Illiquid Assets and Smoothed Returns

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Ohio State

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Considering Alternatives Investments? Factor in Illiquidity

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By Michael A. Pollock

Updated June 14, 2015 11:13 pm ET

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- Illiquidity induces (at least) two major issues

- This talk will focus on (1)

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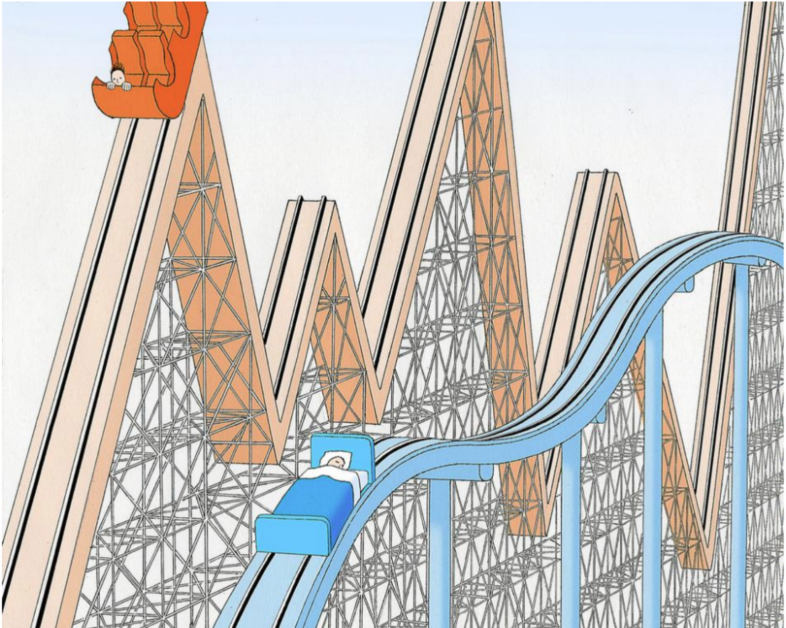

Low Volatility in Appraised Values

- Key point: low volatility due to (artificially) smoothed returns

Low Volatility in Appraised Values

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By Chris Cumming

May 26, 2022 6:30 am ET | WSJ PRO

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(Artificially) Smoothed Returns

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(1) What are smoothed returns?

(2) How do we detect smoothed returns empirically?

(3) Do illiquid assets display smoothed returns empirically?

(4) What is the main driver of smoothed returns?

(5) How do smoothed returns impact performance measurement?

(6) How can we solve this issue (at least partially)?

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Outline

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Defining Smoothed Returns

(1) What are smoothed returns?

- If that is the case, use market values to calculate returns!

- For illiquid assets, we very often do not observe market values
- So, illiquid assets often display smoothed returns
- But, ultimately, they are driven by the lack of market values
- Defining return terminology:

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$$log(V^o_{t+1}) \quad = \quad (1-\theta) \cdot log(V_{t+1}) + \theta \cdot log(V^o_t)$$

V is the economic value of the asset

V^o is the reported value of the asset

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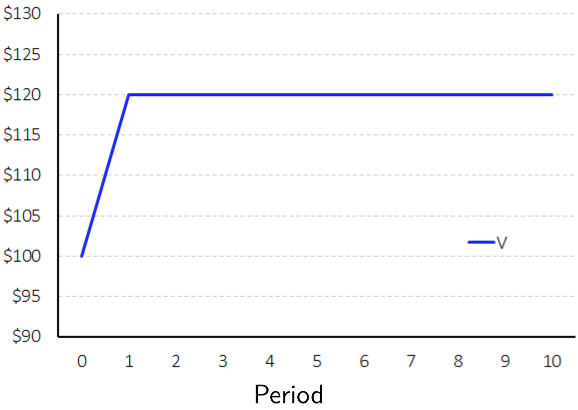

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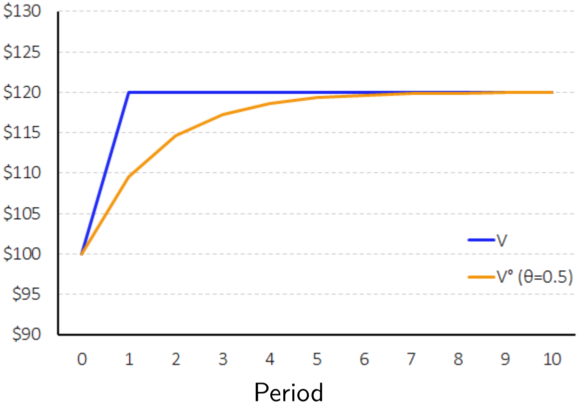

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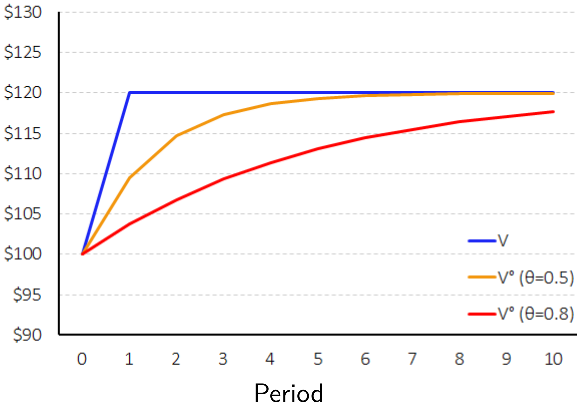

