

The Investment Properties of Real Asset Funds: A Survey of Prior Work and New Findings *

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Abstract

We study the risk and return properties of private real estate, infrastructure, and natural resources funds using a large dataset on private real assets. Employing quarterly index series, we first develop an ARMA–Dimson unsmoothing procedure that restores realistic volatility and co-movement with liquid public benchmarks while preserving buy-and-hold performance. Second, at the fund-level, we find that core and generalist real estate funds, on average, underperform listed REITs over the full sample. However, in contrast to prior findings using data from before the Global Financial Crisis, performance since then has outstripped public markets, driven primarily by the strong performance of value-add and opportunistic funds. Private infrastructure funds have, on average, outperformed publicly traded infrastructure equities. Natural resources funds generally lag their public counterparts throughout our sample. Taken together, once smoothing is stripped out and returns are evaluated against appropriate public comparators, we find that manager, style, and vintage selection are central to realized outcomes.

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1 Introduction

Institutional investment in private real assets has expanded dramatically over the last two decades. Global allocations to these asset classes have risen from less than \$200 billion in 2005 to more than \$1.6 trillion in 2023, reflecting a shift in portfolio strategy toward income stability, inflation protection, and diversification relative to public markets (Andonov et al., 2021; Andonov et al., 2018b). Yet, despite their scale and strategic importance, the historical risk-adjusted performance of private real assets remains a topic of ongoing debate. Differences in sample construction, benchmark selection, and smoothing bias have produced inconsistent findings regarding both average returns and systematic risk exposures. Consequently, the academic and practitioner communities continue to disagree on a basic question: Do private real assets deliver superior risk-adjusted returns and/or provide meaningful diversification benefits?

Prior studies have provided valuable but fragmented insights. The real estate literature documents the pervasive influence of appraisal smoothing, showing that reported returns substantially understate true volatility and distort correlations with public markets (Geltner, 1993; Geltner and Goetzmann, 1998). Similar concerns apply to infrastructure and natural resource funds, whose reported net asset values (NAVs) exhibit serial dependence. Meanwhile, broader research on private fund performance has introduced methodological techniques to properly compare private fund performance to public market benchmarks. For example, studies of private equity and venture capital use the public market equivalent (PME) method of Kaplan and Sensoy (2005) and the direct alpha method of Gredil et al. (2023).¹

To fully understand the performance of real asset funds requires addressing two measurement challenges persist. The first is the estimation of unsmoothed, market-consistent returns obtained from returns derived from appraisal-based NAVs. Reported NAV-based returns are typically auto-correlated because NAVs do not reflect market prices in a timely manner, consequently it can take several quarters for reported returns to fully incorporate market prices. This inherent smoothing of reported returns generates estimates of risk and correlation with other assets that are too low. The longer the lag in appraisal-based value estimates, the greater the serial correlation and decline in observed volatility which mask sensitivity to market factors (Dimson, 1979). The second chal-

¹The academic literature has recently introduced some more sophisticated model such as the generalized PME model of Korteweg and Nagel (2016) which we discuss in Section 6, but do not use in our primary analysis

lenge lies in selecting appropriate benchmarks. Unlike buyout or venture capital funds, whose performance can be linked to broad equity indices, private real assets span a very heterogeneous mix of strategies and geographies—from core stabilized office portfolios to regulated concessions and renewable energy development projects—that each respond differently to macroeconomic and market conditions. As a result, broad market indices fail to capture the more nuanced risk exposures of these investments.

This paper seeks to address these challenges by developing an integrated framework to measure and interpret the performance of private real asset funds. Specifically, our analysis has four main contributions.

First, we collect what we believe to be the most comprehensive dataset of private real asset funds studied to date. We rely primarily on the MSCI Private Capital Universe of closed-end investment funds though we also consider open-end real estate funds in some of our analysis. The MSCI dataset provides the net performance experience of fund limited partners (LPs) including complete cash flows and quarterly net asset values for a set of 1,877 funds we examine representing 2.2 trillion USD in committed capital. Our dataset covers vintage years from 1992 through 2019 and is current through the first quarter of 2025.² The sample includes 1,210 real estate funds, 255 infrastructure funds, and 357 natural resource funds.

Second, we introduce a novel unsmoothing model for quarterly fund returns that unifies appraisal-based unsmoothing and multi-lag beta estimation within a single econometric framework. Building on models that use autoregressive, moving-average (ARMA) models such as Geltner (1993), Dimson (1979), and the *MSCI Private Infrastructure Factor Model* (Bustos and DeMond, 2025), we estimate unsmoothed quarterly return series for time-weighted return series based on the MSCI private fund data. The model simultaneously accounts for serial correlation in reported returns and lagged responses to public-market shocks, enabling us to recover volatility and beta estimates that reflect contemporaneous market dynamics. We call this method the ARMA-Dimson model.

Third, we examine the correlations of these unsmoothed return series with publicly traded comparators (e.g., REITs, listed infrastructure companies, and natural resource companies) and

²As is typical in the academic literature, we exclude vintages from 2020 onward because most funds are still in their investment periods and reported returns are unlikely to reflect much information about realized performance. Performance of funds from the more recent vintages is available from MSCI Private Capital Universe by subscription.

macro variables such as expected inflation and real interest rates. This cross-asset perspective allows us to quantify the extent to which smoothing biases understate correlations and to isolate asset-specific risk channels. For instance, real estate funds are expected to exhibit large sensitivities to public equities and real interest rates, while natural resource funds may load heavily on inflation expectations (Peng and Newell, 2007; ?).

Fourth, we investigate cross-sectional performance and dispersion across strategies and vintages. Using the MSCI fund-level data, we examine the distribution of the total value to paid-in capital ratio (TVPI), the internal rate of return (IRR), the Kaplan-Schoar public market equivalent (PME), and the direct alpha (Gredil et al., 2023) metrics across all real asset categories. We then analyze whether unsmoothed performance differentials persist once adjusted for market exposure. This exercise complements recent large-sample studies of private capital performance (Brown et al., 2024) by extending comparable methodologies to more granular subsets of real assets.

Our findings generate several key insights. At the index level, unsmoothing raises measured volatility by roughly a factor of two or more, bringing private real asset risk much closer to that of their liquid counterparts. The annualized standard deviation of our real assets composite index increases from about 7% to roughly 16%, and the volatility of core open-end real estate indices rises from about 6% to 15%, making them comparable to listed real estate and infrastructure equities. Unsmoothing materially changes the correlation structure. Correlations with corresponding public benchmarks typically rise by 0.2–0.4: the correlation of private infrastructure with listed infrastructure, for instance, increases from roughly 0.5 in reported data to about 0.7 after adjustment, while the correlation of private real estate with global REITs moves from the low 0.3s to the mid 0.4s. This reduces the apparent diversification benefits implied by smoothed NAVs and yields a more realistic measure of systematic risk.

As we move to the fund level and compare net performance to public benchmarks using PME and direct alpha, we find that private real assets, in aggregate, have delivered modest outperformance relative to public markets. For all real asset funds we find a median PME of 1.03, and a median direct alpha of 0.9% per year as compared to public REITs. Real estate funds as a group have a median PME of just 1.01, with core/generalist strategies slightly underperforming value-add and opportunistic funds. However, we document consistently good market-adjusted performance of

real estate funds post-GFC, and especially so for value-add and opportunistic strategies. Private infrastructure funds exhibit somewhat stronger performance: PMEs of 1.05–1.10, and median Direct Alphas on the order of 1–2% per year relative to listed infrastructure benchmarks. In contrast, natural resources funds show the weakest net performance: their median PME is well below one (0.81) and their median Direct Alpha is negative (–3.9% per year), indicating underperformance relative to public natural resources equities.

We document pronounced cross-sectional and cyclical heterogeneity. Value-add and opportunistic real estate and infrastructure funds exhibit wider dispersion in IRR, TVPI, PME, and Direct Alpha than core strategies, with economically large gaps between top- and bottom-quartile managers. Vintage analysis shows that GFC-era cohorts (circa 2007–2010) experience the broadest left tails and the largest shortfalls relative to public markets, while post-2012 vintages display tighter outcome distributions, PMEs closer to or slightly above one, and modestly positive median direct alphas, especially in infrastructure and higher-risk real estate strategies. Finally, our ARMA-Dimson factor analysis reveals that infrastructure funds have the strongest and most stable link to macroeconomic variables, particularly real interest rates and expected inflation, whereas natural resources funds exhibit the widest underlying return dispersion and cyclicality, with returns highly sensitive to commodity prices. Real estate lies between these extremes, with core strategies showing lower betas and tighter distributions than value-add and opportunistic funds. Together, these results suggest that once smoothing is removed and appropriate benchmarks are applied, private real assets behave like risk assets with meaningful exposure to public markets and macro factors, and that investors' realized outcomes are driven as much by manager and vintage selection as by any aggregate asset-class premium.

The contribution of this paper is both empirical and methodological. Empirically, it provides a comprehensive characterization of private real asset performance across strategies, vintages, and geographies, using a consistent dataset and common benchmarking framework. Methodologically, it demonstrates how ARMA–Dimson unsmoothing can bridge the gap between traditional appraisal-based adjustments and modern factor models, extending recent innovations in private capital measurement (Bustos and DeMond, 2025; Brown et al., 2024). The resulting analysis offers a unified perspective on how smoothing, factor exposure, and fund heterogeneity interact to shape observed performance in private real assets.

Ultimately, this research seeks to clarify the economic meaning of reported private real asset returns. By isolating the statistical artifacts introduced by appraisal smoothing and aligning fund-level data with market-based risk factors, we contribute to a clearer understanding of whether these assets offer genuine diversification or simply the illusion of stability (i.e., volatility laundering). Our findings aim to guide both institutional investors and researchers toward more accurate, comparable, and transparent assessments of private real asset performance.

2 Literature Review

The literature on private real assets lies at the intersection of work on private capital performance, appraisal-based valuation and index construction, and asset-class-specific studies of real estate, infrastructure, and natural resources. This section synthesizes those strands and highlights how our analysis builds on and extends prior work.

2.1 Benchmarking Tools from Private Equity

Early empirical research on private equity and venture capital established many of the measurement challenges that also apply to private real assets. Phalippou and Gottschalg (2008) document that, net of fees, the average private equity fund only modestly outperforms public equity benchmarks, with substantial dispersion across managers. ? likewise find economically meaningful performance persistence and strong procyclicality of fundraising and commitments, while Harris et al. (2013) provide a broad synthesis showing that, on average, buyout funds have historically outperformed public markets, with weaker evidence for venture capital. Gredil et al. (2023) propose the "direct alpha" measure that converts relative performance into a continuously compounded excess return, allowing for a more direct comparison with alphas estimated using factor models. Korteweg and Nagel, 2016 extend these ideas to venture capital, deriving riskadjusted returns that account for non-linear payoffs and timing option value. Building on these contributions, Brown et al. (2024) use the MSCI data to estimate aggregate "private capital alpha" across strategies, documenting modest positive abnormal performance on average, with substantial dispersion and important differences across buyout, venture, and real estate investments. Our

³Sorensen and Jagannathan (2015) formalize the Public Market Equivalent (PME).

fund-level analysis follows this benchmarking tradition by anchoring private real asset funds to investable public comparators.

2.2 Appraisal Smoothing, Liquidity, and Index Properties

The real estate literature has long emphasized that appraisal-based indices understate true volatility and distort correlations with public markets. Geltner (1993) shows that appraised values can be viewed as a smoothed transformation of underlying market values, and derives unsmoothing filters that recover more realistic return dynamics using a first-order autocorrelation structure (i.e., AR(1) model). Geltner and Goetzmann (1998) construct an NCREIF-based index that exploits independent appraisals to document commercial property returns while accounting for smoothing and reporting lags. Subsequent work refines the interpretation of smoothing and the statistical design of real estate indices. Childs et al. (2002) model the optimal valuation of noisy real assets, highlighting how information frictions and partial adjustment affect observed prices. Fisher et al. (2003) show that variable market liquidity materially affects commercial real estate price indices, and propose methods to disentangle genuine price movements from liquidity-induced noise. Geltner and Ling (2006) discuss practical considerations in designing investable real estate indices, including sample selection, leverage, and appraisal frequencies. Horrigan et al. (2009) argue that REIT-based property return indices can serve as high-frequency, tradable proxies for underlying commercial real estate, bridging the gap between smoothed appraisal series and marketpriced exposures.

Research examining public market illiquidity has also developed methods useful for analysis of private fund returns. Dimson (1979) demonstrates, in the context of thinly traded equities, that infrequent trading requires multi-lag regressions to obtain unbiased beta estimates, a logic that carries over directly to appraisal-based real asset returns. Getmansky et al. (2004) formalize this idea for hedge funds by developing an econometric model in which serial correlation arises from return smoothing and illiquidity, and by showing how smoothing distorts volatility and Sharpe ratios and how to recover smoothing-adjusted risk measures. More recently, Couts et al. (2024) show that funds investing in similar illiquid assets share a common source of spurious autocorrelation that standard unsmoothing filters fail to remove; they propose a generalized unsmoothing

procedure that better recovers systematic risk exposures and risk-adjusted performance for hedge funds and private commercial real estate funds. Complementary work on hedge funds by Dichev and Yu (2011) highlights that money-weighted (IRR) investor returns can lag time-weighted fund returns when capital flows are poorly timed, underscoring the importance of distinguishing fund-level performance from investor-level outcomes in illiquid vehicles.

