Advanced Methods for Private Fund Performance Analysis

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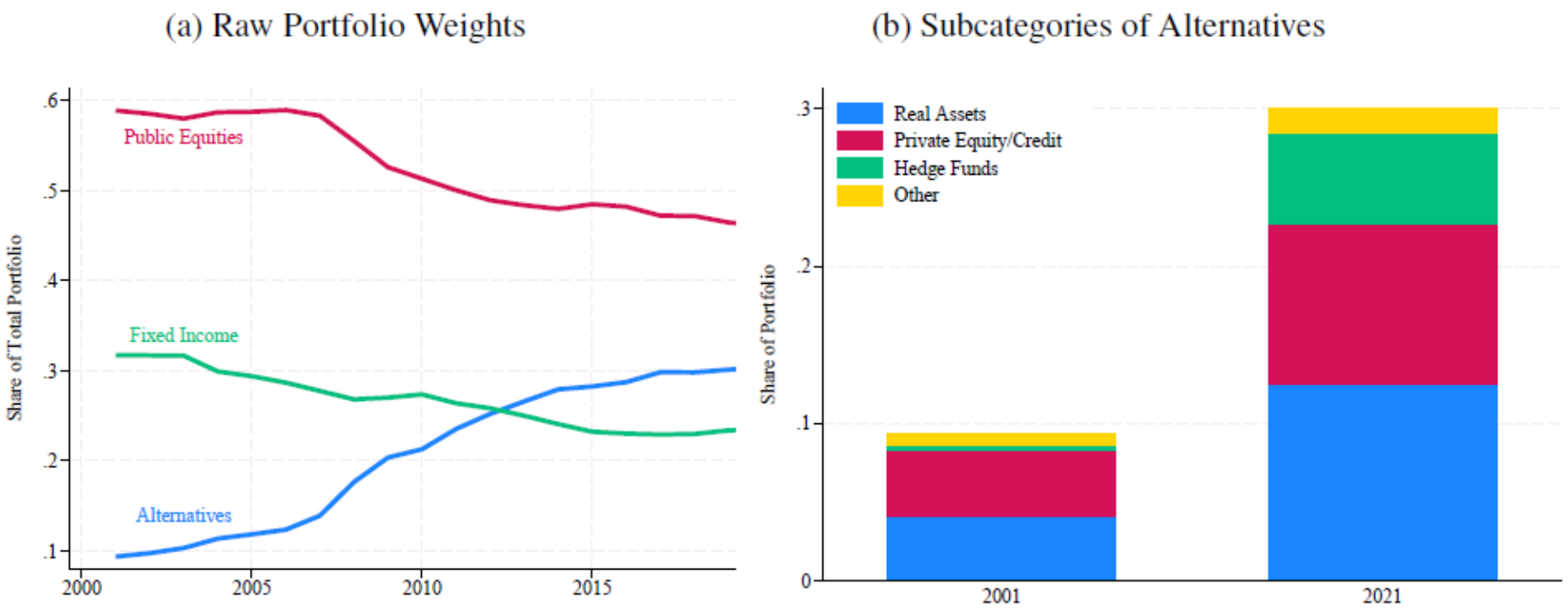
U.S. Public Pension Portfolios

Figure 2: The Composition of the Aggregate Risky Portfolio



Notes: Panel (a) of the figure plots the target share of public equities, alternatives, and fixed income (including cash) for the aggregate U.S. public pension portfolio. Panel (b) shows the aggregate portfolio shares of different categories of alternatives for 2001 and 2021. Panel (c) plots the share of alternatives in the risky portfolio. All data are based on the PPD. Risky investments are defined as any holding outside of fixed income and cash.

