[69] Vassalou, Maria and Yuhang Xing, Default Risk in Equity Returns, The Journal of Finance, 2004

[70] Ulf Axelson, Per Stromberg, and Michael S. Weisbach, Why are Buyouts Levered: The Financial Structure of Private Equity Funds, 2007, NBER Working Paper No. 12826

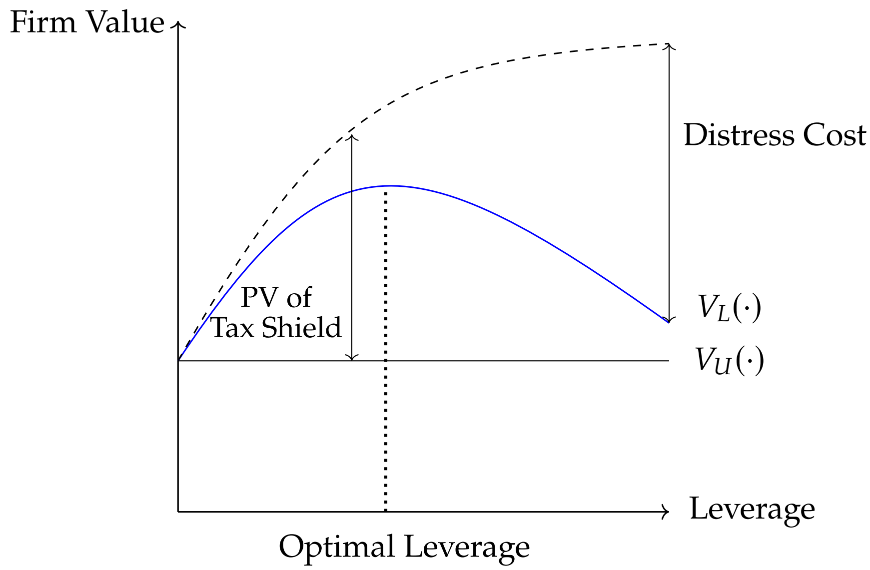
Appendix A. Data

Table A1: Variable Definition

Variable	Definition
Total Debt	Debt in Current Liabilities + Total Long-term Debt
Leverage Ratio	Total Debt Net of Cash /Total Assets
Long-term Debt Ratio	Total Long-term Debt Net of Cash /Total Assets
Debt Maturity	Total Long-term Debt/Total Debt
EBITDA Margin	Earnings Before Interest, Taxes Depreciation and Amortization/Operating Revenue
Return on Assets	Earnings Before Interest and Taxes/Assets
Cash Ratio	Cash/Current Liabilities
Tangibility	Share of Tangible Assets
Enterprise Value to Assets	(Equity + Long term debts + Loans minus cash and cash equivalent)/ Total Assets
Return on Capital Employed	(Net income + Interest paid) / (Shareholder funds + Non-current liabilities) * 100