Our ARMA–Dimson model is directly motivated by this body of work. We treat reported private real asset returns as the outcome of an appraisal filter applied to underlying economic shocks, estimate that filter via low-order ARMA models, and use Dimson-style regressions to recover long-horizon betas to public benchmarks Geltner (1993); Dimson (1979); Geltner and Goetzmann (1998).

2.3 Real Estate Funds, Style Differentiation, and Portfolio Role

Within private real estate, the literature documents differences across strategies and evaluates real estate's role in multi-asset portfolios. Fisher and Hartzell (2016) show that value-added and opportunistic funds deliver disappointing returns and also exhibit greater risk relative to core strategies. Pagliari (2020) further examines whether value-added and opportunistic funds have "pulled their weight," finding that nominal return premia largely reflect leverage, cyclicality, and timing rather than alpha. Pagliari (2016) shows that real estate's diversification benefits depend on whether one uses appraisal-based, REIT-based, or unsmoothed indices. These portfolio-level findings align with the smoothing literature: appraisal indices understate volatility and overstate diversification (Geltner and Ling, 2006). Once unsmoothed, real estate appears as a risk asset with meaningful exposure to growth, inflation, and real-rate shocks.

2.4 Infrastructure Funds: Risk, Return, and Investor Demand

Private infrastructure investing has attracted little attention in the academic literature. Work by Peng and Newell (2007) and Newell et al. (2010) emphasizes infrastructure as a source of long-dated, inflation-linked cash flows with low public-market correlation. Using a large sample of infrastructure funds, Andonov et al. (2021) show that average PMEs relative to broad public markets are near or below one. Andonov et al. (2018b) argue that rapid institutional inflows reflect

regulatory and political objectives rather than strong historical outperformance. Further work investigates performance drivers. Haran et al. (2019) analyze cash flows and valuation drivers; Shen and Blanc-Brude (2022) study how infrastructure behaves in portfolios. Bustos and DeMond (2025) develop the MSCI Private Infrastructure Factor Model, showing exposures to public equity growth, real rates, credit spreads, and a latent private factor. Our empirical work follows this literature by examining risk and return properties using a larger and more recent dataset.

2.5 Natural Resources and Direct Real Asset Investments

Natural resource investments—including energy, mining, timber, and agriculture—are promoted as inflation hedges, but empirical evidence shows substantial cyclicality and tail risk. Newell et al. (2010) and Peng and Newell (2007) show that unlisted infrastructure and resource vehicles can improve outcomes in moderate inflation regimes while transmitting commodity shocks. Cremers (2013) examine direct ownership of natural resource and real asset investments, concluding that outcomes depend heavily on asset selection and timing.

2.6 Investor Base, Governance, and Capital Allocation to Real Assets

Recent research links investor behavior to private real asset markets. Andonov et al. (2021) and Andonov et al. (2018b) highlight the central role of pension funds and policy-driven investors in infrastructure growth. Andonov et al. (2018a) show that governance structures influence alternative-asset allocations. Bernstein et al. (2013) document sovereign wealth fund investment patterns. Lerner et al. (2005) identify an LP performance puzzle, suggesting that realized outcomes depend critically on governance and manager selection. Our fund-level dispersion results are consistent with this literature: while median returns are close to public benchmarks, performance across funds varies widely both within and across fund vintages.

Taken together, the extant literature cited above suggests that (i) private fund performance is often, but not always, better than public markets yet there is substantial variation across fund types, individual funds and time; (ii) appraisal smoothing and index construction meaningfully affect measured volatility and correlations; (iii) real estate, infrastructure, and natural resources each exhibit distinct risk–return and macro-exposure profiles; and (iv) institutional behavior and

governance likely shape real asset realized performance of LPs in real asset markets.

3 Data

We combine proprietary and public sources to construct a comprehensive quarterly dataset covering private real asset fund performance, open-end core real estate benchmarks, global public market indices, and macroeconomic series. In our analysis, all return series are measured at a quarterly frequency and expressed in percentage terms unless otherwise noted. Descriptive statistics for all return series are reported in Table 1, and summary distributions of fund-level outcomes (TVPI, IRR, PME, and Direct Alpha) appear in Table 2. A complete list of variable definitions and data sources is provided in Table 6.

We include in our analysis almost all primary real asset funds in the MSCI Private Capital Universe data with vintages from 1990 to 2019 for which complete cash flow and NAV data are available. As in most large-sample empirical studies of private markets, we trim the most recent vintages whose investment periods are still ongoing and for which the number of realizations is limited; this avoids downward-biased volatility due to appraisal smoothing and partial exit histories. Unless noted otherwise, the majority of our analysis uses private fund cash flows and reported NAVs through the first quarter of 2025.

3.1 Private Real Asset Fund Data

Our primary dataset comprises capitalization-weighted, quarterly time-weighted (TWRR, QTD) return aggregates from the MSCI Private Capital Universe. We focus on real assets broadly defined—private real estate, infrastructure, and natural resources—and classify funds according to the MSCI Private Capital Classification System (MSCI, 2025). To ensure representativeness, we require at least five active underlying funds in a given calendar quarter for that quarter to enter the aggregate. The first observation in each time series is defined as the first quarter for which that quarter and all subsequent quarters meet this coverage criterion.

The resulting real estate and natural resource aggregates begin in 1990Q1, while infrastructure series start 2000Q2, reflecting the gradual maturation of the asset class. Each aggregate captures net-of-fee returns across funds within its respective segment, value-weighted by fund size. All

private return aggregates in this paper are constructed at the global level. That is, each composite pools funds investing across regions worldwide and is value-weighted by fund size wherever they operate. We define the *Real Assets – All* composite pools all private real asset funds (real estate, infrastructure, and natural resources) satisfying the coverage rule. It serves as a capitalization-weighted benchmark for the private real asset universe. We construct three mutually exclusive private real estate indices: (1) *Real Estate – All*, which includes all private real estate funds; (2) *Real Estate – Core*, comprising funds classified as generalist, "not elsewhere classified," or unknown under the MSCI style taxonomy, consistent with income-oriented core strategies; and (3) *Real Estate – VA/Opp*, combining value-added and opportunistic strategies emphasizing capital expenditures and development-driven returns. Because of limited fund counts, we merge value-added and opportunistic subtypes into a single VA/Opp category. These series span 1990Q1–2025Q1, capturing multiple real estate cycles. As shown in Table 1, core strategies exhibit lower volatility and narrower tails than VA/Opp funds.

Infrastructure series are built analogously. We distinguish between (1) *Infrastructure – All*, which includes all private infrastructure funds; (2) *Infrastructure – Core*, representing mature, contracted, or regulated assets, and (3) *Infrastructure – VA/Opp*, covering higher-risk, development-oriented vehicles. Because of limited fund counts at finer levels of disaggregation, we again merge value-added and opportunistic subtypes into a single VA/Opp category. The resulting aggregates start in the early 2000s.

The Natural Resources composite includes private funds investing in energy, timber, agriculture, mining, and related activities. We employ the broad composite rather than finer subcategories to preserve sample depth, since some sub-categories are thinly populated.

To complement the closed-end MSCI Private Capital Universe data series, we include three open-end, core (and core-adjacent) property fund benchmarks: (1) the NCREIF Fund Index – Open-End Diversified Core Equity (NFI–ODCE), a capitalization-weighted index of institutional U.S. property funds with core mandates and moderate leverage; (2) the MSCI/PREA ACOE Quarterly Property Fund Index (Unfrozen), covering large U.S. open-end core commingled funds; and (3) the MSCI/PREA AFOE, an analogous index for funds with broader, core-plus mandates. All three series are used on a gross-of-fee, quarterly total return basis over their available sample periods. The NCREIF series starts in 1990 and the ACOE and AFOE start in 2008Q1.

3.2 Benchmarks

A key component of our empirical approach is the careful selection and construction of public-market benchmarks against which private real asset returns can be evaluated. Because private real assets comprise heterogeneous sub-sectors—real estate, infrastructure, and natural resources—each with distinct liquidity profiles and reporting conventions, benchmark choice necessarily involves balancing conceptual alignment, historical coverage, and data continuity. Our objective is to represent the investable opportunity set of each private strategy while ensuring consistent frequency and global comparability. Table 6 lists all benchmark series and sources.

For the primary analysis, we employ global, investable indices that span the major public markets corresponding to each private real asset category. Specifically:

- Global Public Equities: Developed markets value-weighted market return from the Fama– French data library.⁴
- Global Public Bonds: Bloomberg-Barclay Global Aggregate Total Return Index (USD), covering sovereign, quasi-sovereign, and investment-grade corporate bonds across both developed and emerging markets.
- Global Public Real Estate: FTSE EPRA Nareit Developed Total Return Index (USD), a listed REIT benchmark capturing the performance of global, income-producing property companies.
- Global Public Infrastructure: MSCI World Infrastructure Gross Total Return Index (USD),
 a broad listed infrastructure equity index emphasizing regulated utilities, transport, and energy assets.
- Global Public Real Assets: S&P Global Real Assets Equity Index where available; otherwise, a 50/50 linear combination of the global listed real estate and infrastructure indices, ensuring continuity prior to the S&P series' inception.
- Public Natural Resources: An equity-based proxy formed from the Ken French 48-industry portfolios corresponding to Agriculture, Gold, Mines, Coal, and Oil, combined with the S&P

⁴We thank Ken French for making several public equity and industry series available at https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

GSCI Total Return Index to capture direct commodity exposure.

Commodities and Currencies: The S&P GSCI Total Return Index for broad commodity futures and the Bloomberg U.S. Dollar Index (DXY) to capture dollar appreciation or depreciation over the quarter.

Our benchmark choices are designed to reflect the public analogues of the private strategies analyzed: (1) Listed REITs serve as the most direct liquid proxy for private real estate holdings, sharing exposure to underlying property valuations, leverage, and regional cycles. (2) Listed infrastructure equities capture the cash flow and regulatory dynamics of private infrastructure funds. (3) Public natural resources and commodities mirror private exposure to energy and extraction sectors, providing direct sensitivity to commodity price shocks. (4) The composite Global Real Assets series integrates these exposures and thus serves as a natural public benchmark for our aggregate Real Assets – All private series.

In parallel, we include a set of interest-rate and inflation series that summarize the macroe-conomic discount rate environment facing real asset investors. Specifically, we track the yield on the 5-year U.S. Treasury note (5yr US Treasury) as a nominal risk-free rate at an intermediate horizon, the 5-year breakeven inflation rate (5yr Breakeven Inflation) constructed as the yield spread between nominal 5yr US Treasuries and the "real yield" on 5-year Treasury Inflation-Protected Securities (TIPS). These series provide a compact decomposition of nominal discount rates into real and expected-inflation components. All three are sampled at quarter-end and expressed in percentage terms; they are used in our summary statistics and correlation analysis to characterize how both private and public real asset returns co-move with changes in real rates and inflation expectations over time.

Public indices available at daily or monthly frequencies, depending on the vendor are compounded to quarterly total return indices. Yield-based series enter as quarter-end levels when used as conditioning variables, and as first differences when used in our explanatory analysis. The use of global indices reflects the global nature of the private fund data. In total, our benchmark and factor panel combines ten broad asset-class and style indices with five benchmark and rate series. The resulting dataset provides a robust and globally representative basis for comparing private and public real asset performance. Its breadth allows us to examine both short-term co-movement

and long-horizon adjustment dynamics, serving as the foundation for the unsmoothing in Section 4.

3.3 Summary Statistics

Table 1 reports the quarterly distributional properties for all private real asset indices, fund benchmarks, and public market and macro series used in this study. Across private strategies, reported volatility is markedly lower than that of public benchmarks, reflecting the influence of appraisal-based valuations, infrequent pricing, and the inherently smoothed nature of private NAVs. The broad Real Assets – All composite exhibits a mean quarterly return of roughly 1.8% with a standard deviation of 3.7%, implying annualized volatility near 7.4%. Within real estate, the dispersion across sub-strategies is meaningful: Core funds average 1.4% per quarter with thinner tails and moderate skewness (–1.4), while Value-Add / Opportunistic (VA/Opp) funds generate higher mean returns (1.9%) but with substantially greater kurtosis (12.2) and deeper left tails, consistent with episodic write-downs and capital-intensive repositioning strategies. Infrastructure funds deliver slightly higher mean returns (1.9–2.3%) with lower skewness (–0.3 to 0.8) and moderate kurtosis, indicating more symmetric but still fat-tailed outcomes. Natural resources show the highest mean quarterly return (2.5%) and the broadest dispersion (standard deviation 4.8%), driven by cyclical commodity exposure and leverage to energy prices.