Source: Begenau, Liang, and Siriwardane, 2024, “The Rise of Alternatives”

Risk-adjustment

- Why?
  - Assess portfolio exposure to risk factors
  - Separate GP skill from risk-taking
    - Leverage (credit lines, NAV loans)

- How?
  - No regularly observed fund returns: standard asset pricing methods cannot be directly applied
  - Benchmarks
    - Drawbacks: subjective, gaming
  - Recent academic literature has focused on stochastic discount factor (SDF) pricing
    - Duality with factor models such as CAPM, Fama-French

Stochastic Discount Factor

- Value of a future cash flow:

$$V_t = E_t[M_{t+1} \cdot C_{t+1}]$$

(random) cash flow at time *t+1*

- *M* is the stochastic discount factor (SDF)
  - Random variable (as of time *t*)
  - Closely related to investor's marginal utility and *state prices*

If investor feels $\begin{Bmatrix} \text{rich} \\ \text{poor} \end{Bmatrix}$ at t+1, then M_{t+1} is $\begin{Bmatrix} \text{low} \\ \text{high} \end{Bmatrix}$

- Note: Unlike factor model-based discount rates, SDF does not vary by security/fund

Private Capital Fund Pricing

- The value of a sequence of random cash flows is the sum of their values
- ➔ For a fund with J cash flows, its net present value at inception (time *t*) is

$$NPV = E_t \sum_{j=1}^{J} M_{t+\tau(j)} C_j$$

time of cash flow j since inception

- Notes:
  - Normalize cash flows (for example, to a $1 commitment) to compare across funds
  - Distributions are positive cash flows, capital calls are negative
  - For not-yet-liquidated funds, add product of final NAV and SDF (treating final NAV as a pseudo-distribution)

Public Market Equivalent (PME)

- Economic content is in the specification of the SDF
  - SDF is a function of risk factors
- Example: PME

$$M = \frac{1}{R^M} \Rightarrow PME = \sum_{j=1}^{J} \frac{1}{R^M_{t+\tau(j)}} C_j$$

E[PME] is the NPV of the fund

- Close, but not exactly the Kaplan-Schoar PME
  - KS-PME is a *ratio* of discounted distributions and capital calls
  - If fund matches public market performance, then KS-PME = 1, whereas the above PME = 0
- Advantages of this version:
  - Better behaved statistically
  - The PME of a portfolio is the weighted average of its constituents' PMEs (unlike KS-PME)

Generalized PME (GPME)

- PME is limiting:
  - Public market performance is the only risk factor
  - Restricted functional form of SDF (log-utility investors)
- Korteweg-Nagel (2016) assume *exponential-affine SDF*

$$M_{t+1} = \exp(\delta - \gamma \cdot r^{M}_{t+1})$$

log market return from *t* to *t+1*

- This SDF compounds nicely over time and has a theoretical basis (power utility; γ is risk aversion coeff.)
- If PE payoffs can be replicated by a (possibly levered) portfolio of the benchmark assets in the SDF (here: the public market), then $\mathrm{E}[GPME] = 0$.
- PME is a special case: $\delta = 0, \gamma = 1$ ➔ $M_{t+1} = 1/R^{M}_{t+1}$
- Can add additional factors (Fama-French, etc.)

Code: https://sites.google.com/view/korteweg/home/data-code

(G)PME Estimates

- Korteweg-Nagel (2024) show results using MSCI-Burgiss data (<2017 vintages, cash flows until 2022Q1)

	Venture Capital (N=1,630)			Buyout (N=1,073)		
	α	GPME	PME	α	GPME	PME
Mean		-0.184	0.258		0.190	0.161
Percentiles:						
10th		-1.193	-0.683		-0.740	-0.305
25th		-0.632	-0.392		-0.334	-0.106
Median		-0.293	-0.059		-0.138	0.098
75th		-0.005	0.370		0.238	0.350
90th		0.735	1.256		1.446	0.671
St.Dev.		1.377	1.407		1.632	0.553
Skewness		6.106	5.092		5.696	7.774
Kurtosis		79.148	41.365		66.053	146.155
p-value		0.195	0.000		0.304	0.000
SDF Parameters						
δ		0.182	0		0.211	0
		(0.061)			(0.068)	
γ		3.613	1		3.780	1
		(0.690)			(0.648)	