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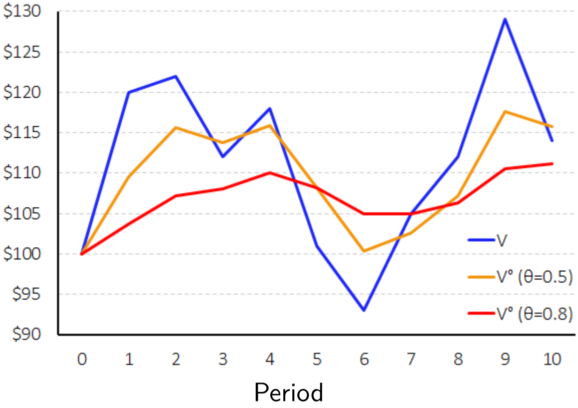

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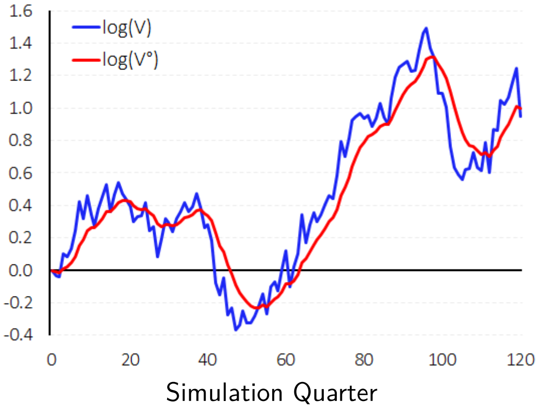

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- A simple way to think about smoothed returns is to consider

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r is the economic log return on the asset

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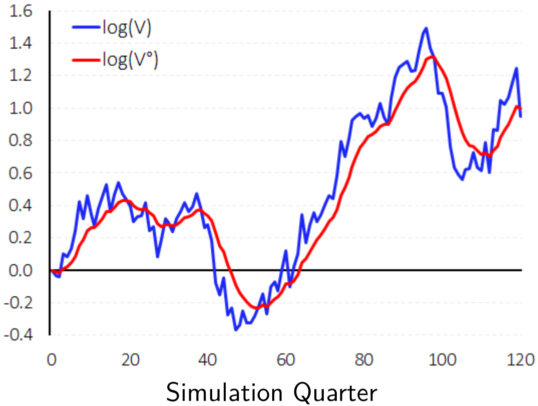

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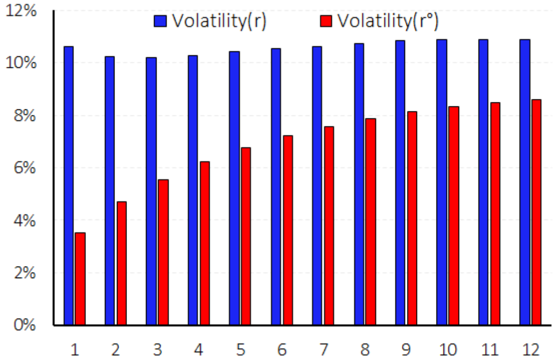

Returns Frequency (in Quarters)

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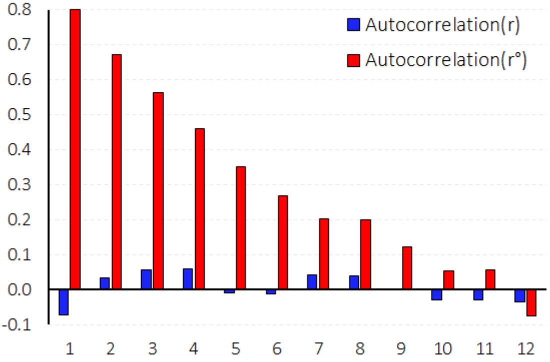

Autocorrelation Lag (in Quarters)

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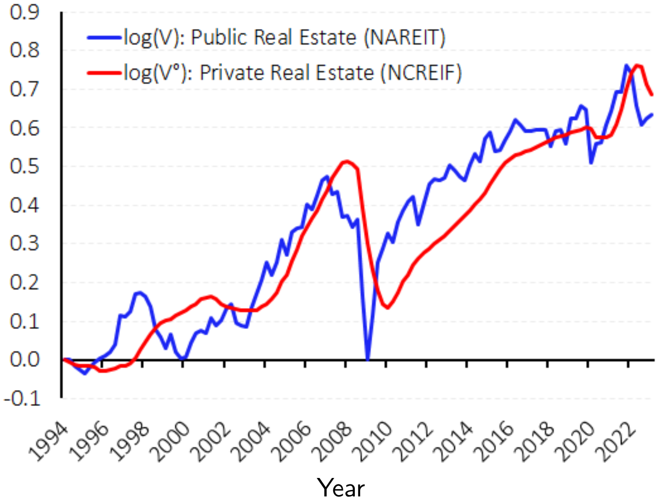

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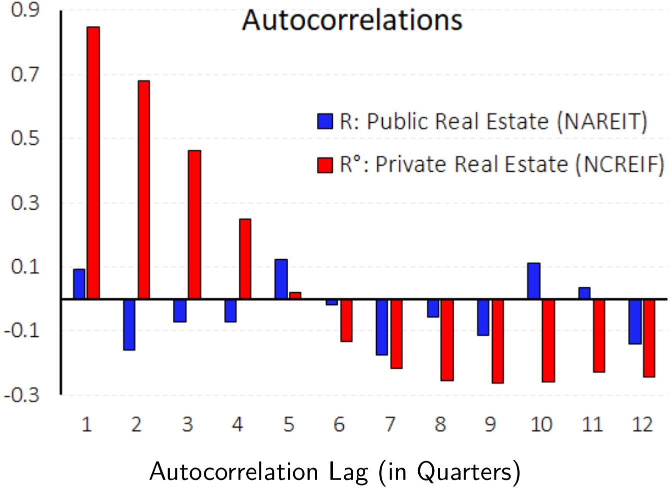