Open-end core real estate indices—NFI–ODCE, MSCI ACOE, and AFOE—display the smoothest return profiles in the dataset. The NFI–ODCE's 3.0% quarterly standard deviation is roughly one-third that of listed REITs, underscoring the persistence of valuation lag in open-end structures. These benchmark series serve as useful low-frequency comparators for calibrating unsmoothing procedures later in the paper.

Public indices exhibit much wider return dispersion and heavier tails. Global equities deliver a mean quarterly return of 2.9% with an 8.4% standard deviation and mild negative skewness (–0.6), while global public real assets—combining listed real estate and infrastructure—display even higher volatility (8.7%) and kurtosis (>1). Listed real estate (FTSE EPRA NAREIT) shows the largest downside extremes, with a minimum quarterly return of –32.4%, corresponding to the global financial crisis. Infrastructure equities are somewhat less volatile (7.8%), reflecting

regulated-asset exposure, whereas the broad commodities index (S&P GSCI) remains the most erratic, with 12.4% volatility and extreme tails (minimum –43.9%, maximum +46.7%). The public natural resources proxy sits between these, with volatility around 11% and left-skewness consistent with energy drawdowns.

Rate series show relatively muted dispersion, as expected. The five-year U.S. Treasury yield averages 3.7% with a 2.2% standard deviation, while breakeven inflation and real yields fluctuate narrowly around 2.5% and 1.2%, respectively. These variables enter later analyzes as conditioning factors in macro-sensitive specifications.

Comparative distributional broad patterns emerge from these statistics. First, the relative smoothness of private-market series compared with their public counterparts highlights the role of appraisal smoothing and the limited mark-to-market frequency inherent to private valuation processes. Second, private real asset returns are distinctly non-normal: almost all series display negative skewness and excess kurtosis, reflecting asymmetric downside risk and rare but severe drawdowns. Third, the tails of public indices, particularly listed real estate and commodities, are much fatter, providing an external reference point for assessing unsmoothing corrections and factor betas. These contrasts set the stage for our subsequent Dimson-type beta estimations and multi-factor decompositions.

Table 2 summarizes fund-level performance outcomes across strategies using the cross-sectional quartiles of Total Value to Paid-In (TVPI), net internal rate of return (IRR), public-market equivalent (PME), and Direct Alpha (DA). The median Real Assets – All fund reports a TVPI of 1.31 and a net IRR of 7.2%, implying moderate value creation after fees. However, dispersion is substantial: the interquartile range of TVPI spans roughly 0.6, corresponding to more than 600 basis points of annualized IRR spread.

By asset class, Real Estate dominates the sample (over 1,200 funds), with median IRRs around 7% but tighter dispersion for core and generalist vehicles (IQR 13 percentage points) than for VA/Opp funds. Infrastructure exhibits slightly higher medians (8–9%) and narrower left tails, consistent with stable cash flow profiles and regulated revenue streams. Natural Resources funds show the widest spread—median IRR 6% but with heavy tails extending from strongly negative to double-digit positive outcomes—reflecting commodity cyclicality and concentration risk.

Across the entire sample of 1,877 funds, the median PME is close to 1.0, indicating parity with

listed public-market benchmarks on an aggregate basis. The median Direct Alpha of roughly 1% suggests that private real asset funds, as a group, generate modest positive abnormal performance relative to public indices, albeit with substantial heterogeneity. Importantly, the 25th percentile Direct Alpha is negative across all strategies, implying that a material share of funds underperform public equivalents even after accounting for liquidity premia.

Taken together, these cross-sectional and time-series characteristics provide a detailed picture of the return dynamics underlying the private real asset universe. They reveal economically significant heterogeneity in both risk and performance across strategy types, justify the need for unsmoothing and factor adjustment procedures, and motivate the empirical analyzes that follow in Sections 4.

TABLE 1: Raw Time Series Data (Quarterly Returns)

This table reports quarterly distributional properties for all private real asset indices, open-end fund benchmarks, and public market and macro series used in the analysis. Private fund returns are capitalization-weighted, quarter-on-quarter TWRR aggregates from the MSCI Private Capital Universe data and related index providers. Public indices and macro factors are transformed to a consistent quarter-end frequency via within-quarter compounding or end-of-quarter level changes, as appropriate. For each series, we report the number of observations (*N*), mean, standard deviation, skewness, kurtosis, selected quantiles (1%, 5%, 25%, median, 75%, 95%, 99%), and the minimum and maximum quarterly returns. All returns are expressed in percent unless noted otherwise; full series definitions are provided in Table 6.

Series	N	Mean	SD	Skew	Kurt	Min	1%	5%	25%	Med	75%	95%	99%	Max
Private Funds														
Real Assets – All	141	1.8%	3.7%	-1.4	9.6	-19.7%	-8.7%	-4.8%	0.6%	1.8%	3.2%	6.3%	11.7%	14.7%
Real Estate – All	141	1.6%	4.3%	-1.7	11.4	-24.9%	-11.3%	-5.8%	0.7%	1.9%	3.6%	7.2%	11.8%	17.6%
Core	141	1.4%	3.9%	-1.4	7.0	-19.6%	-10.3%	-5.4%	0.4%	1.6%	3.2%	6.6%	9.9%	14.6%
VA/Opp	141	1.9%	4.6%	-1.6	12.2	-26.4%	-12.0%	-3.2%	0.7%	2.0%	3.6%	7.4%	13.7%	18.9%
Infrastructure – All	100	1.9%	3.9%	-0.3	3.1	-11.3%	-9.8%	-4.3%	0.5%	2.1%	3.5%	7.2%	12.1%	14.5%
Core	82	2.3%	4.7%	0.8	9.9	-13.6%	-13.4%	-4.6%	0.8%	2.2%	3.9%	8.1%	15.3%	26.0%
VA/Opp	90	2.1%	2.9%	0.8	4.7	-7.1%	-5.7%	-2.1%	0.8%	2.0%	3.5%	5.8%	13.4%	13.4%
Natural Resources	141	2.5%	4.8%	-0.7	4.1	-20.9%	-10.9%	-3.6%	0.1%	2.3%	5.1%	10.8%	12.5%	15.8%
NFI-ODCE	141	1.4%	3.0%	-2.0	7.0	-13.9%	-10.3%	-3.7%	0.5%	2.0%	3.2%	4.8%	6.9%	7.7%
MSCI-ACOE	69	1.0%	3.8%	-1.8	4.2	-13.9%	-12.0%	-6.5%	0.2%	1.8%	3.0%	5.0%	7.4%	7.7%
MSCI-AFOE	69	1.0%	4.0%	-1.8	4.7	-15.0%	-13.0%	-6.9%	0.4%	2.0%	3.1%	5.4%	7.7%	8.0%
Public Benchmarks & Facto	ors													
Global Public Equities	141	2.9%	8.4%	-0.6	0.7	-22.2%	-19.0%	-14.6%	-0.4%	3.9%	7.6%	15.9%	20.9%	22.9%
Global Public Bonds	141	1.2%	3.3%	-0.1	0.1	-8.3%	-7.0%	-3.6%	-0.9%	1.2%	3.3%	6.9%	8.1%	8.9%
Global Public Real Assets	141	2.0%	8.7%	-0.5	1.2	-27.9%	-23.7%	-13.2%	-2.5%	3.1%	7.6%	14.2%	22.1%	23.1%
Global Public Real Estate	141	2.2%	9.6%	-0.4	2.0	-32.4%	-25.8%	-11.4%	-3.8%	3.7%	7.5%	15.6%	24.3%	35.9%
Global Public Infrastructure	106	1.4%	7.8%	-0.2	0.2	-17.7%	-17.6%	-12.4%	-2.9%	2.6%	4.8%	15.2%	17.9%	21.2%
GS Commodity Index	141	1.5%	12.4%	-0.2	2.6	-43.9%	-36.0%	-14.3%	-5.1%	1.5%	8.7%	19.0%	29.0%	46.7%
Public Natural Resources	141	3.0%	11.3%	-0.4	1.7	-34.9%	-31.5%	-15.8%	-2.8%	3.9%	8.4%	21.7%	28.1%	34.2%
5yr US Treasury	141	3.7%	2.2%	0.0	0.0	0.3%	0.3%	0.9%	1.7%	3.7%	5.3%	7.3%	8.6%	8.6%
5yr Breakeven Inflation	141	2.5%	1.0%	0.0	0.0	-0.3%	0.6%	1.3%	1.7%	2.4%	3.3%	4.1%	5.0%	5.0%
5yr Real Interest Rate	141	1.2%	1.4%	0.0	0.0	-1.6%	-1.6%	-1.4%	0.2%	1.5%	2.1%	3.2%	3.7%	3.7%

TABLE 2: Fund-level Performance Data (Quartile Breakpoints)

This table summarizes the cross-sectional distribution of fund-level outcomes by strategy. For each real asset category, we report the number of funds and the 25th, 50th, and 75th percentiles of Total Value to Paid-In (TVPI), net IRR, public market equivalent (PME), and direct alpha (DA), all computed using standard MSCI Private Capital Universe data cash flow and benchmarking conventions. The quartiles provide a compact view of performance dispersion within and across strategies, distinguishing relatively tight distributions for core vehicles from the wider spread of outcomes observed for value-add, opportunistic, and natural resources funds.

Series	N		TVPI			IRR			PME		Di	Direct Alpha		
		25%	50%	75%	25%	50%	75%	25%	50%	75%	25%	50%	75%	
Real Assets – All	1,877	1.01	1.31	1.60	0.4%	7.2%	12.9%	0.76	1.03	1.24	-5.8%	0.9%	6.4%	
Real Estate – All	1,210	0.98	1.29	1.56	-0.2%	7.2%	13.4%	0.76	1.01	1.22	-5.8%	0.5%	6.0%	
Generalist-Core	334	1.02	1.29	1.57	0.5%	7.2%	13.5%	0.81	1.01	1.22	-5.3%	0.2%	5.9%	
VA/Opp	876	0.97	1.29	1.55	-0.5%	7.3%	13.4%	0.76	1.02	1.23	-6.1%	0.6%	6.1%	
Infrastructure – All	255	1.17	1.38	1.63	4.1%	8.6%	12.7%	0.90	1.06	1.22	-2.4%	1.4%	5.7%	
Generalist-Core	150	1.20	1.37	1.66	4.4%	8.5%	12.1%	0.91	1.04	1.20	-2.1%	1.1%	5.0%	
VA/Opp	105	1.08	1.38	1.57	2.3%	8.8%	13.6%	0.86	1.07	1.24	-3.6%	1.8%	6.8%	
Natural Resources	357	1.00	1.37	1.73	0.2%	5.9%	12.1%	0.59	0.81	1.13	-9.7%	-3.9%	2.9%	

4 Aggregate Performance Analysis

In this section, we apply a reduced-form autoregressive and moving-average (ARMA) framework to aggregate private real asset indices to quantify how appraisal practice and reporting conventions reshape their observed risk and factor properties. We begin by documenting the extent of serial correlation and smoothing in reported net asset value based indices, then construct unsmoothed counterparts that are anchored to liquid public benchmarks and disciplined by time series diagnostics. These unsmoothed series preserve the long horizon performance actually earned by investors but reveal volatility and correlation patterns that are much closer to those of public analogs. The contrast between reported and unsmoothed behavior therefore speaks directly to the central question of the paper, whether private real assets truly provide distinct return and diversification benefits or instead derive much of their apparent stability from infrequent pricing and appraisal smoothing. The remainder of this section summarizes the evidence on smoothing, describes the construction of the unsmoothed indices, and compares their comovement with public market and macroeconomic factors. Table 3 presents the ARMA evidence on smoothing in the raw reported data.

Specifically, Table 4 summarizes the resulting Dimson+ARMA unsmoothing specifications and parameter estimates. Table 5 compares correlations of raw and unsmoothed returns with public and macro series. Finally, Figure 1 provides a visual comparison of reported, unsmoothed, and

benchmark indices. Together, these analyzes illustrate both the mechanics and the empirical consequences of removing appraisal smoothing.

4.1 Reported-Return Dynamics and ARMA Diagnostics

We begin by estimating univariate ARMA(p, q) models on the quarterly reported returns for each private index. The purpose of this step is diagnostic: it reveals the extent and form of serial correlation that arises from appraisal smoothing and reporting lags. Table 3 summarizes these fits.

For most categories especially Real Assets (All) and Real Estate (All, Core, VA/Opp) the first-order autoregressive coefficients are large (typically between 0.4 and 0.8) and highly significant, confirming strong persistence in reported returns. Moving-average components are often significant as well, suggesting a layered smoothing process in which both lagged fundamentals and lagged appraisal errors contribute to measured returns. Open-end indices such as NFI–ODCE, MSCI ACOE, and MSCI AFOE show the most pronounced smoothing: AR(1) coefficients approach unity, and MA(1) terms remain sizable, with R^2 frequently exceeding 0.75. These patterns are consistent with appraisal conventions and quarterly valuation cycles that distribute valuation shocks over time rather than recording them immediately.