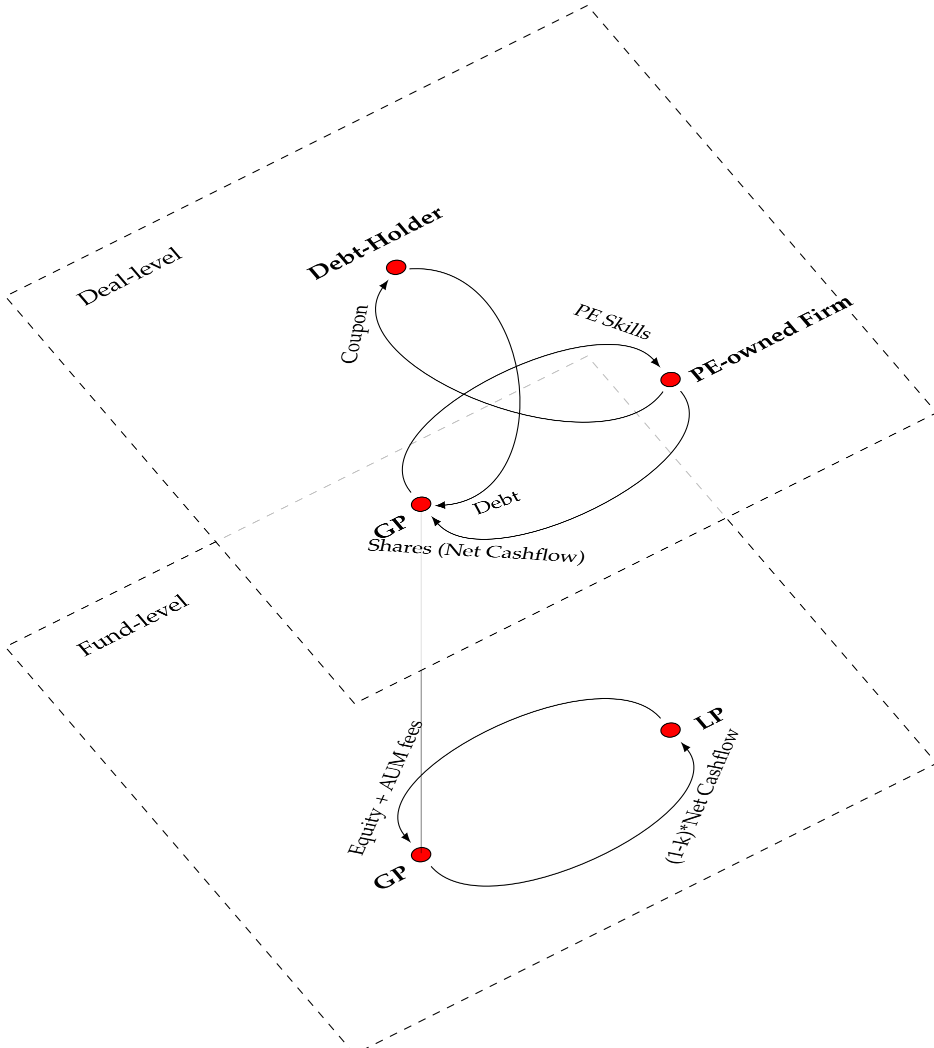
Appendix B. Model Details

Figure A1: Simplified Trade-off Theory



Notes: This figure is a simplified static illustration of a trade-off model. It shows optimal leverage depends on the present value of tax shields and distress costs of debt. V_U refers to the value of a firm without leverage, V_L with leverage. In addition to higher tax benefits, lower distress costs can raise value-maximising (optimal) leverage. Note the figure is not drawn to scale.

Figure A2: Flows between GP, LP, Debt-Holder and Portfolio Company



B1. Deriving Payoff to Debt-Holders: Over each time interval $[t, t + dt]$ the firm is servicing its debt holders by coupon at the rate C. Now consider any claim on the PE-firm that perpetually pays a non-negative coupon, C, per interval of time when the firm is solvent. If $A_t < C_t$, the company is in distress, and receives equity injection from the PE fund to finance liquidity shortfall; if the company cannot get external financing or chooses not to, it defaults.

As Leland (1994) outline, under the assumption of time-independence of coupon payouts, the valuation or HJB equation for debt-holders can be reduced to the following ordinary differential equation:

$$rV(a) = C + \mu AV'(a) + \frac{1}{2}\sigma^2 A^2 V''(a) \tag{25}$$

with general solution taking the form:

$$V(a) = K_0 + K_\gamma A^{-\gamma} + K_\eta A^\eta \tag{26}$$

where the coefficients are determined by boundary conditions. The two power parameters are roots to the fundamental quadratic equations:

$$\frac{1}{2}\sigma^2 x^2 + (\mu - \frac{1}{2}\sigma^2)x - r = 0 \tag{27}$$

where we assume $V(a) = a^x$, implying $V'(a) = xa^{x-1}, V''(a) = x(x-1)a^{x-2}$. For debt, the flow payoff can be expressed as:

$$D(A) = \frac{C}{r} + K_\gamma A^{-\gamma} + K_\eta A^\eta \tag{28}$$

Let ρ represent the fraction of asset value A_B which is lost in the event of bankruptcy. There are two boundary conditions. When $A = \infty$, default never occurs so $D = \frac{C}{r}$ perpetuity. Hence $K_\eta = 0$ otherwise debt value goes to infinity. Absolute priority rule applies, and debt-holder get the value of company's assets if the firm declares bankruptcy. When $A = A_B$, debt value is $\frac{(1-\rho)A_B}{r-\mu_a}$.[29] Substituting into Eq. (12) and solving for K_γ:

$$K_\gamma = \frac{\frac{(1-\rho)A_B}{r-\mu_a} - \frac{C}{r}}{A_B^{-\gamma}} \tag{29}$$

We can then derive closed-form analytical solution for debt value:

$$D(A) = (\frac{A}{A_B})^{-\gamma}\frac{(1-\rho)A_B}{r-\mu_a} + (1 - (\frac{A}{A_B})^{-\gamma})\frac{C}{r} \tag{30}$$

where $\gamma = (r - \delta - 0.5\sigma^2 + [(r - \delta - 0.5\sigma^2)^2 + 2\sigma^2 r]^{0.5})/\sigma^2$.