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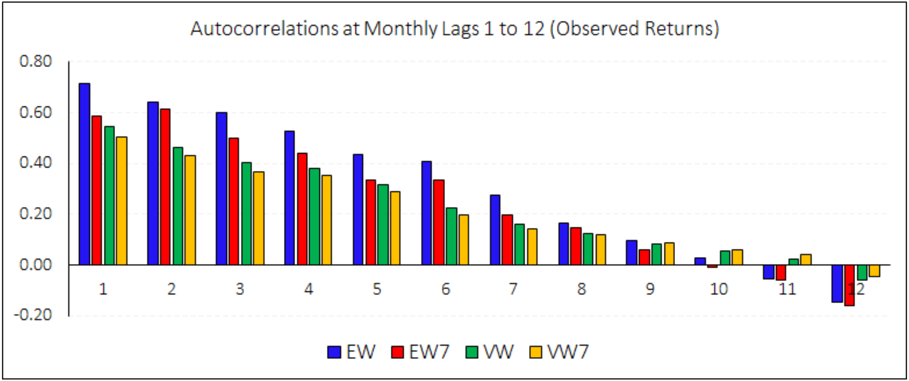

Source: Couts, Gonçalves (2024)

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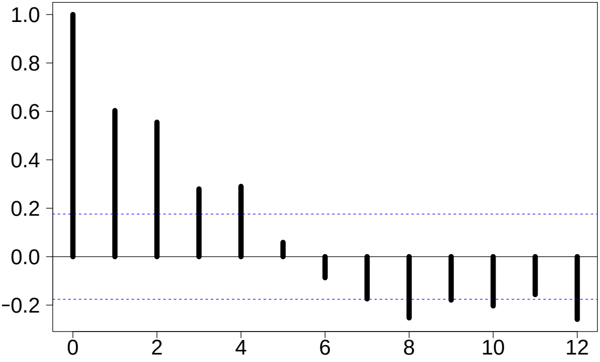
- For Private Equity Funds (Real Estate):



Source: Brown, Gonçalves, Hu (2024)

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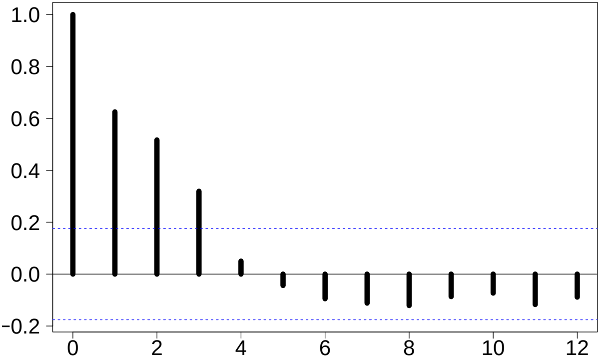

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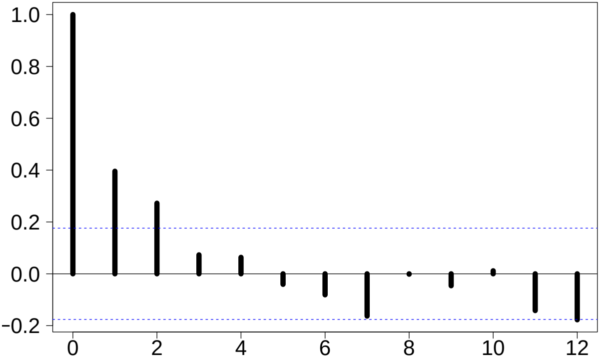
- For Private Equity Funds (Buyout):



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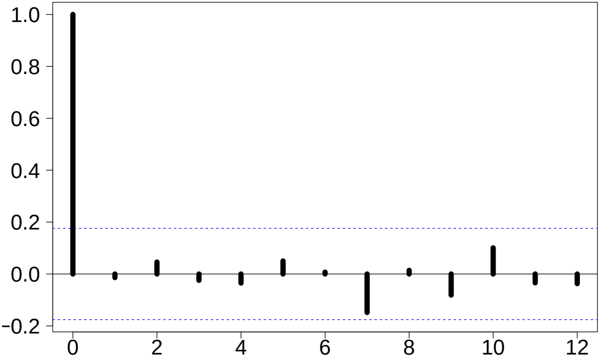
- For Public Equities:



Source: Brown, Gonçalves, Hu (2024)

Detecting Smoothed Returns (Evidence)

(3) Do illiquid assets display smoothed returns empirically?

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- It is not only about private vs public markets
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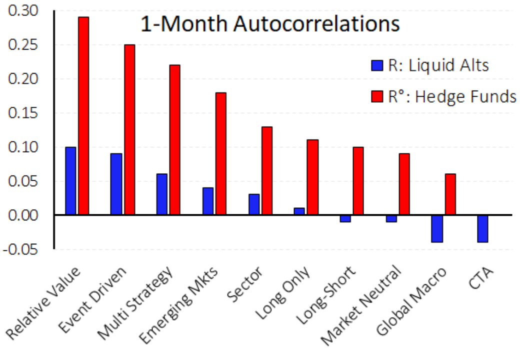

Source: Couts, Gonçalves, Rossi (2020)

Outline

Main Drivers of Smoothed Returns

(4) What is the main driver of smoothed returns?