By contrast, Infrastructure and Natural Resources composites exhibit smaller and often insignificant AR and MA coefficients, with R^2 values below 0.1 in many cases. These series therefore appear closer to a martingale difference process. Table 3 thus establishes a clear ordering of smoothing intensity: strongest for open-end real estate funds, moderate for private real estate composites, weaker for aggregate real assets, and minimal for infrastructure and natural resources. This heterogeneity motivates a model-based unsmoothing procedure that can adapt to different degrees of persistence rather than applying a uniform filter.

4.2 Unsmoothed Series Construction

Interpreting reported private real-asset returns requires disentangling economic information from appraisal mechanics. Because net asset values are marked periodically using lagged transactions and smoothing conventions, the resulting quarter-on-quarter series exhibit subdued volatility and

delayed responses to aggregate shocks. Left unadjusted, these features bias inferences about risk, factor exposures, and diversification (e.g., Geltner, 1993; Geltner and Goetzmann, 1998). Our objective is therefore to recover a return series that moves more contemporaneously with market information while preserving the long-run performance that investors actually earned. We follow the literature by treating reported returns as the outcome of an appraisal filter applied to an underlying economic process and then estimating and inverting that filter in a disciplined way.

Conceptually, we view the reported private return r_t^P as the superposition of a systematic response to a suitable public benchmark and a low-pass appraisal filter that spreads shocks across adjacent quarters. To recover the total systematic exposure in the presence of reporting lags, we estimate a multi-lag "Dimson" regression of private returns on the contemporaneous and several lagged benchmark returns (Dimson, 1979). Let r_t^B denote the public benchmark. We regress

$$r_t^P - \bar{r}^P = \alpha + \sum_{k=0}^K \beta_k (r_{t-k}^B - \bar{r}^B) + u_t,$$

where K is a small integer capturing delayed pass-through. The sum $\beta^{\text{Dimson}} = \sum_{k=0}^{K} \beta_k$ measures the long-horizon benchmark sensitivity after allowing for reporting frictions. This step anchors the private series to economically appropriate comparators, which is essential for credible factor interpretations.

The residual component u_t captures serial dependence induced by appraisal practice rather than by fundamentals. Consistent with classic and modern treatments, we model u_t as a low-order ARMA process,

$$u_t = \sum_{i=1}^p \phi_i u_{t-i} + \sum_{j=1}^q \theta_j \varepsilon_{t-j} + \varepsilon_t,$$

imposing stationarity and invertibility so that the filter is well-defined and economically interpretable. Positive autoregressive parameters ϕ_i represent persistence typical of appraisal smoothing; moving-average terms θ_j accommodate short-memory averaging in reported marks. Estimating this ARMA jointly with the Dimson regression allows us to attribute serial correlation to the filter rather than to spurious dynamics in the economic signal.

Unsmoothing corresponds to inverting the estimated appraisal filter and restoring the timing and amplitude of shocks. For a low-order ARMA, the zero-frequency (long-run) gain of the filter

is

$$g = \frac{1 + \sum_{j} \theta_{j}}{1 - \sum_{i} \phi_{i}},$$

which characterizes the extent to which innovations are attenuated and smeared over time. We construct an unsmoothed return by combining the fitted systematic component from the Dimson regression with the innovation sequence implied by the ARMA residuals, scaled by *g* so that shock magnitudes reflect economic variability rather than reporting conventions. In practice, this yields a quarterly series that reacts more promptly to benchmark innovations and exhibits volatility closer to market-priced analogs, a pattern also emphasized by Bayesian state-space formulations for private markets (e.g., Bustos and DeMond, 2025).

A key identification principle is that unsmoothing should not manufacture alpha. We therefore re-scale the unsmoothed path multiplicatively so that its compounded total return over the estimation window matches that of the reported series. This "total-return anchoring" ensures that we change only the intraperiod timing and dispersion of returns, not the buy-and-hold outcome that investors realized. Economically, the adjustment keeps the level of the private index intact while reallocating variation across quarters in a way that is consistent with observed public information and the estimated appraisal dynamics.

Model selection balances parsimony and fit. We search over small ARMA orders and choose specifications that satisfy stability constraints, reduce residual autocorrelation, and improve information criteria, while yielding unsmoothed volatility that remains plausible relative to listed comparators. The preferred models produce Dimson betas that align with asset-class narratives—moderate long-horizon loadings for core real estate and regulated infrastructure, larger equity and credit sensitivities for value-add and opportunistic strategies—and contemporaneous correlations with public benchmarks that rise meaningfully from raw levels yet remain below unity, consistent with partial segmentation.

This framework connects three strands of the literature. Appraisal-based pricing explains the serial correlation and volatility suppression in reported returns; thin-trading and lead–lag results motivate the use of multi-lag benchmark regressions to recover long-horizon betas; and modern factor models for private markets formalize the mapping between unsmoothed private returns and public risk drivers (Geltner, 1993; Dimson, 1979; Bustos and DeMond, 2025). By integrat-

ing these ideas, our ARMA–Dimson approach produces quarterly private return estimates that are better suited for risk–return comparison, factor attribution, and portfolio construction, while remaining faithful to realized long-run performance.

4.3 Correlation Structure with Public and Macro Factors

We next examine how unsmoothing alters the relationship between private real asset returns, public markets, and macroeconomic conditions. Table 5 reports unconditional correlations of each private series, in both raw and unsmoothed form, with a set of key public indices and macro variables. These include global equities, global bonds, a broad commodity index, public real assets, listed real estate and infrastructure, public natural resources, five year breakeven inflation, the five year real rate, and gold. This comparison allows us to assess whether private real assets truly represent distinct sources of systematic risk or whether appraisal practice simply obscures their connections to familiar priced factors.

First, unsmoothing systematically increases correlations with liquid benchmarks. For the Real Estate and Infrastructure composites, correlations with global equities, public real estate, and public infrastructure generally move higher, often by 10 to 15 percentage points. Infrastructure Core, for example, exhibits substantially stronger co-movement with listed infrastructure and broad equities once smoothing is removed. Economically, this pattern is consistent with the view that investors in these private vehicles are exposed to the same underlying cash flow and discount rate shocks that drive public markets, but that infrequent pricing and appraisal smoothing mask this co-movement in reported returns.

Second, the unsmoothed series also display more pronounced linkages to inflation expectations and commodity risk. Real Assets - All and Natural Resources show higher correlations with the broad commodity index and with five-year breakeven inflation, while correlations with real rates tend to become more negative or less positive. These shifts indicate that smoothing dampens the observable macro exposures of private real assets and that the latent returns are more clearly pro-cyclical and inflation sensitive. From an asset allocation perspective, the unsmoothed indices therefore look more like traditional inflation hedges, with returns that move in line with the real side of the economy and with changes in expected inflation, rather than appearing artificially

stable.

Third, For NFI–ODCE, ACOE, and AFOE, raw correlations with public markets are very low, reflecting heavy smoothing and appraisal based reporting. After unsmoothing, correlations with global equities, listed real estate, and public real assets rise meaningfully, although they remain well below one-for-one. This outcome is consistent with partial segmentation and with differences in leverage, property-type mix, and valuation conventions, rather than any complete insulation from aggregate market risk. In other words, open-end core real estate appears less like a distinct asset class and more like an illiquid, appraisal smoothed claim on the same underlying risk factors that drive public real estate and broader equity markets.

Overall, the correlation evidence supports viewing private real assets, especially once unsmoothed, as risk exposures that are more tightly linked to familiar public market and macro drivers than suggested by reported returns alone. The unsmoothing procedure reveals a risk and return profile that is more consistent with basic asset pricing intuition, in which investors who earn equity like long-run returns are in fact bearing meaningful exposure to aggregate growth, discount rate, and inflation risks, even if those exposures are muted in appraisal based index data.

4.4 Interpretation and Model Quality

The evidence in Tables 3 through 5 and Figure 1 points to a coherent and robust characterization of the role of appraisal smoothing in these series. The strong and systematic autoregressive patterns in Table 3 make clear that an explicit unsmoothing step is necessary if we hope to use these indices for standard risk and factor analysis. Our reliance on BIC to select relatively simple but stable ARMA specifications keeps the adjustment grounded in the data, rather than in an arbitrary filtering choice. The implied unsmoothed volatilities and correlations fall in a range that is economically sensible when compared with liquid counterparts, which supports the view that the extra persistence in reported returns is largely mechanical and not driven by slow moving fundamentals. Figure 1 reaches the same conclusion from a different perspective, since the unsmoothed indices replicate the same cumulative performance as the reported series, while exhibiting more pronounced and better timed responses to public benchmarks, which provides a more realistic picture of the risk and return properties of private real assets.

Taken together, these results show that appraisal smoothing materially distorts the reported risk profile of private real asset indices, particularly for open end core real estate, while leaving long-horizon performance essentially unchanged. Once we recover unsmoothed series that are more tightly linked to liquid public benchmarks and macroeconomic factors, the diversification properties of these indices look more conventional and more consistent with their underlying economic exposures. At the same time, the cross category differences we document make clear that not all private real assets are affected to the same degree, with infrastructure and natural resources already behaving more like public assets in reported form and therefore requiring more modest adjustments. These index level patterns motivate a closer look at the underlying fund universe. In Section 5, we therefore turn to the fund level, where we ask directly whether investors have been compensated for bearing the unsmoothed risk profile and how much dispersion in risk adjusted performance remains once we place private real assets on a more comparable footing with public market alternatives.

TABLE 3: ARMA Summaries by Fund Type

This table reports reduced-form ARMA(p,q) estimates for quarterly reported returns on each private real asset index and open-end fund benchmark. For each specification, we present the estimated constant and AR and MA coefficients (with p-values in parentheses), along with R^2 , AIC, and BIC. These diagnostics quantify the degree of serial correlation and smoothing in reported returns by strategy, and serve as inputs for the more structured Dimson+ARMA unsmoothing models summarized below.