Observe that while Eq. (14) is convenient for numerical estimation, it can also be expressed in the familiar intuitive form:

$$D(A) = \mathbb{E}\,[\int_0^{\tau_B} e^{-rs} C ds + e^{-r\tau_B}\frac{(1-\rho)A_B}{r-\mu}] \tag{31}$$

$$= \mathbb{E}\,[\frac{C}{r}(1 - e^{-r\tau_B}) + e^{-r\tau_B}\frac{(1-\rho)A_B}{r-\mu}] \tag{32}$$

Eq. (16) expresses payoff to debt holders as a function of the likelihood of solvency and default and respective payouts in each states of the world. Following previous studies, τ_B

[29]The denominator follows from standard Gordon Growth formula.

is the first passage of time when cash flows fall below the bankruptcy-triggering level.

B2. Deriving the Payoff to the Equity-Holder (GP): Following standard derivation steps for the Black-Scholes-Merton PDE and assuming time-independence (i.e. $\frac{\partial E}{\partial t} = 0$), the GP's HJB equation reduces to the following ODE:

$$(r+\lambda)E(a) = f(a) + E^{'}(a)(\mu - \delta) + \frac{1}{2}\sigma^2(a)E^{''}(a) \tag{33}$$

Following Lan et al. (2013), the left-hand side of Eq. (19) changes from the standard discount rate r to $(r+\lambda)$, where is the mean arrival rate of a jump process. When the jump occurs, the deal matures and the investment in liquidated and committed capital is returned to the LP. For simplicity, I do not assume a hurdle rate since it will not change the optimization problem (i.e. I assume the hurdle rate is 0). The first term on the right hand side represents the cash flow generated by the firm per unit of time net of debt repayment as well as the management fee the GP receives. The second and third term capture the expected change in equity value caused by a fluctuation in the firm's asset value A_t. Without loss of generality, I set the share of profit the GP receives equal to 1. Next, I outline $f(a)$ to the GP as follows:

$$(r+\lambda)E(a) = mI_0 + [A_t - (1-\tau)C] + (r+\alpha-\delta)AE^{'}(a) + \frac{1}{2}\sigma^2 A^2 E^{''}(a) \tag{34}$$

Eq. (32) shows that payoffs to the GP's interest is different from the standard Black-Scholes-Merton PDE due to three terms: (i) $\frac{mt_0}{r}$, which represents the GP's inflow of ongoing asset management fees (ii) excess return and (iii) λ the stochastic deal exit probability. It is well-known that the general solution to $E(a)$ is given by the following expression:

$$E(a) = \frac{mI_0}{r'} + k\big[\frac{A_t}{r'-\mu} - \frac{(1-\tau)C}{r'} + K_\gamma A^{-\gamma} + K_\eta A^\eta\big] \tag{35}$$

where, for ease of notation, I define $r^{'} = r + \lambda$

The first boundary condition is when A approaches ∞. Equity value cannot grow faster than first best firm value which is linear in A, meaning $K_\eta = 0$. The second boundary condition is when $A = A_B$; GP still receives asset management fees $\frac{mI_t}{r}$, which are senior in nature and resemble a risk-free annuity per interval of time. This boundary condition captures the option-like structure of the GP's payoffs. It is worth recalling even without the GP's asset management fee, the payoff would resemble a plain vanilla call-option, an observation first made by Merton (1974). However, once could plausibly argue the GP's incentive to increase risk is higher compared to benchmark equity holders since in bankruptcy the former still receives a positive fee.

$$E(A_B) = \frac{A_B}{r' - \mu} - \frac{(1-\tau)C}{r'} + K_\gamma A^{-\gamma} + K_\eta A^\eta = \frac{mI_0}{r} \tag{36}$$

Using boundary conditions and rearranging yields:

$$K_\gamma = \frac{mI_0/r + \frac{(1-\tau)C}{r'} - \frac{A_B}{r'-\mu}}{A_B^{-\gamma}} \tag{37}$$