- Managers report AUM based on the value of their asset
- But managers only observe noisy signals of illiquid asset values
- So, optimal estimates of illiquid AUM imply smoothed returns (we formalize this statement when discussing return unsmoothing)
- But managers may manipulate reported AUM to "look good"
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Table 2. Lyxor and Main Fund Smoothing

MA(1) model	Main fund	Lyxor	Difference
Average θ_1	0.182	0.121	0.061
t-Statistic	(9.22)	(6.28)	(4.35)

Source: Cao, Farnswoth, Liang, Lo (2017)

Internal vs External Appraisals of Fund Properties

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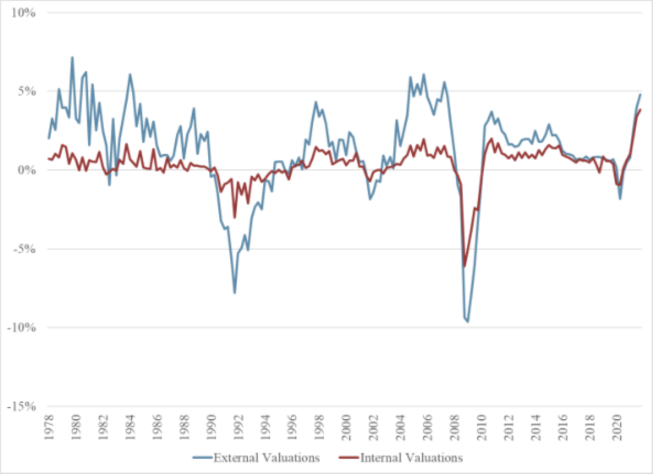

Figure 1
External vs. Internal Indexes (all observations)

Source: Couts (2024)

Internal vs External Appraisals of Fund Properties

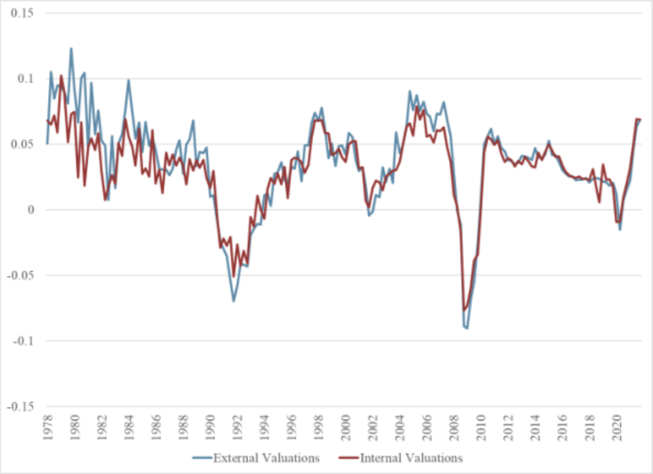

Figure 2
External vs Internal Indexes (no lame valuations)

Source: Couts (2024)

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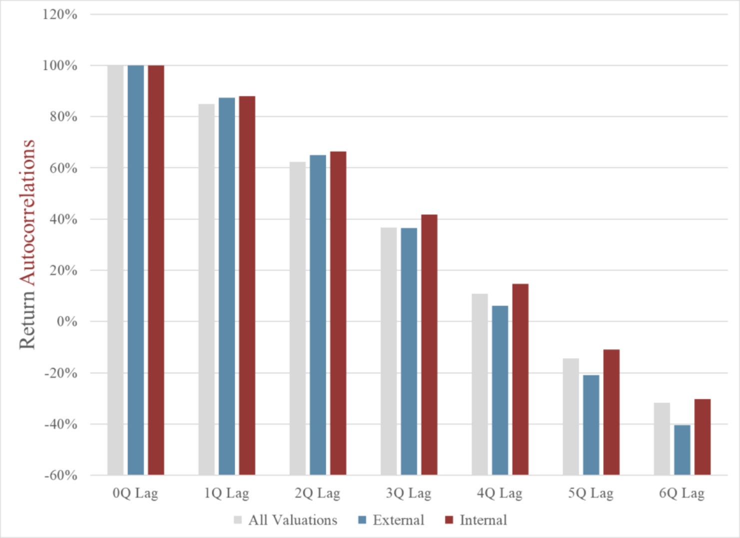

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Outline

Effect of Smoothed Returns on Performance Measurement

(5) How do smoothed returns impact performance measurement?

- Consider a simple example:

- In this case, the risk exposure estimated using R_t^o is

- And the estimated alpha is

- So, $\beta^o < \beta$ and $\alpha^o > 0$

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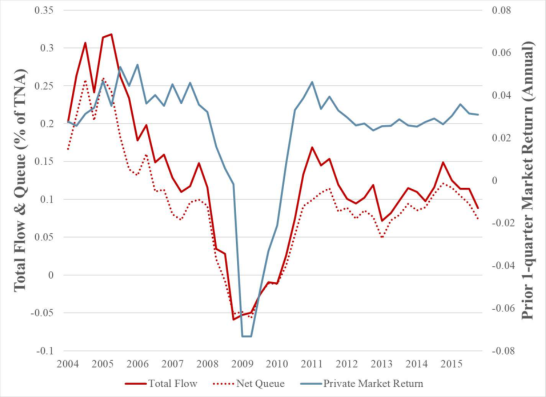

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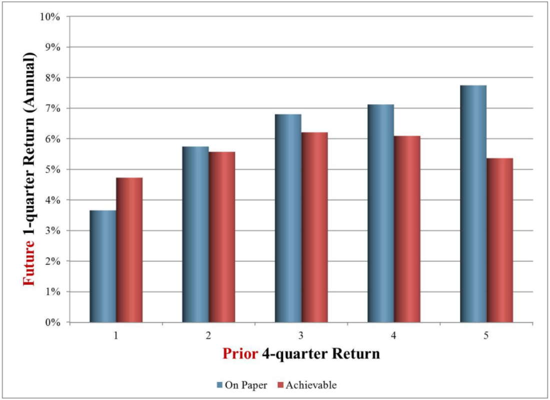

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- Identification: $R_{j,t} = \mu_j + \eta_{j,t} \quad \text{with} \quad \eta_{j,t} \overset{iid}{\sim} \mathcal{N}(0, \sigma_j^2)$

- This is a MA(H): moving average process of order H

- Estimate μ_j and $\theta_j^{(h)}$ by MLE (Getmansky, Lo, Makarov (2004))
  (MLE typically imposes $\theta_j^{(0)} = 1$, so divide estimates by $\sum_{h=0}^{H} \theta_j^{(h)}$)