Fund / Term	AR(1)	AR(2)	MA(1)	MA(2)	ARMA(1,1)	ARMA(2,1)	ARMA(1,2)	ARMA(2,2)
					<u> </u>			<u> </u>
Real Assets								
Const	0.018 (0.000)	0.018 (0.000)	0.018 (0.000)	0.018 (0.000)	0.018 (0.000)	0.018 (0.000)	0.018 (0.000)	0.018 (0.000)
AR(1)	0.564 (0.000)	0.427 (0.000)			0.759 (0.000)	-0.149 (0.357)	0.699 (0.000)	-0.250 (0.160)
AR(2)		0.234 (0.007)	0.246 (0.000)	0.500 (0.000)	0.202 (0.011)	0.567 (0.000)	0.244 (0.000)	0.524 (0.002)
MA(1)			0.346 (0.000)	0.529 (0.000)	-0.293 (0.011)	0.630 (0.000)	-0.244 (0.080)	0.768 (0.000)
$MA(2)$ R^2	0.210	0.257	0.102	0.401 (0.000)	0.246	0.086 (0.541)	0.104 (0.595)	0.207
AIC	0.318 -581.512	0.357 -587.553	0.193 -557.922	0.345 -585.076	0.346 -585.302	0.386 -591.970	0.351 -584.464	0.386 -589.996
BIC	-575.614	-578.707	-552.025	-576.229	-576.456	-580.175	-572.669	-575.253
DIC	070.011	070.707	002.020	0,0.22)	070.100	000.170	072.009	070.200
Real Estate - Const	- All 0.016 (0.000)	0.016 (0.000)	0.016 (0.000)	0.016 (0.000)	0.016 (0.000)	0.016 (0.000)	0.016 (0.000)	0.016 (0.000)
AR(1)	0.624 (0.000)	0.454 (0.000)	0.016 (0.000)	0.016 (0.000)	0.782 (0.000)	-0.086 (0.487)	-0.295 (0.030)	-0.243 (0.061)
AR(1)	0.024 (0.000)	0.265 (0.009)			0.762 (0.000)	0.625 (0.000)	-0.293 (0.030)	0.386 (0.003)
MA(1)		0.200 (0.00)	0.364 (0.000)	0.705 (0.000)	-0.264 (0.017)	0.596 (0.000)	0.933 (0.000)	0.910 (0.000)
MA(2)			0.001 (0.000)	0.577 (0.000)	0.201 (0.017)	0.050 (0.000)	0.677 (0.000)	0.446 (0.000)
R^2	0.389	0.433	0.223	0.452	0.416	0.474	0.465	0.497
AIC	-551.198	-559.541	-517.751	-564.111	-555.616	-567.911	-565.549	-572.058
BIC	-545.301	-550.694	-511.854	-555.265	-546.770	-556.116	-553.754	-557.314
D 15.4								
Real Estate - Const	- Core 0.014 (0.000)	0.014 (0.000)	0.014 (0.000)	0.014 (0.000)	0.014 (0.000)	0.014 (0.000)	0.014 (0.000)	0.014 (0.000)
AR(1)	0.587 (0.000)	0.363 (0.000)	0.014 (0.000)	0.014 (0.000)	0.818 (0.000)	-0.077 (0.423)	0.230 (0.076)	-0.169 (0.149)
AR(2)	0.507 (0.000)	0.373 (0.000)			0.010 (0.000)	0.645 (0.000)	0.230 (0.070)	0.520 (0.000)
MA(1)		0.070 (0.000)	0.318 (0.000)	0.544 (0.000)	-0.358 (0.000)	0.539 (0.000)	0.304 (0.014)	0.725 (0.000)
MA(2)			0.010 (0.000)	0.508 (0.000)	0.000 (0.000)	(0.000)	0.432 (0.000)	0.244 (0.016)
R^2	0.344	0.438	0.184	0.424	0.399	0.482	0.424	0.492
AIC	-569.064	-588.466	-538.472	-585.023	-579.247	-597.830	-583.155	-598.314
BIC	-563.167	-579.620	-532.574	-576.176	-570.400	-586.035	-571.360	-583.570
Real Estate	- Value-Added	l/Opportunisti	c					
Const	0.019 (0.000)	0.019 (0.000)	0.019 (0.000)	0.019 (0.000)	0.019 (0.000)	0.019 (0.000)	0.019 (0.000)	0.019 (0.000)
AR(1)	0.592 (0.000)	0.460 (0.000)	` ,	` ′	0.757 (0.000)	-0.123 (0.403)	0.375 (0.179)	-0.278 (0.086)
AR(2)		0.215 (0.058)				0.577 (0.000)		0.331 (0.061)
MA(1)			0.366 (0.000)	0.650 (0.000)	-0.258 (0.040)	0.625 (0.000)	0.180 (0.511)	0.894 (0.000)
MA(2)				0.489 (0.000)			0.235 (0.211)	0.393 (0.009)
R^2	0.351	0.381	0.213	0.389	0.373	0.412	0.371	0.424
AIC	-527.693	-532.434	-500.909	-533.989	-530.511	-537.420	-528.034	-538.215
BIC	-521.795	-523.587	-495.012	-525.143	-521.665	-525.625	-516.239	-523.471
Infrastructu	re – All							
Const	0.019 (0.000)	0.019 (0.000)	0.019 (0.000)	0.019 (0.000)	0.019 (0.000)	0.019 (0.000)	0.019 (0.000)	0.019 (0.000)
AR(1)	0.290 (0.001)	0.280 (0.002)			0.593 (0.011)	-0.380 (0.632)	0.701 (0.074)	0.181 (0.829)
AR(2)		0.016 (0.868)				0.140 (0.602)		0.370 (0.318)
MA(1)			0.295 (0.001)	0.295 (0.001)	-0.347 (0.182)	0.688 (0.362)	-0.410 (0.301)	0.117 (0.884)
MA(2)	0.070	0.070	0.077	0.009 (0.914)	0.000	0.004	-0.104 (0.528)	-0.365 (0.120)
R^2	0.079	0.079	0.077	0.077 -369.674	0.082 -370.201	0.084	0.087	0.098 -368.126
AIC BIC	-371.870 -366.660	-369.893 -362.077	-371.666 -366.456	-369.674 -361.859	-370.201 -362.385	-368.465 -358.044	-368.846 -358.426	-355.100
DIC	-500.000	-304.077	-500.450	-301.037	-302.363	-550.044	-550.420	-555.100
Infrastructu		0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
Const	0.023 (0.000)	0.023 (0.000)	0.023 (0.000)	0.023 (0.000)	0.023 (0.000)	0.023 (0.000)	0.023 (0.000)	0.023 (0.000)
AR(1)	0.215 (0.016)	0.236 (0.011)			-0.105 (0.813)	-0.774 (0.009)	-0.776 (0.938)	-1.017 (0.029)
AR(2)		-0.082 (0.615)	0.240 (0.007)	0.240 (0.000)	0.246 (0.416)	0.204 (0.150)	1.027 (0.917)	-0.630 (0.009)
MA(1) MA(2)			0.249 (0.007)	0.240 (0.009) -0.032 (0.804)	0.346 (0.416)	0.993 (0.016)	0.199 (0.934)	1.247 (0.002) 0.801 (0.008)
14111(2)				0.002 (0.004)			0.177 (0.304)	0.001 (0.000)

Fund / Term	AR(1)	AR(2)	MA(1)	MA(2)	ARMA(1,1)	ARMA(2,1)	ARMA(1,2)	ARMA(2,2)
R^2	0.046	0.052	0.053	0.054	0.053	0.047	0.053	0.094
AIC	-269.821	-268.345	-270.394	-268.493	-268.479	-265.993	-266.396	-268.712
BIC	-265.008	-261.125	-265.581	-261.273	-261.259	-256.366	-256.770	-256.679
	ıre – Value-Ad	ded/Opportun	istic					
Const	0.021 (0.000)	0.021 (0.000)	0.021 (0.000)	0.021 (0.000)	0.021 (0.000)	0.021 (0.000)	0.021 (0.000)	0.021 (0.000)
AR(1)	0.062 (0.477)	0.054 (0.562)			0.692 (0.103)	0.484 (0.352)	0.603 (0.181)	0.492 (0.673)
AR(2)		0.124 (0.079)				0.096 (0.356)		0.074 (0.951)
MA(1)			0.050 (0.588)	0.051 (0.551)	-0.601 (0.192)	-0.437 (0.367)	-0.559 (0.186)	-0.446 (0.688)
MA(2)	2.224	0.000	0.002	0.130 (0.050)	0.047	2.222	0.090 (0.400)	0.024 (0.983)
R^2	0.004	0.020	0.003	0.019	0.016	0.023	0.023	0.023
AIC	-376.676	-376.084	-376.608	-376.045	-375.758	-374.397	-374.412	-372.405
BIC	-371.676	-368.585	-371.608	-368.546	-368.258	-364.398	-364.413	-359.906
Natural Res		0.005 (0.000)	0.005 (0.000)	0.00= (0.000)	0.00= (0.000)	0.005 (0.000)	0.005 (0.000)	0.005 (0.000)
Const	0.025 (0.000)	0.025 (0.000)	0.025 (0.000)	0.025 (0.000)	0.025 (0.000)	0.025 (0.000)	0.025 (0.000)	0.025 (0.000)
AR(1)	0.297 (0.000)	0.258 (0.000)			0.684 (0.000)	0.623 (0.237)	0.660 (0.014)	0.193 (0.990)
AR(2)		0.132 (0.076)	0.227 (0.001)	0.245 (0.004)	0.420 (0.020)	0.029 (0.883)	0.412 (0.165)	0.331 (0.975)
MA(1) MA(2)			0.236 (0.001)	0.245 (0.004) 0.139 (0.145)	-0.430 (0.030)	-0.374 (0.495)	-0.413 (0.165) 0.022 (0.859)	0.059 (0.997) -0.201 (0.976)
R^2	0.088	0.103	0.069	0.139 (0.143)	0.107	0.107	0.022 (0.839)	0.107
AIC	-468.155	-468.563	-465.230	-466.339	-469.171	-467.205	-467.213	-465.175
BIC	-462.257	-459.717	-459.332	-457.492	-460.324	-455.410	-455.418	-450.431
	102.207	10,11	107.002	107.172	100.021	100.110	100.110	100.101
NFI ODCE	0.014 (0.000)	0.014 (0.000)	0.014 (0.000)	0.014 (0.000)	0.014 (0.000)	0.014 (0.000)	0.014 (0.000)	0.014 (0.000)
Const	0.014 (0.000)	0.014 (0.000) 1.197 (0.000)	0.014 (0.000)	0.014 (0.000)	0.014 (0.000) 0.778 (0.000)	0.014 (0.000) 1.521 (0.000)	0.014 (0.000) 0.713 (0.000)	0.014 (0.000) 1.721 (0.000)
AR(1) AR(2)	0.637 (0.000)	-0.387 (0.000)			0.778 (0.000)	-0.663 (0.000)	0.713 (0.000)	-0.788 (0.000)
MA(1)		-0.387 (0.000)	0.750 (0.000)	1.067 (0.000)	0.350 (0.000)	-0.410 (0.064)	0.461 (0.000)	-0.599 (0.000)
MA(2)			0.750 (0.000)	0.570 (0.000)	0.550 (0.000)	-0.410 (0.004)	0.173 (0.037)	-0.206 (0.067)
R^2	0.745	0.783	0.570	0.723	0.774	0.786	0.780	0.787
AIC	-775.670	-796.149	-702.985	-762.595	-790.995	-796.335	-792.547	-794.814
BIC	-769.772	-787.303	-697.088	-753.748	-782.149	-784.540	-780.752	-780.070
MSCI ACO Const	0.010 (0.032)	0.010 (0.032)	0.010 (0.032)	0.010 (0.032)	0.010 (0.032)	0.010 (0.032)	0.010 (0.032)	0.010 (0.032)
AR(1)	0.850 (0.000)	1.243 (0.000)	01010 (0100_)	(0.00-2)	0.741 (0.000)	1.457 (0.000)	0.666 (0.000)	1.634 (0.000)
AR(2)	(,	-0.471 (0.000)			()	-0.646 (0.002)	(,	-0.755 (0.000)
MA(1)		()	0.797 (0.000)	1.150 (0.000)	0.468 (0.001)	-0.292 (0.379)	0.574 (0.000)	-0.415 (0.258)
MA(2)			, ,	0.576 (0.000)	` /	, ,	0.180 (0.163)	-0.235 (0.336)
R^2	0.722	0.786	0.585	0.727	0.774	0.788	0.780	0.791
AIC	-339.691	-355.568	-314.333	-340.175	-352.032	-354.012	-351.588	-353.679
BIC	-335.223	-348.866	-309.865	-333.473	-345.330	-345.075	-342.651	-342.508
MSCI AFO	E							
Const	0.010 (0.033)	0.010 (0.033)	0.010 (0.033)	0.010 (0.033)	0.010 (0.033)	0.010 (0.033)	0.010 (0.033)	0.010 (0.033)
AR(1)	0.843 (0.000)	1.266 (0.000)	(-/	, ,,	0.745 (0.000)	1.362 (0.000)	0.671 (0.000)	1.634 (0.000)
AR(2)	(-/	-0.489 (0.000)			` '/	-0.569 (0.006)	, ,	-0.754 (0.000)
MA(1)		(-/	0.801 (0.000)	1.221 (0.000)	0.491 (0.000)	-0.130 (0.680)	0.614 (0.000)	-0.379 (0.288)
MA(2)			` ,	0.614 (0.000)	` /	` '	0.183 (0.143)	-0.264 (0.282)
R^2	0.730	0.798	0.591	0.743	0.787	0.798	0.793	0.802
AIC	-333.107	-350.579	-306.739	-335.887	-347.134	-348.655	-346.905	-348.731
BIC	-328.639	-343.876	-302.271	-329.184	-340.432	-339.719	-337.968	-337.560

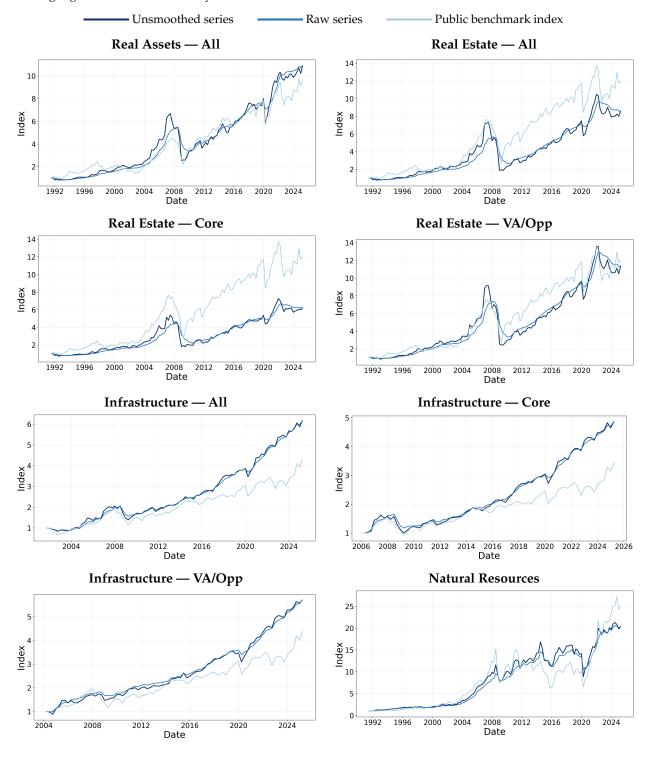
TABLE 4: Unsmoothing Models

This table summarizes the preferred unsmoothing specifications and resulting properties for each private index. For each series, we select a benchmark public index and estimate a Dimson-style regression of quarterly private returns on contemporaneous and four lags of the benchmark, combined with an ARMA(p,q) process for the regression residuals. The Dimson β sum aggregates the slope coefficients on the benchmark lags. The unsmoothed series is constructed as the fitted $\beta_{\text{sum}} \times$ benchmark return plus an ARMA-based residual component scaled by the long-run gain, $g = (1 + \sum \theta_j)/(1 - \sum \phi_i)$, to recover the implied underlying innovations. We then re-anchor this unsmoothed return series so that its cumulative total return matches the raw private index over the common sample. The table reports the chosen ARMA orders, annualized means and volatilities for the benchmark, raw, and unsmoothed series, the key unsmoothing parameters (Dimson β sum, AR and MA coefficients with p-values), and the fit statistics and correlations with both the public benchmark and original reported series.