Substituting the expression for K_γ back into the Black-Scholes-Merton ODE gives the solved payoff function for the GP.

$$E(A) = \frac{mI_0}{r'} + [\frac{A_t}{r'-\mu} - \frac{(1-\tau)C}{r'}] + (\frac{mI_0}{r'} + \frac{(1-\tau)C}{r'} - \frac{A_B}{r'-\mu})(\frac{A}{A_B})^{-\gamma} \tag{38}$$

Note that the γ is now different since it is elevated by the α. We can then derive the optimal default barrier using standard smooth-pasting condition, where the solution to the default barrier is given in section 2:

$$h(c, A_B) = \frac{\partial E(A, A_B)}{\partial A}\Big|_{A=A_B} = 0 \tag{39}$$

Appendix C. Verifying LBO Sample Representatibility

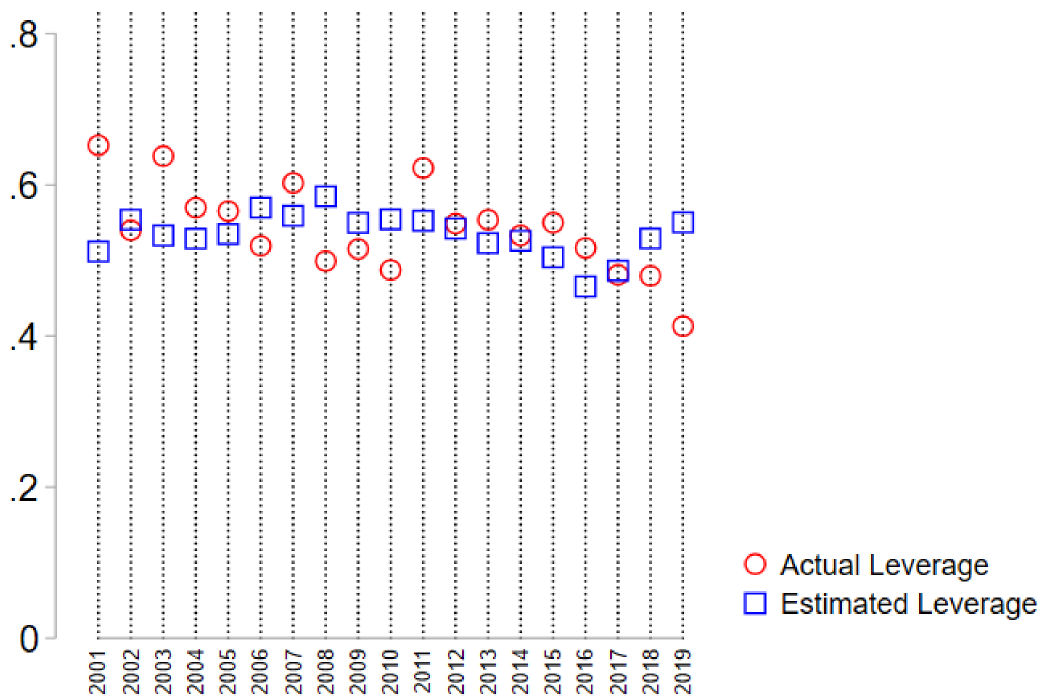
An important identification challenge is verifying that all deals retrieved are indeed leveraged buyouts undertaken by PE funds and not any other form of majority-owned private equity transaction. Ayash and Rastad (2018) survey the literature on LBOs, and suggest that researchers have difficulty differentiating between leveraged buyouts and other private equity investments. Since data providers typically cannot see deal leverage, they assume all buyouts are leveraged buyouts. They suggest it is possible a growth equity transaction can be majority-owned (i.e. a "growth equity buyout") and hence advocate using some form of a cut-off approach to filter out any non-LBO transactions, either based on transaction values or debt if capital structure is observable. Since I can observe firm-level debt, I use a cut-off approach based on debt itself, yielding a total of 1958 unique LBOs. I require that the level of debt scaled by asset rises in a company following the LBO is higher than the pre-LBO year. Next, I go through my entire sample of deals and confirm that the acquirers are all PE funds.