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- Identification: $R_{j,t} = \mu_j + \eta_{j,t} \quad \text{with} \quad \eta_{j,t} \overset{iid}{\sim} \mathcal{N}(0, \sigma_j^2)$

- This is a MA(H): moving average process of order H

- Estimate μ_j and $\theta_j^{(h)}$ by MLE (Getmansky, Lo, Makarov (2004))
(MLE typically imposes $\theta_j^{(0)} = 1$, so divide estimates by $\sum_{h=0}^{H} \theta_j^{(h)}$)

- Get $\eta_{j,t}$ from MA(H) residuals to obtain unsmoothed returns:

MA(H) Unsmoothing

- Reported returns (R^o) reflect past economic returns (R):

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AR(1) Unsmoothing

- Reported returns (R^o) reflect past economic returns (R):

$$R^o_{j,t} = (1-\theta_j) \cdot R_{j,t} + \theta_j \cdot R^o_{j,t-1}$$

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- Identification: $R_{j,t} = \mu_j + \eta_{j,t}$ with $\eta_{j,t} \overset{iid}{\sim} \mathcal{D}ist(0, \sigma_j^2)$

- This is an AR(1): autoregressive process of order 1

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Autocorrelation Functions: MA(1) and AR(1)

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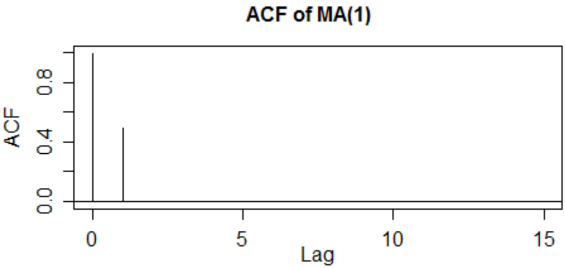
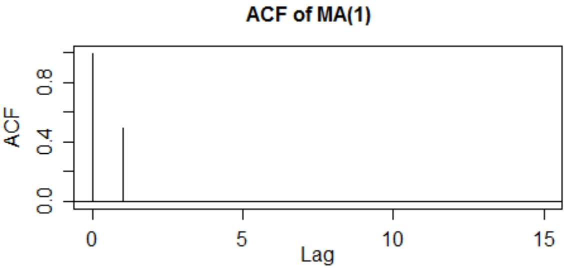
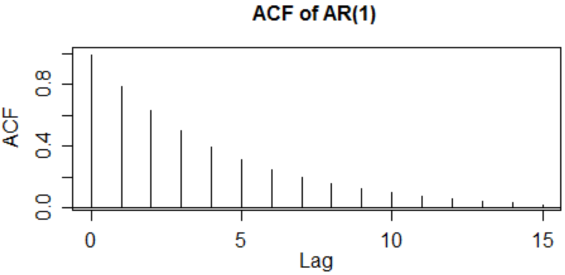


Autocorrelation Functions: MA(1) and AR(1)





Outline

Motivation: Autocorrelations (1 Lag)

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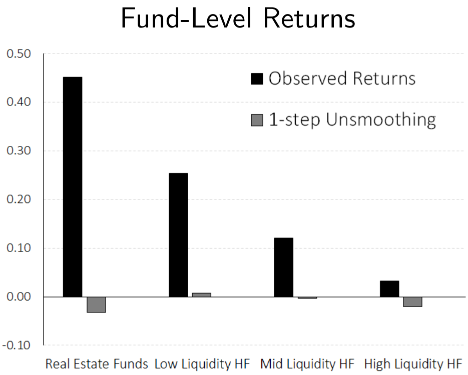

Source: Couts, Gonçalves, Rossi (2020)

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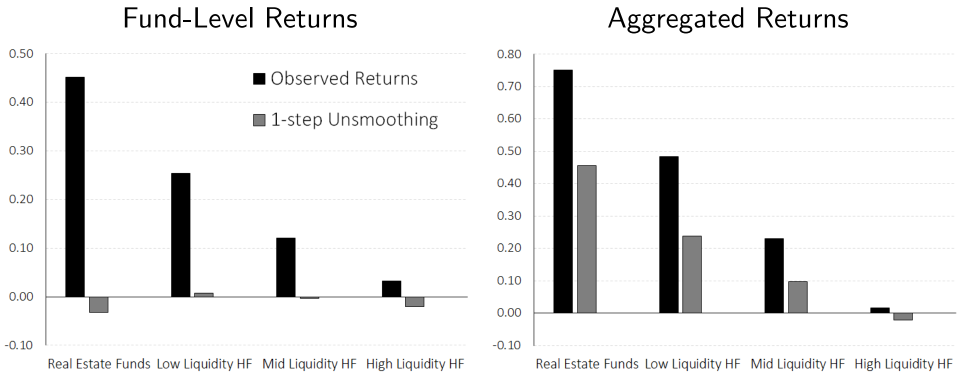

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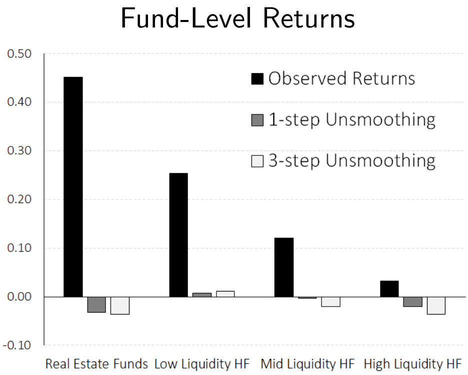