	Real Assets		ructure	Infrast- ructure VA/Opp	Natural Resources	Real Estate All	Real Estate Core	Real Estate VA/Opp	NFI ODCE	MSCI ACOE	MSCI AFOE
Best ARMA(p,q)											
(p,q) Benchmark	1,5	0,5	0,4	0,5	0,4	1,4	1,4	1,4	1,1	1,0	1,0
Mean Returns (Annual, %)	6.98	5.72	6.56	7.82	10.38	7.40	7.40	7.40	7.40	2.42	2.42
Std Dev (Annual, %) Raw (reported)	17.20	14.90	13.50	13.60	22.80	19.20	19.20	19.20	19.20	21.20	21.20
Mean Returns (Annual, %)	7.40	7.82	9.09	8.67	9.09	6.56	5.30	7.40	5.72	3.65	4.06
Std Dev (Annual, %)	7.40	7.40	9.20	5.90	9.50	8.80	7.90	9.20	6.10	7.40	7.90
Unsmoothed (model-impl	ied)										
Mean Returns (Annual, %)	7.40	7.82	9.09	8.67	9.09	6.56	5.30	7.40	5.72	3.65	4.06
Std Dev (Annual, %)	16.50	11.90	13.30	11.00	16.20	19.40	17.50	19.00	15.00	17.70	19.20
Unsmoothing Parameters	(coefficie	ents with	n <i>p-</i> valu	es in para	intheses)						
Dimson β sum	0.377	0.536	0.714	0.413	0.598	0.369	0.271	0.454	0.260	0.420	0.445
ϕ (AR)	0.537	_	_	_	_	0.496	0.531	0.379	0.752	0.773	0.783
ψ (ΓΠζ)	(0.057)	0.1.15	0.000	0.405	0.070	(<0.001)	. ,	, ,	. ,	(<0.001)	(<0.001)
θ_1	-0.237 (0.413)	0.145 (0.206)	0.088 (0.715)	0.125 (0.265)	0.073 (0.575)	-0.100 (0.528)	-0.194 (0.227)	-0.015 (0.921)	0.260 (0.007)	_	_
	0.216	-0.095	-0.313	0.236	0.099	0.277	0.227)	0.275	(0.007)		
θ_2	(0.069)	(0.291)	(0.040)	(0.026)	(0.354)	(0.011)	(0.002)	(0.026)	_	_	-
0	0.021	0.254	0.270	0.177	0.040	-0.053	-0.032	-0.018			
θ_3	(0.850)	(0.006)	(0.089)	(0.226)	(0.669)	(0.517)	(0.672)	(0.858)	_	_	_
$ heta_4$	0.523	0.243	0.345	0.120	0.194	0.598	0.447	0.588	_	_	_
-4	(<0.001)	. ,	(0.044)	(0.282)	(0.115)	(<0.001)	(<0.001)	(<0.001)			
θ_5	-0.026 (0.906)	0.183 (0.023)	_	0.372 (0.012)	_	_	_	_	_		_
Fit and Correlations	(0.700)	(0.023)		(0.012)							
$\overline{R^2}$	0.630	0.540	0.512	0.392	0.580	0.666	0.599	0.647	0.803	0.806	0.814
Corr. vs. public	0.461	0.688	0.744	0.520	0.844	0.441	0.373	0.515	0.400	0.590	0.583
Corr. vs. private	0.757	0.920	0.897	0.921	0.881	0.725	0.763	0.751	0.535	0.563	0.562

FIGURE 1: Cumulative Returns of Private Funds and Public Benchmarks

This figure compares, for each private real asset index, the reported (raw) return index, the model-implied unsmoothed index, and the corresponding public benchmark. The unsmoothed series are generated from the selected Dimson+ARMA specifications: private returns are regressed on contemporaneous and lagged benchmark returns with ARMA(p, q) residuals, the Dimson β -sum and ARMA coefficients are used to infer the underlying innovations, and the resulting unsmoothed returns are re-scaled so that their cumulative performance matches the raw index over the strict common sample. All indices are normalized to 1 at the start of the overlapping window to highlight differences in volatility and co-movement rather than level shifts.



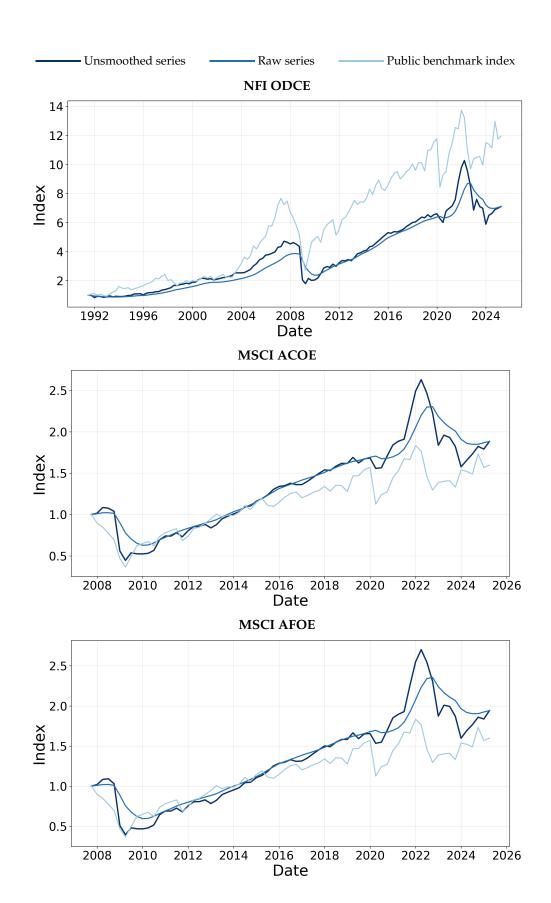


TABLE 5: Correlations with Public Markets: Raw vs. Unsmoothed Fund Returns

This table reports correlations between each private real asset index and a broad set of public market and macro series, separately for raw reported returns (Panel A) and model-implied unsmoothed returns (Panel B). Public comparators include global equities, global bonds, public real asset indices, commodities, natural resource equities, gold, and quarter-on-quarter changes in 5-year breakeven inflation and 5-year real yields. The unsmoothed correlations are computed using the preferred Dimson+ARMA-based series, rescaled to match the raw index total return over the estimation window. Comparing Panels A and B highlights how removing return smoothing increases the alignment of private index behavior with economically related public markets, particularly for infrastructure, natural resources, and open-end real estate.

Panel A: Raw Data										
	Global Eq.	Glob. Bonds	GSCI	Pub. RA	Pub. RE	Pub. Infra	Pub. NatRes	5Y BE	Real Rate	Gold
Real Assets - All	0.335	-0.009	0.376	0.421	0.395	0.415	0.429	0.361	0.005	0.100
Real Estate - All	0.261	-0.020	0.232	0.313	0.314	0.307	0.279	0.243	0.018	0.061
Real Estate - Core	0.234	-0.023	0.258	0.295	0.297	0.314	0.294	0.280	0.008	0.069
Real Estate - Value-Added	0.276	0.004	0.205	0.320	0.319	0.304	0.273	0.218	0.005	0.050
Infrastructure - All	0.467	0.091	0.374	0.582	0.500	0.537	0.402	0.388	-0.040	0.099
Infrastructure - Core	0.392	0.187	0.443	0.515	0.463	0.526	0.452	0.334	-0.111	0.218
Infrastructure - Value Added	0.418	0.110	0.375	0.559	0.477	0.467	0.461	0.381	-0.034	0.134
Natural Resources	0.276	-0.040	0.581	0.433	0.337	0.350	0.597	0.417	-0.004	0.167
NRI ODCE	0.010	-0.198	0.126	0.060	0.065	0.069	0.050	-0.039	0.155	-0.019
MSCI ACOE	0.020	-0.236	0.110	0.058	0.057	0.044	-0.008	-0.078	0.169	0.016
MSCI AFOE	0.027	-0.238	0.112	0.059	0.056	0.048	-0.001	-0.077	0.173	0.013
Panel B: Unsmoothed Fund F	Returns									
	Global Eq.	Glob. Bonds	GSCI	Pub. RA	Pub. RE	Pub. Infra	Pub. NatRes	5Y BE	Real Rate	Gold
Real Assets - All	0.417	0.047	0.465	0.461	0.439	0.421	0.423	0.538	-0.074	-0.013
Real Estate - All	0.384	0.115	0.302	0.431	0.441	0.372	0.308	0.449	-0.071	-0.007
Real Estate - Core	0.319	0.094	0.334	0.369	0.373	0.333	0.324	0.474	-0.113	-0.002
Real Estate - Value-Added	0.425	0.166	0.272	0.506	0.515	0.409	0.320	0.375	-0.082	-0.007
Infrastructure - All	0.561	0.209	0.368	0.698	0.607	0.688	0.502	0.464	-0.079	-0.022
Infrastructure - Core	0.570	0.281	0.415	0.703	0.641	0.744	0.536	0.415	-0.116	-0.005
Infrastructure - Value Added	0.500	0.161	0.376	0.587	0.492	0.520	0.509	0.429	-0.073	-0.008
Natural Resources	0.495	0.054	0.727	0.603	0.479	0.508	0.844	0.537	-0.143	-0.044
NRI ODCE	0.354	-0.095	0.341	0.350	0.400	0.237	0.190	0.276	0.016	0.013
NRI ODCE MSCI ACOE	0.354 0.484	-0.095 0.055	0.341 0.452	0.350 0.549	0.400 0.590	0.237 0.405	0.190 0.283	0.276 0.291	0.016 -0.050	0.013 -0.008

5 Fund-Level Performance Analysis

In the aggregate analysis above, we used ARMA Dimson unsmoothing and factor benchmarking to show that reported private real asset indices are best viewed as smoothed, lagged reflections of underlying economic exposures to public markets and macro variables. That exercise clarified the systematic risk that investors bear when they allocate to real estate, infrastructure, and natural resources, once we strip out appraisal mechanics and align the indices with appropriate public comparators. It does not, however, speak to how much heterogeneity exists across individual funds and vintages around those aggregate relationships.

This section turns to the fund level, using the same benchmarking logic to examine the cross sectional distribution of nominal outcomes, TVPI and IRR, and market adjusted outcomes, PME and Direct Alpha, across strategies and vintages. By reading the boxplots and vintage stacks in

Figures 2 through 6, we document how wide the dispersion in realized performance is, how much it varies by style and cycle, and how alpha sits alongside economically large selection risk for limited partners. The core messages mirror prior evidence on private markets—material heterogeneity, cyclical timing effects, and a meaningful role for public-market comovement and appraisal practice—while extending to the full real-assets spectrum.⁵

5.1 Cross-Sectional Dispersion and Central Tendency

Considering nominal performance as measured by TVPI and IRR, the pooled boxplots display a broadly similar central tendency across strategies. Median TVPIs fall in the range of 1.3 to 1.5, and median IRRs range from high single digits to low teens, yet the dispersion around these medians differs sharply between categories. Real Estate Core and Infrastructure Core exhibit the tightest interquartile ranges and comparatively short lower whiskers, which is consistent with portfolios of stabilized assets, contracted revenue streams, and relatively conservative leverage. In contrast, Real Estate Value Added and Natural Resources funds display visibly wider interquartile ranges and longer left tails. Dispersion is most pronounced for Natural Resources, where outcomes range from deep capital losses to very high multiples, a pattern that aligns with exposure to commodity price cycles and project level concentration risk documented in prior work such as Newell et al., 2010.

Turning to market adjusted outcomes, the PME and Direct Alpha panels sharpen these contrasts. Median PMEs are near or modestly above one for Real Assets All, Real Estate All, and both Infrastructure cohorts, whereas Natural Resources centers below one on PME and below zero on Direct Alpha. In economic terms, most real estate and infrastructure funds deliver net outcomes that are broadly in line with liquid public comparators, with median Direct Alpha on the order of zero to two percentage points. By comparison, Natural Resources funds generate negative performance relative to public benchmarks on average, even though the upper quartile of the distribution remains quite strong. This pattern echoes the earlier evidence in Section 2 that the typical real asset fund offers only a limited aggregate advantage over public markets, while the cross sectional dispersion in outcomes remains economically large, so that manager selection is central to realized alpha. (Brown et al., 2024)

⁵Kaplan and Sensoy, 2005; Sorensen and Jagannathan, 2015; Brown et al., 2024; Andonov et al., 2021.

Across all four metrics, the distributions exhibit longer left tails than right tails, particularly for IRR and Direct Alpha. This asymmetry highlights the downside risk associated with project timing, the use of leverage, and the dynamics of write downs in stressed states. The presence of extreme right tail outcomes in Value Added real estate and Natural Resources indicates that a small set of very successful deals can more than offset a portfolio of middling performers, which is consistent with option like payoff structures and exposure to rare positive shocks. However, the frequency of such outcomes is low, so investors who pursue these segments effectively trade a higher probability of moderate underperformance for a small chance of very high payoffs. This trade off is central to how limited partners should think about sizing and diversifying allocations across real asset strategies.