In addition, I take the following steps. First, as mentioned, I follow the standard practice in the literature to retrieve and clean accounting data from Orbis in order to ensure national representability following instructions in Kalemli-Ozcan (2015). Second, I compare moments in my sample with a benchmark paper that is known to have reliable data. Specifically, I compared mean and median firm size and leverage in my sample with the sample in Brown (2021) that uses high-quality propriety data from StepStone and a large international bank. I find firm size and leverage post-buyout are comparable: median and inter-quartile values of levarage are nearly identical. However, my dataset has time-series accounting information for several years pre and post-buyout while Brown (2021) observe data at deal entry and exit.

Appendix D. Additional Results

Appendix D1: Model Validation

Figure A1: Time-Series of Actual vs. Estimated Leverage



Notes: The chart above reports results from model-implied optimal leverage and actual leverage in the post-LBO sample from 2001-2019. Actual leverage is measured by Net Debt/Asset.

Table A2: Parameter Estimates

	Post-PE		Pre-PE	
Parameter	Estimate	SE	Estimate	SE
α	1.898	0.833	-	-
ln(size)	-0.134	4.00E-03	-0.246	5.00E-03
Tangibility	0.001	2.10E-04	0.001	2.00E-03
Profitability	-0.014	9.19E-05	-0.013	5.89E-05
Profit Vol.	0.002	9.19E-05	0.004	2.00E-03
Debt/Value	0.004	1.22E-04	0.004	1.42E-04
Constant	1.41	5.40E-02	2.895	8.90E-02
Predicted Value	Mean	Std. Dev	Mean	Std. Dev
σ_a	0.219	0.052	0.471	0.160

Notes: This table reports parameter estimates of Eq. (12) and Eq. (13). The first row reports the estimates of Excess return (α) according to the method proposed in Section 3. The estimate is in percent. The remaining rows provide estimates of the parameterization of asset volatility. Estimation procedure utilizes a Markov Chain Monte Carlo simulation with Gibbs sampling using diffuse priors. I use a total of 12500 iterations with a burn-in sample of 2500. Robustness checks have been carried out to ensure results are not sensitive to alternate specifications. The table also reports Monte Carlo Standard Errors (MCSE).

Table A3: Non-Targeted Moments

Moment	Data	Model
Credit Spread, p(50)	306	302
Cost of Debt, Mean	7.1	5.5
Cost of Debt, p(75)	4.6	6.1

Notes: This table reports specific moments in the data and those simulated by the model, in order to test external validity. The first column compared credit spreads in the actual data with model-implied spreads. Model-estimated spreads are restricted to the first 5 years after buyout to capture the median holding period of Brown (2021). Since my sample does not have information on spreads, I use data from Axelson et al. (2013) who report debt pricing over Libor in Table II. I use their reported values for Senior Term Loans B and C. The second and third row shows data from the sample used in this paper where cost of debt is computed as interest paid over current liabilities and long-term debt.

Table A4: Regression of Leverage on Value in Post-LBO sample

	Actual Data	Simulated Data
β	-0.0297***	-0.0231***
Standard Error	(0.0055)	(0.0042)
R^2	75.17	74.21
FE	Y	Y

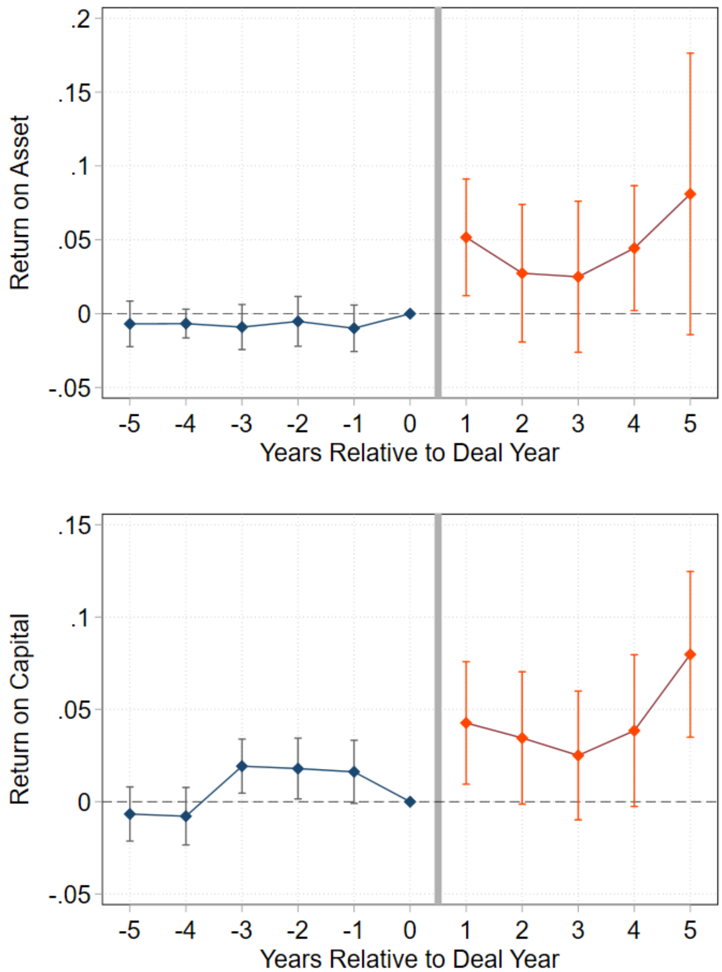
Notes: To validate the model, this table reports fixed-effects regression estimates of the specification taking the following form using both simulated data and actual data:

$$Leverage_{it} = \beta_0 + \beta_1 Value + \alpha_i + \epsilon_{it} \tag{40}$$

In the actual data, leverage is defined as before as Net Debt/Asset. Value is defined as the log of operating profits. In the simulated data, leverage is the optimal debt over firm value. Value is simulated directly from the model. Both regressions control for firm-specific fixed effects.

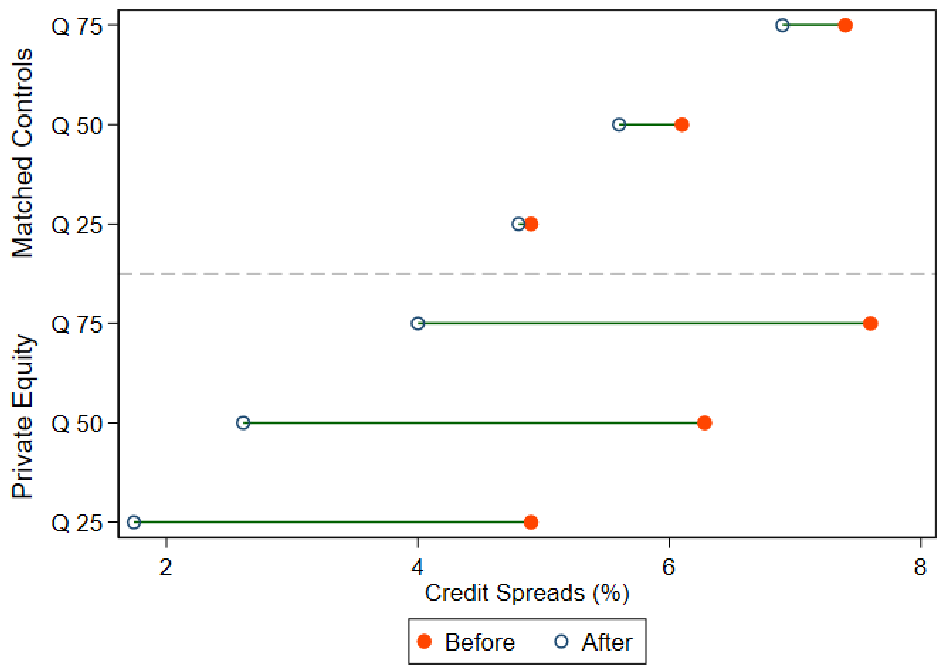
Appendix D2: Parallel Trends

Figure A2: Impact of PE-Ownership on ROA and ROCE and Evidence of Parallel Trends



Notes: This figure reports the dynamic difference-in-differences estimates specified in Eq. (24) around the PE buyout as well as its confidence interval estimated in the full sample. The dependant variable is Return on Capital Employed. The thick grey line separates the post-buyout sample from the pre-buyout sample. Standard errors are clustered by firm and time.

Figure A3: Structural model Credit Spreads Estimates



Notes: This figure reports model-estimated values of Credit Spreads. The estimation window is restricted to $t = [-5, 5]$. Credit Spreads is calculated as the difference in interest rate on debt paid by firm j and the risk-free rate. To estimate asset volatility, I deploy a Markov Chain Monte Carlo simulation with Gibbs sampling using diffuse priors. I use a total of 12500 iterations with a burn-in sample of 2500. Robustness checks have been carried out to ensure results are not sensitive to alternate specifications for asset risk as well as a variety of plausible priors for the simulation.