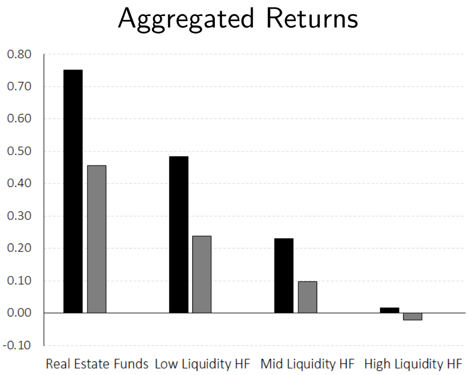

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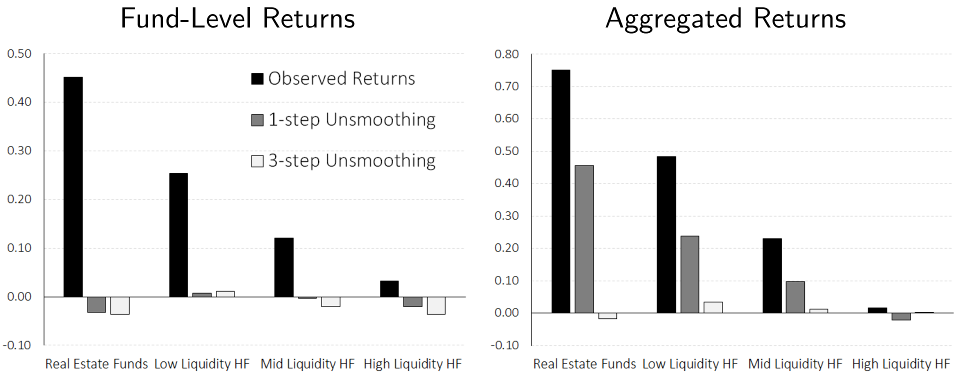

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3-Step MA(H) Unsmoothing

- R^o reflects past aggregate ($\overline{R}$) and relative ($\widetilde{R}$) returns:

- **Step 1**: $\overline{\eta}_t$ from MA(H) on $\overline{R}^o_t$

- **Step 2**: $\widetilde{\eta}_{j,t}$ from MA(H) on $\widetilde{R}^o_{j,t}$ with $\overline{\eta}s$ as covariates

- **Step 3**: $R_{j,t} = \mu_j + \widetilde{\eta}_{j,t} + \overline{\eta}_t$

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$$R^o_{j,t} = \sum_{h=0}^{H} \phi_j^{(h)} \cdot \widetilde{R}_{j,t-h} + \sum_{h=0}^{H} \pi_j^{(h)} \cdot \overline{R}_{t-h}$$

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&= \mu_j + \sum_{h=0}^{H} \phi_j^{(h)} \cdot \widetilde{\eta}_{j,t-h} + \sum_{h=0}^{H} \pi_j^{(h)} \cdot \overline{\eta}_{t-h}
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$$\overline{R}^o_{j,t} \approx \mu_j + \sum_{h=0}^{H} \pi_j^{(h)} \cdot \overline{\eta}_{t-h}$$

- **Step 2**: $\widetilde{\eta}_{j,t}$ from MA(H) on $\widetilde{R}^o_{j,t}$ with $\overline{\eta}$s as covariates

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- **Step 1**: $\overline{\eta}_t$ from MA(H) on $\overline{R}^o_t$

$$\overline{R}^o_{j,t} \approx \mu_j + \sum_{h=0}^{H} \overline{\pi}^{(h)} \cdot \overline{\eta}_{t-h}$$

- **Step 2**: $\widetilde{\eta}_{j,t}$ from MA(H) on $\widetilde{R}^o_{j,t}$ with $\overline{\eta}$s as covariates

- **Step 3**: $R_{j,t} = \mu_j + \widetilde{\eta}_{j,t} + \overline{\eta}_t$

3-Step MA(H) Unsmoothing

- R^o reflects past aggregate ($\overline{R}$) and relative ($\widetilde{R}$) returns:

$$
\begin{aligned}
R^o_{j,t} &= \sum_{h=0}^{H} \phi_j^{(h)} \cdot \widetilde{R}_{j,t-h} + \sum_{h=0}^{H} \pi_j^{(h)} \cdot \overline{R}_{t-h} \\
&= \mu_j + \sum_{h=0}^{H} \phi_j^{(h)} \cdot \widetilde{\eta}_{j,t-h} + \sum_{h=0}^{H} \pi_j^{(h)} \cdot \overline{\eta}_{t-h}
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- **Step 2**: $\widetilde{\eta}_{j,t}$ from MA(H) on $\widetilde{R}^o_{j,t}$ with $\overline{\eta}s$ as covariates

$$\widetilde{R}^o_{j,t} = \mu_j + \sum_{h=0}^{H} \phi_j^{(h)} \cdot \widetilde{\eta}_{j,t-h} + \sum_{h=0}^{H} \psi_j^{(h)} \cdot \overline{\eta}_{t-h}$$

- **Step 3**: $R_{j,t} = \mu_j + \widetilde{\eta}_{j,t} + \overline{\eta}_t$

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3-Step AR(1) Unsmoothing

- R^o reflects past aggregate ($\overline{R}$) and relative ($\widetilde{R}$) returns:

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\begin{aligned}
R^o_{j,t} &= (1-\phi_j)\cdot \widetilde{R}_{j,t} + \phi_j \cdot \widetilde{R}^o_{j,t-1} \;+\; (1-\pi_j)\cdot \overline{R}_t + \pi_j \cdot \overline{R}^o_{j,t-1} \\
&= \mu_j \;+\; \phi_j\cdot(\widetilde{R}^o_{j,t-1} - \widetilde{\mu}_j) \;+\; \pi_j \cdot (\overline{R}^o_{j,t-1} - \overline{\mu}_j) \;+\; \epsilon_{j,t}
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- **Step 1**: $\epsilon_{j,t}$ are residuals in the time-series regression above
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&= \mu_j \;+\; \phi_j\cdot(\widetilde{R}^o_{j,t-1} - \widetilde{\mu}_j) \;+\; \pi_j\cdot(\overline{R}^o_{j,t-1} - \overline{\mu}_j) \;+\; \epsilon_{j,t}
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- **Step 3**: $R_{j,t} = \mu_j + \widetilde{\eta}_{j,t} + \overline{\eta}_t$ where