5.2 Vintage Dynamics and Cyclical Timing

Across all asset groups, the by-vintage stacks display three regimes:

- Pre-crisis build-up (early/mid-2000s). TVPI and IRR medians are elevated with wide spreads; the dispersion reflects aggressive underwriting, readily available leverage, and—in Natural Resources—the commodity super-cycle.
- Crisis vintages (≈2007–2010). Distributions widen materially, lower whiskers extend deep into negative IRR and DA territory, and median PMEs often dip toward or below parity, especially where exits were delayed or marks reset.
- 3. Post-2012 normalization. IQRs tighten, medians move back toward the 1.3–1.5 TVPI / \sim 8–12% IRR zone, and market-adjusted medians (PME, DA) rise toward or slightly above parity, consistent with improved discipline, lower entry leverage, and a more predictable exit environment.

Real Assets—All. The aggregate mirrors the pattern above. Pre-2006 vintages show higher medians and broader spreads; 2007–2009 cohorts display the fattest left tails in IRR and DA, with a notable fraction of funds delivering PMEs below one. Beginning with ~2012 vintages, dispersion compresses: median PME drifts marginally above 1 and median DA turns positive, while

lower-tail outcomes become less severe. This post-crisis convergence is consistent with the return of underwriting discipline and the gradual healing of exit markets—a theme echoed across private-capital studies of the same era (Brown et al., 2024).

Real Estate—All. Real estate exhibits clear bimodality by style embedded within the all-in composite. Crisis-era vintages show the largest widening of the IRR and DA distributions, with many funds realizing write-downs that depress PMEs below unity. From ~2011 forward, medians recover and spreads narrow. By mid-2010s vintages, PME medians sit at or just above 1.0 and DA medians hover around zero to modestly positive, but the upper quartile remains meaningfully positive—suggesting persistent room for manager selection to add value even as market beta explains more of the cross-section. The stabilization is particularly visible in TVPI and PME, whose IQRs shrink notably relative to pre-2008 cohorts.

Infrastructure—All. Relative to real estate, infrastructure's dispersion is narrower throughout, and the crisis-era widening is less extreme. IRR and DA medians remain positive for most vintages, with PME medians generally above one after ~2011. The interquartile bands contract visibly in the 2012–2016 vintages, consistent with the asset class' contractual cash flows, a maturing GP universe, and more standardized project finance structures. The upper quartile of DA, however, still reaches into mid-single-digits, indicating that differentiated sourcing, regulatory skill, and measured development risk can generate excess returns even in a "core"-leaning universe (Bustos and DeMond, 2025; Andonov et al., 2021). Within-infrastructure style effects are also evident in the overview boxplots: Core funds present lower dispersion and slightly lower medians in nominal terms, while Value-Added/Opportunistic funds display a wider IQR and a higher upper tail in IRR/TVPI but only a mildly higher DA median—suggesting that part of the headline nominal premium reflects higher market and credit beta rather than pure alpha.

Natural Resources. Natural resources remain the outlier. Vintage stacks show structural cyclicality—cohorts aligned with commodity upswings post stronger medians and very high upper quartiles; those intersecting downcycles exhibit broad left tails with PME medians below one and DA medians negative. Even in post-2012 vintages, dispersion compresses less than in other

real-asset categories. The combination of price volatility, operational leverage, and binary exploration/development outcomes sustains fat-tailed distributions that do not wash out with maturation. This echoes earlier findings that resource strategies deliver inflation sensitivity but also amplify drawdowns tied to energy and metals cycles (?Newell et al., 2010).

5.3 Nominal vs. Market-Adjusted Lenses

Comparing the nominal measures of performance, IRR and TVPI, to the market adjusted metrics, PME and Direct Alpha, across vintages highlights two recurrent themes. The first theme concerns the gap that opens during the global financial crisis period. For the 2007 through 2010 vintages, particularly in Real Estate and Natural Resources, many funds report IRRs that appear respectable, while their PMEs cluster below one and Direct Alpha falls below zero. This pattern indicates that a substantial share of the apparent nominal performance in these cohorts reflects compensation for market beta and the subsequent rebound in public markets rather than persistent manager specific value creation.

The second theme is a re-anchoring of outcomes in the period after 2012. In the Real Assets All, Real Estate All, and Infrastructure All aggregates, median PMEs drift above one and median Direct Alpha turns positive for the bulk of mid 2010s vintages, and the interquartile ranges narrow. These features suggest a return to outcomes that are roughly in line with public markets, with modest but positive skill premia. This evolution is consistent with a more competitive GP landscape and increasingly institutional and disciplined LP due diligence.

5.4 Selection Risk and the Economics of Dispersion

Taken together, the cross sectional distributions of fund outcomes underscore the economic importance of selection risk. For many strategies and vintages, the interquartile range of IRRs is on the order of ten to fifteen percentage points, and Direct Alpha interquartile ranges of five to eight percentage points are common even when the median is near zero. These spreads imply that the difference between selecting a top quartile and a bottom quartile manager is economically large relative to the average premium of the asset class over public markets. The visual evidence therefore suggests that manager selection and style tilts dominate average asset class effects in shaping

realized outcomes for limited partners. In particular:

- Core vs. VA/Opp (Real Estate, Infrastructure): Value Added cohorts display higher nominal dispersion and greater upside potential in the right tail. However, their median Direct Alpha is only modestly above that of Core strategies. This pattern underscores the role of market timing, greater leverage, and exposure to development risk in generating wider nominal spreads.
- Natural Resources: The mass of the distribution in the left tail remains large across cohorts.
 This feature makes sizing and diversification decisions central for portfolio construction, including diversification across basins, commodities, and development stages, for investors who wish to contain the probability of severe losses.
- Aggregate Real Assets: The compression in cross-sectional spreads for post-2012 vintages
 reduces the probability of extremely poor outcomes, but it also narrows the distribution of
 alpha. This pattern is consistent with increased competition in the general partner universe,
 stronger governance by limited partners, and more transparent benchmarking of performance.

5.5 Measurement Considerations

Two measurement cautions are worth emphasizing before we interpret these cross sectional patterns more broadly. First, recent vintages in the late 2010s still have partial exit histories and may continue to reflect residual appraisal smoothing. The relatively tight interquartile ranges for these cohorts therefore should not be read as evidence of a structural and permanent decline in risk, but rather as a reminder that some of the underlying assets have yet to be marked or realized. Second, benchmark choice remains central for the interpretation of public market equivalent and Direct Alpha. Our preferred benchmarks rely on broad, investable public analogues that do a reasonable job of capturing market-wide movements. However, when the mix of strategies within a category shifts over time, for example toward more core plus real estate or more energy transition oriented infrastructure, local public market equivalents can differ from those computed against a single global comparator, as discussed in Section 3. These considerations parallel the identification issues highlighted in the literature, for example in Kaplan and Sensoy, 2005; Korteweg and Nagel,

2016, and they reinforce the motivation for the unsmoothing and factor based aggregate analysis developed in the prior section.

5.6 Synthesis

Viewed through the lens of Figures 2 through 6, the fund-level evidence points to three central conclusions. First, material dispersion that depends on strategy is a defining feature of private real assets, with tails and interquartile ranges that differ sharply across Real Estate Core versus Value Added or Opportunistic funds, Infrastructure Core versus Value Added or Opportunistic funds, and Natural Resources. Second, cyclical timing is pivotal. Vintages that intersect the global financial crisis display the widest left tails and the largest gaps between nominal and market adjusted metrics, while post-2012 cohorts exhibit narrower spreads, public market equivalents with medians at or above one, and Direct Alpha medians at or above zero for Real Assets, Real Estate, and Infrastructure. Third, alpha is modest, on average, but valuable in the right tail. Median Direct Alpha is near zero to slightly positive for most strategies, and negative for Natural Resources, yet the upper quartile remains economically meaningful, which is consistent with the view that realized outperformance is scarce, persistent for some managers, and magnified by style and timing choices.

These findings align closely with the unsmoothing and factor results in Section 4. Once we remove appraisal dynamics and benchmark private real asset returns against appropriate public comparators, much of the apparent stability in reported performance gives way to a more conventional risk profile that is clearly linked to equity, credit, and macroeconomic factors. What remains, especially the cross-sectional spread in PMEs and Direct Alpha, belongs to the domain of genuine manager skill, asset selection, and cycle navigation. In this sense, the aggregate analysis clarifies the systematic risks that investors bear when they allocate to private real assets, and the fund-level analysis clarifies how unevenly the associated rewards are distributed across strategies and vintages, setting the stage for the portfolio level assessment that follows.

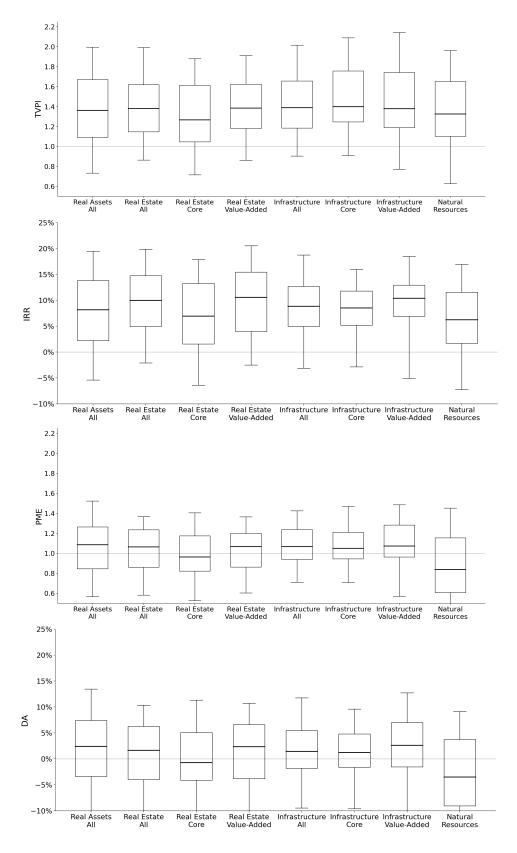


FIGURE 2: Boxplots of fund-level outcomes across strategies. Each panel shows the cross-sectional distribution of TVPI, IRR, PME, and DA by major real asset category. Medians (bold lines) and interquartile ranges illustrate the central tendency and dispersion across funds.

Real Assets All 2.5 2.0 1.0 0.5 0.0 2004 1994 1999 2009 2014 2019 40% 35% 30% 25% 20% 15% 10% 5% 0% -5% -10% -15% -20% -25% 1994 1999 2004 2009 2014 2019 3.0 2.5 2.0 0.5 1994 1999 2004 2009 2014 2019 40% 35% 30% 25% 20% 15% 10% М 5% 0% -5% -10%

FIGURE 3: Real Assets – All: by-vintage distributions of TVPI, IRR, PME, and DA. Each box represents the cross-sectional spread for funds of the same vintage year.

2009

2014

2019

2004

-15% --20% --25% -1994

1999

Real Estate All

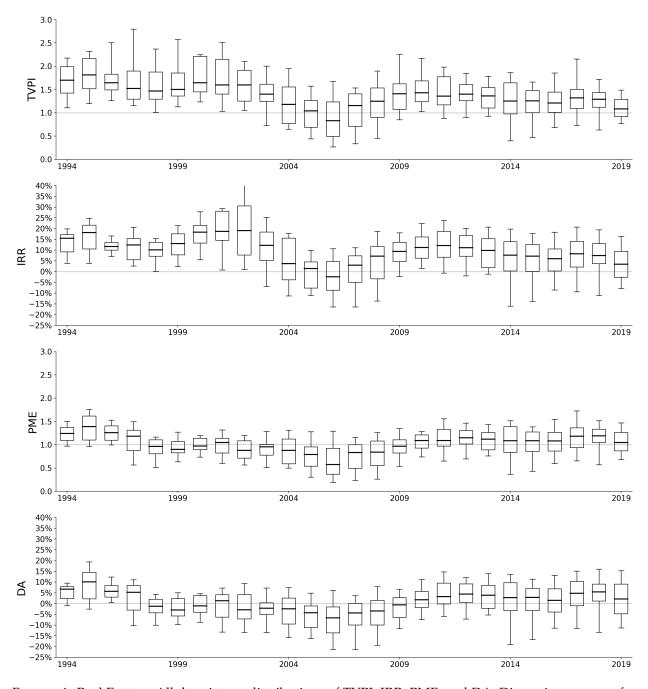


FIGURE 4: Real Estate – All: by-vintage distributions of TVPI, IRR, PME, and DA. Dispersion narrows for recent vintages, reflecting increased consistency in fund execution.