Relation to Factor Models

- SDF pricing and standard factor models like CAPM and Fama-French are closely related
- With the additional assumption of *lognormal returns,* the exponential-affine SDF implies the log-CAPM

$$\log E[R_t] - r_f = \beta\left(\log E[R_{mt}] - r_f\right)$$

- Benchmarking interpretation:
  - If the SDF pricing equation holds (GPME=0), then a security's return can be replicated by a (levered) investment in the benchmark portfolios (here: the public market portfolio)
- Advantage of the SDF approach relative to log-CAPM is that it does not require the lognormality assumption

Leverage

- Subscription lines, NAV loans increase fund-level leverage
  - Modigliani-Miller: leverage increases fund beta
- GPME properly accounts for effects of leverage
  - No implicit assumption about fund betas
- PME does not
  - implicitly assumes beta = 1

leverage	PME	GPME
k	Log-utility CAPM	CAPM
	(1)	(2)
-0.8	-0.053	-0.021
	(0.016)	(0.026)
	[0.001]	[0.428]
-0.6	-0.027	-0.041
	(0.023)	(0.041)
	[0.234]	[0.320]
0	0.048	-0.103
	(0.061)	(0.042)
	[0.430]	[0.015]
1	0.174	-0.205
	(0.131)	(0.121)
	[0.182]	[0.089]
2	0.301	-0.308
	(0.201)	(0.209)
	[0.135]	[0.141]

Korteweg-Nagel (2016) Table 3
Standard errors in parentheses
p-values in square brackets

Fund-level Alpha

- The GPME/SDF approach works well at the *asset class* level, taking averages across many funds/vintages
- GPME for an *individual* fund is unbiased but noisy
- Korteweg-Nagel (2024) develop a fund alpha measure that is consistent with GPME but less noisy for individual funds
- Idea is to find a benchmark factor model return that has same pricing implication as GPME
  - Produces beta estimates
- Additional assumptions:
  1. All funds have the same factor loadings (“betas”)
  2. Returns are lognormal

Alpha Estimates

	Venture Capital (N=1,630)			Buyout (N=1,073)		
	α	GPME	PME	α	GPME	PME
Mean	-0.184	-0.184	0.258	0.190	0.190	0.161
Percentiles:						
10th	-0.920	-1.193	-0.683	-0.289	-0.740	-0.305
25th	-0.566	-0.632	-0.392	-0.085	-0.334	-0.106
Median	-0.282	-0.293	-0.059	0.130	-0.138	0.098
75th	-0.027	-0.005	0.370	0.379	0.238	0.350
90th	0.469	0.735	1.256	0.719	1.446	0.671
St.Dev.	0.965	1.377	1.407	0.563	1.632	0.553
Skewness	5.638	6.106	5.092	7.790	5.696	7.774
Kurtosis	59.430	79.148	41.365	147.151	66.053	146.155
p-value		0.195	0.000		0.304	0.000
β	2.373			0.908		
SDF Parameters						
δ		0.182	0		0.211	0
		(0.061)			(0.068)	
γ		3.613	1		3.780	1
		(0.690)			(0.648)	

Other SDF Specifications

- SDF/GPME approach accommodates multiple factors
  - Fama-French 3 factor, 5 factor models, etc.
    - VC behaves like small-growth firms
    - Buyout tends to loads on value
  - Exposure to Pastor-Stambaugh liquidity factor
    - Franzoni-Nowak-Phalippou, 2012, Ang et al., 2018
  - Idiosyncratic risk
  - Some evidence of PE-specific factor
    - Korteweg-Sorensen 2010, Ang et al. 2018
- Gredil, Sorensen, Waller (2020) use consumption CAPM based SDFs
  - External habit formation
  - Long-run risks
  - VC appears to be a good hedge for post-crisis consumption shocks

    ➔ VC performance not as bad as CAPM suggests.

Term Structure

- Gupta and Van Nieuwerburgh (2019) show that the term structure of interest rates matters.
  - For example, management fee is a short bond (from LPs' perspective).
  - They also consider other equity exposures, such as growth and value stocks, and REITs.

- Methodologically, instead of evaluating PE cash flows at SDF realizations and taking expectations, they use *state prices (strips)*.
  - Pro: Avoid estimating expectations over short time series and small cross-sections (especially when looking at subsamples).
  - Con: more structure needed for the asset pricing model.