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Outline

Smoothed Returns with Bayesian Fund Manager: MA(1)

- Consider a simple framework for $v_{j,t} = log(V_{j,t})$:

- At t, the manager learns $v_{j,t-1}$ and observes the noisy signal

- The manager reports the Bayesian estimate for the fund value,

- In this case, the reported return is

Smoothed Returns with Bayesian Fund Manager: MA(1)

- Consider a simple framework for $v_{j,t} = log(V_{j,t})$:

$$v_{j,t} = \mu_j + v_{j,t-1} + \eta_{j,t} \qquad \text{with} \quad \eta_{j,t} \sim \mathcal{N}(0, \sigma_j^2)$$

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- At t, the manager learns $v_{j,t-1}$ and observes the noisy signal

$$\widehat{\eta}_{j,t} \; = \; \eta_{j,t} \; + \; u_{j,t} \qquad \text{with} \quad u_{j,t} \sim \mathcal{N}(0, \widehat{\sigma}_j^2)$$

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$$v^o_{j,t} \;\; = \;\; \mu_j \; + \; v_{j,t-1} \; + \; \underbrace{\mathbb{E}[\eta_{j,t} | \widehat{\eta}_{j,t}]}_{\theta_j^{(0)} \cdot \widehat{\eta}_{j,t}}$$

where $\theta_j^{(0)} = (1/\widehat{\sigma}_j^2)/(1/\sigma_j^2 + 1/\widehat{\sigma}_j^2)$

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$$r^o_{j,t} \quad = \quad v^o_{j,t} \; - \; v^o_{j,t-1} \; = \; (v_{j,t-1} - v_{j,t-2}) \; + \; \theta_j^{(0)} \cdot (\widehat{\eta}_{j,t} - \widehat{\eta}_{j,t-1})$$

$$= \; \theta_j^{(0)} \cdot r_{j,t} \; + \; (1 - \theta_j^{(0)}) \cdot r_{j,t-1} \; + \; \xi_{j,t}$$

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$$v_{j,t}^o = \mu_j + v_{j,t-1} + \underbrace{\mathbb{E}[\eta_{j,t} | \widehat{\eta}_{j,t}]}_{\theta_j^{(0)} \cdot \widehat{\eta}_{j,t}}$$

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$$= \theta_j^{(0)} \cdot r_{j,t} + (1 - \theta_j^{(0)}) \cdot r_{j,t-1} + \xi_{j,t}$$

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$$v_{j,t} = \mu_j + v_{j,t-1} + \eta_{j,t} \qquad \text{with} \quad \eta_{j,t} \sim \mathcal{N}(0, \sigma_j^2)$$

- At t, the manager learns $v_{j,t-1}$ and observes the noisy signal

$$\widehat{\eta}_{j,t} = \eta_{j,t} + u_{j,t} \qquad \text{with} \quad u_{j,t} \sim \mathcal{N}(0, \widehat{\sigma}_j^2)$$

- The manager reports the Bayesian estimate for the fund value,

$$v_{j,t}^{o} = \mu_j + v_{j,t-1} + \underbrace{\mathbb{E}[\eta_{j,t} | \widehat{\eta}_{j,t}]}_{\theta_j^{(0)} \cdot \widehat{\eta}_{j,t}}$$

where $\theta_j^{(0)} = (1/\widehat{\sigma}_j^2)/(1/\sigma_j^2 + 1/\widehat{\sigma}_j^2)$

- In this case, the reported return is

$$\begin{aligned} r_{j,t}^{o} &= v_{j,t}^{o} - v_{j,t-1}^{o} = (v_{j,t-1} - v_{j,t-2}) + \theta_j^{(0)} \cdot (\widehat{\eta}_{j,t} - \widehat{\eta}_{j,t-1}) \\ &= \theta_j^{(0)} \cdot r_{j,t} + (1 - \theta_j^{(0)}) \cdot r_{j,t-1} + \xi_{j,t} \end{aligned}$$

where $\xi_{j,t} = \theta_j^{(0)} \cdot (u_{j,t} - u_{j,t-1})$

Smoothed Returns with Bayesian Fund Manager

- Our Bayesian smoothing process is a MA(1) except for $\xi_{j,t}$
- We can ignore the $\xi_{j,t}$ term (Couts, Gonçalves, Rossi (2020))
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Outline

Hedge Funds

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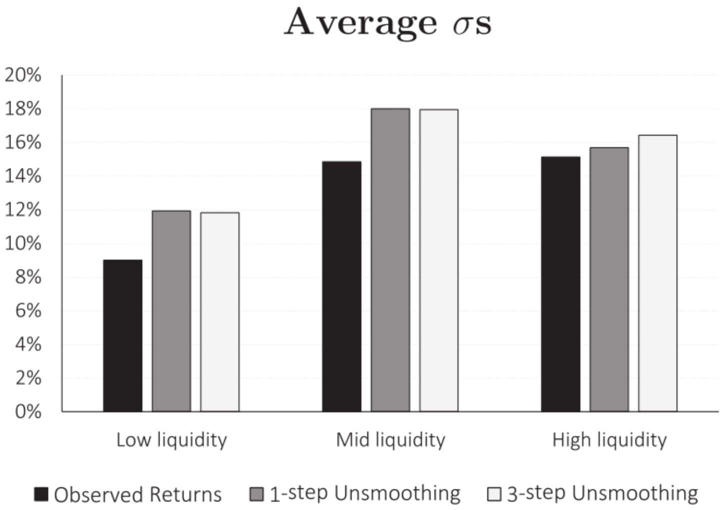