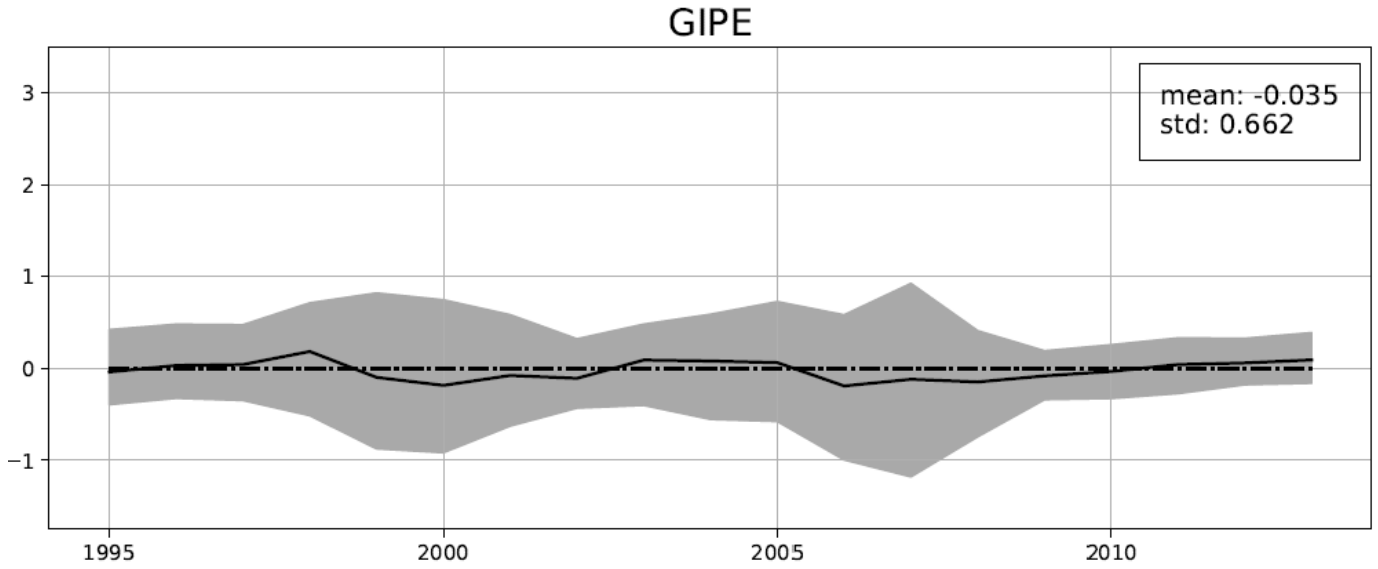
Heterogeneity in Betas

- Time-varying betas?
  - Over vintages
    - E.g., over the macro-cycle
  - Over fund life
    - E.g., in buyout: market beta drops as debt is paid down
    - Nonlinear (option-like) payoffs: option betas are highly (time-)varying
  - In principle not an issue for GPME

- Fund-specific betas?
  - Also not an issue for GPME
  - Brown-Ghysels-Gredil (2023) provide estimates of fund-specific betas.

Investor Portfolio Equivalent

- LP-specific pricing
  - All portfolios are not created equal
  - How does private capital fit into a specific LP's portfolio?
- (Generalized) Investor Portfolio Equivalent (Korteweg-Panageas-Systla, 2024)
  - Use investor's own return in SDF, instead of public equity market
  - Apply to U.S. public pension plans



Conclusion

- Much progress in understanding risk and return in PE.
  - For more details and references, see: “Risk and Return in Private Equity” (in: Handbook of the Economics of Corporate Finance, Vol 1: Private Equity and Entrepreneurial Finance).
- Some risks remain un(der)explored:
  - Liquidity
  - Idiosyncratic risk
  - Existence of PE-specific factor(s)?
- Other open questions remain
  - Skill vs luck in generating alpha?
  - Agency and risk-taking at the fund level?
  - LP portfolio formation?
    - Within PE budget
    - With other asset classes: public equities, fixed income, etc.