Source: Couts, Gonçalves, Rossi (2020)

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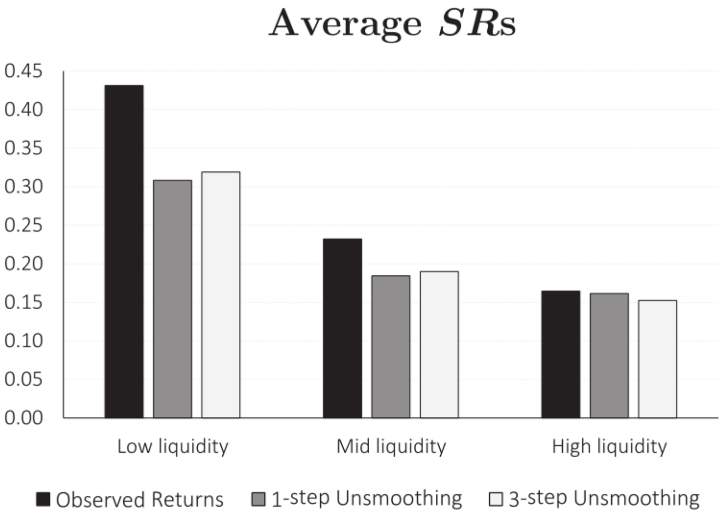

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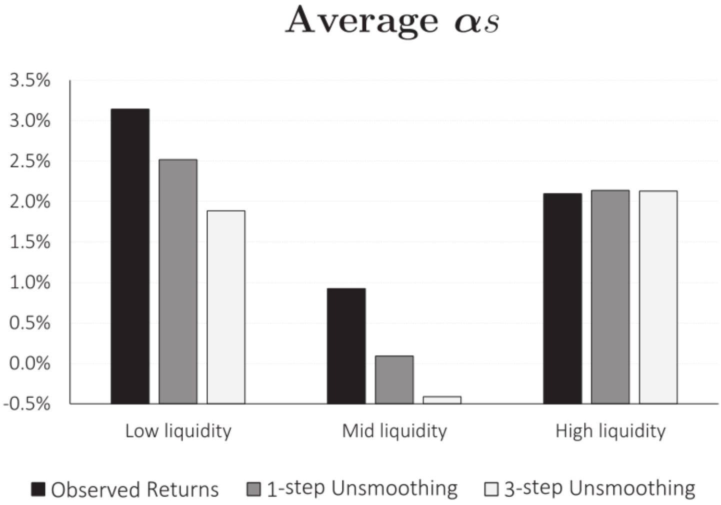

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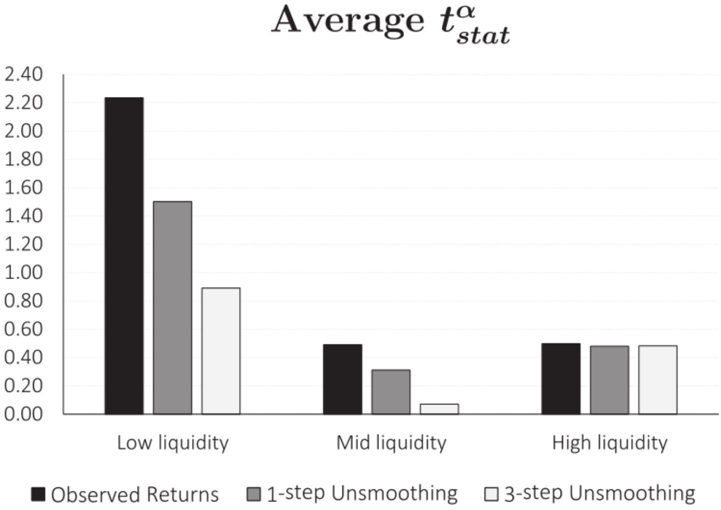

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Commercial Real Estate Funds

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Table 7
Risk and performance of private CRE funds

Statistics are related to	**Raw performance**		
	$\mathbb{E}[r]$	σ	$\mathbb{E}[r]/\sigma$
Observed, R_o	5.0%	13.1%	0.38
1-step, R_{1s}	5.0%	25.3%	0.20
3-step, R_{3s}	5.0%	24.1%	0.21

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Statistics are related to	Raw performance			1-factor model	
	$\mathbb{E}[r]$	σ	$\mathbb{E}[r]/\sigma$	α	β_{re}
Observed, R_o	5.0%	13.1%	0.38	4.3%	0.07
1-step, R_{1s}	5.0%	25.3%	0.20	2.7%	0.22
3-step, R_{3s}	5.0%	24.1%	0.21	1.6%	0.34

Source: Couts, Gonçalves, Rossi (2020)

Commercial Real Estate Funds

Table 7
Risk and performance of private CRE funds

Statistics are related to	Raw performance			1-factor model		2-factor model		
	$\mathbb{E}[r]$	σ	$\mathbb{E}[r]/\sigma$	α	β_{re}	α	β_{re}	β_e
Observed, R_o	5.0%	13.1%	0.38	4.3%	0.07	4.0%	0.02	0.10
1-step, R_{1s}	5.0%	25.3%	0.20	2.7%	0.22	2.4%	0.15	0.15
3-step, R_{3s}	5.0%	24.1%	0.21	1.6%	0.34	0.8%	0.21	0.26

Source: Couts, Gonçalves, Rossi (2020)

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Conclusion

- Illiquidity induces two major issues
- We explored (1) in this talk
- There are alternative approaches to deal with (1)
- Need more work on how (2) affects performance measurement (see Brown, Gonçalves, Hu (2024) for early work on that)

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- Illiquidity induces two major issues
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