Infrastructure All

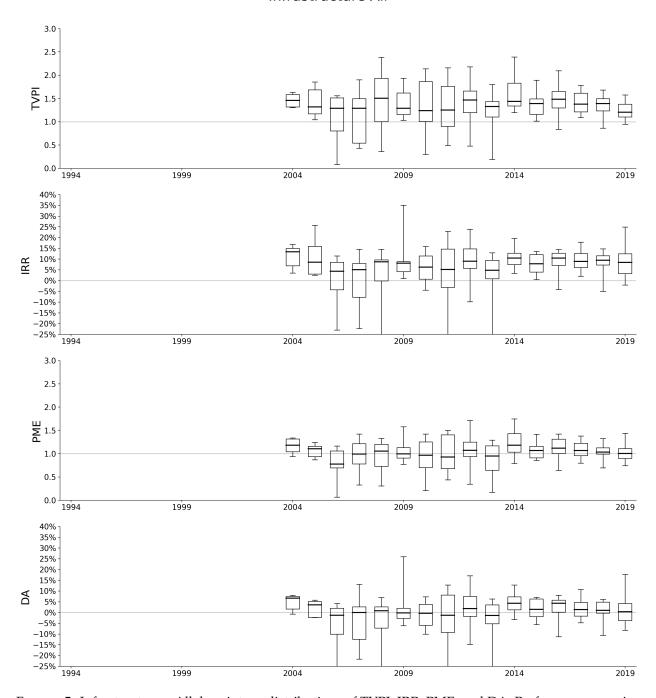


FIGURE 5: Infrastructure – All: by-vintage distributions of TVPI, IRR, PME, and DA. Performance remains stable across vintages, suggesting lower cyclicality relative to Real Estate and Natural Resources.

Natural Resources

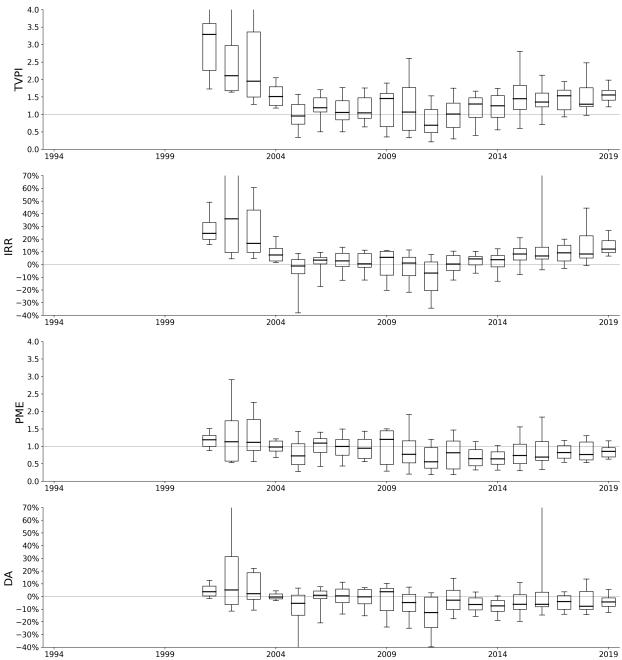


FIGURE 6: Natural Resources: by-vintage distributions of TVPI, IRR, PME, and DA. Wide dispersion across vintages captures the regime sensitivity of commodity-linked investments.

6 Real Assets in Diversified Portfolios

This section evaluates the role of private real assets within diversified multi-asset portfolios, building directly on the empirical insights from Sections 4 and 5. The unsmoothing exercises showed that private real asset returns exhibit substantially higher true volatility and materially stronger co-movement with public benchmarks than reported NAV-based series imply. At the same time, the fund-level dispersion analysis underscored economically significant heterogeneity in cross-sectional outcomes across strategies and vintages. Integrating these insights, we assess how the inclusion of private real assets affects portfolio-level volatility, correlation structure, drawdown characteristics, and inflation sensitivity when compared with portfolios composed solely of liquid public assets.

In doing so, we emphasize two guiding principles that emerge from our earlier results. First, the portfolio value of private real assets depends critically on the use of unsmoothed, economically interpretable return estimates. Using reported returns artificially inflates diversification benefits; using unsmoothed returns reveals their true systematic linkages to growth, inflation, and real-rate shocks. Second, private real assets are not a monolithic category. Real estate, infrastructure, and natural resources carry distinct cash-flow profiles, factor exposures, and macro sensitivities. Their diversification value therefore varies by sub-strategy, consistent with the heterogeneity documented in Sections 3 and 5.

We proceed by analyzing: (1) the unconditional and conditional correlation structure of private real assets relative to core public asset classes; (2) the inflation-hedging properties of private real assets; (3) the contribution of real assets to portfolio drawdowns and recovery dynamics; and (4) the implications for long-horizon strategic allocation.

6.1 Unsmoothed Correlations and the True Degree of Diversification

The evidence in Table 5 demonstrated that unsmoothing increases correlations between private real assets and public benchmarks by 10–40 percentage points across categories. This adjustment reveals that much of the apparent "low beta" behavior in reported real asset returns arises from appraisal smoothing rather than fundamental segmentation.

Unsmoothed real estate returns exhibit correlations of 0.37-0.52 with listed real estate and

global equities—significantly more than the raw values. This is consistent with the literature documenting substantial hidden exposure to public real estate cycles once appraisal lag is removed. Despite higher correlations, real estate maintains moderate idiosyncratic variation, contributing meaningful diversification relative to equities and credit.

Unsmoothed infrastructure returns display the strongest rise in public-market co-movement, with correlations of 0.60–0.74 to listed infrastructure and broad real-asset benchmarks. These magnitudes are consistent with the ARMA–Dimson findings in Table 4, where infrastructure exhibited high long-run betas to public infrastructure indices. Nonetheless, infrastructure retains lower cyclicality than equities, especially in downturns, due to its regulated and contractual cash-flow structure.

Natural resources show the largest increase in commodity-linked correlation—reaching 0.72 with the GSCI and 0.84 with public natural-resource equities—reflecting the dominance of commodity-price shocks in the true economic returns of private resource funds. The high unsmoothed beta and factor sensitivity imply that natural resources primarily enhance diversification through exposure to inflation and supply shocks, rather than through orthogonality to traditional assets.

Overall, the unsmoothed correlations imply that private real assets offer partial, not complete, diversification relative to public markets. Their contribution stems less from low correlation and more from differentiated sensitivity to macro variables, distinct drawdown behavior, and structural cash-flow characteristics.

6.2 Inflation Sensitivity and Real-Rate Dynamics

Section 4 showed that unsmoothed private real asset returns exhibit stronger correlations with breakeven inflation and more negative correlations with real interest rates. These relationships suggest an important portfolio role tied to macro-hedging.

Both Core and VA/Opp real estate series display positive correlations with breakeven inflation (0.37–0.47 unsmoothed). This arises from the partial indexation of rents, the inflation pass-through embedded in lease structures, and the real-option value of new development. However, real estate also demonstrates sensitivity to real rates—reflecting duration-like valuation effects—so its inflation hedging is most effective when inflation shocks coincide with stable or declining real-rate

conditions.

Infrastructure exhibits the clearest inflation-hedging properties in our data. Unsmoothed correlations with breakeven inflation exceed 0.40 across Core and VA/Opp strategies, consistent with regulated or contracted revenue models that include explicit inflation linkages. The negative correlation with real rates is more pronounced than in real estate, providing a natural hedge against rising real-rate environments that challenge traditional fixed-income heavy portfolios.

Natural resources offer the strongest inflation linkage of any private real-asset category. Unsmoothed correlations with both the GSCI and breakevens exceed 0.50, reflecting commodity price pass-through and the deep integration of natural-resource investments into global supply chains. However, their inflation hedging is episodic and highly cyclical: protection is strongest during supply-driven inflation shocks and weakest during demand-driven recessions.

These patterns imply that combining real estate, infrastructure, and natural resources delivers a diversified exposure to different types of inflation risk—contractual, demand-driven, and supply-driven—providing a more complete inflation hedge than any single real-asset category alone.

6.3 Drawdown Behavior and Portfolio Shock Absorption

The analysis in Figure 1 revealed that unsmoothed private real asset series exhibit larger quarteron-quarter shocks than reported returns suggest, yet their drawdown paths show a distinct profile relative to public markets.

Unsmoothed real estate series experience deeper drawdowns than raw reported data indicate, but these drawdowns typically lag public market selloffs by one to two quarters. This staggered pattern provides temporal diversification: real estate does not fully hedge equity crashes, but it spreads drawdown timing over multiple quarters, smoothing the portfolio's peak-to-trough path.

Infrastructure demonstrates the smallest beta to public drawdowns among private real assets. Even when unsmoothed, drawdowns tend to be shallow and quickly reversed, reflecting the inelastic demand for essential services. During equity bear markets, infrastructure reduces the magnitude of portfolio-level drawdowns and shortens recovery horizons.

Natural resources behave as a "crisis amplifier" during supply-driven inflation shocks but

as a "cyclical risk asset" during recessions. This duality makes natural resources a potent but volatile component of diversified portfolios: they hedge specific macro scenarios extremely well but increase tail risk in others. Optimal sizing is therefore essential.

Overall, the drawdown evidence suggests that real assets provide *shock-type diversification* rather than purely statistical diversification. Their value lies in responding differently to specific macroeconomic crises, not in universally low correlations.

6.4 Strategic Allocation and Long-Horizon Implications

Integrating the above findings, the portfolio role of private real assets can be understood through three channels: (1) structural income and contractual cash flows; (2) macro factor diversification; and (3) horizon-dependent risk shaping.

First, real estate and infrastructure add duration-short, income-heavy exposures whose cash flows partially hedge inflation and demonstrate lower sensitivity to earnings cycles than equities. Second, the unsmoothed factor decompositions imply that real assets load meaningfully on inflation, real rates, and commodity shocks—providing orthogonal exposures to the growth-and discount-rate factors that dominate traditional asset classes. Third, because private real assets react to macro shocks with lag and exhibit smoother (though not artificially smooth) adjustment dynamics, they reduce short-horizon portfolio volatility and mitigate cliff-risk during equity drawdowns.

Taken together, these properties support strategic allocations to real assets not for their artificially low reported volatility, but for their distinct macroeconomic exposure profile and their contribution to multi-horizon portfolio stability.

6.5 Synthesis

Real assets meaningfully expand the efficient frontier, but for reasons quite different from those implied by reported appraisal-based returns. Once unsmoothed, their volatility rises to levels comparable to listed analogues, their systematic risk exposures become transparent, and their diversification value shifts from low correlation to macro complementarity. Real estate contributes income stability and moderate inflation protection; infrastructure offers regulated cash flow re-

silience and strong inflation linkage; natural resources provide episodic but powerful hedges against supply-driven inflation and geopolitical shocks. Combined, these exposures enhance the robustness of diversified portfolios to a wider range of macroeconomic environments than traditional assets alone.

In this sense, the integration of private real assets into portfolio construction is best understood not as a pursuit of artificially smoothed return paths but as a deliberate expansion of macro risk coverage—aligning with the empirical adjustments, factor sensitivities, and cross-sectional patterns established in the preceding sections.

7 Conclusion

This paper aimed to clarify the investment properties of closed-end private real asset funds by bringing together three strands of the literature: appraisal-based unsmoothing, factor-based performance evaluation, and large-sample evidence on private fund dispersion. Using the MSCI Private Capital Universe data and a consistent set of public benchmarks and macro factors, we developed an ARMA–Dimson framework that treats reported NAV-based returns as smoothed representations of underlying economic shocks and explicitly links them to listed real estate, infrastructure, and natural resource markets. Our goal was not to "improve" performance, but to recover volatility, comovement, and timing patterns that are more consistent with how markets actually price risk, while preserving the long-run returns investors ultimately realized.

At the aggregate level, the results in Sections 4 and 3 show that appraisal smoothing and reporting lags materially distort the time-series properties of private real asset indices. Reported returns exhibit strong ARMA persistence—particularly for open-end core property funds and core real estate indices—and unusually low volatility compared with their public counterparts. Our ARMA-based unsmoothing procedure, combined with Dimson-style multi-lag factor regressions, raises annualized volatility by a factor of roughly two to three and increases correlations with public benchmarks by 0.2–0.4, depending on asset class. Importantly, these adjustments leave long-run average returns essentially unchanged by construction. Rather than manufacturing alpha, the unsmoothed series reallocate variation across quarters in a way that is consistent with observed public-market shocks and estimated appraisal dynamics, yielding beta estimates that

better reflect the true economic risk embedded in private real assets.

The unsmoothed correlations and factor sensitivities reveal a richer and more intuitive pattern of macroeconomic exposure than raw NAV data suggest. Real estate indices, once unsmoothed, load meaningfully on listed REITs and global equities, with moderate positive correlations to breakeven inflation and modest sensitivity to real rates. Infrastructure funds—especially core strategies—exhibit the clearest link to regulated and contracted cash flows: their unsmoothed returns show strong co-movement with listed infrastructure, significant exposure to inflation expectations, and more negative relationships with real interest rates. Natural resources display the highest unsmoothed beta to commodity indices and public resource equities, reinforcing the idea that they are primarily a vehicle for commodity and inflation risk rather than a low-beta diversifier. Open-end core property funds, which appear almost bond-like in raw data, show substantially higher and more realistic co-movement with listed real estate once their smoothing is stripped away.

Fund-level analysis in Section 5 complements this aggregate picture by documenting substantial cross-sectional dispersion in both nominal and market-adjusted performance. Median TVPIs in the 1.3–1.6 range and median IRRs in the high single to low double digits conceal wide interquartile ranges and long left tails, particularly for value-added/opportunistic real estate and natural resource funds. When benchmarked against broad public comparators via PME and Direct Alpha, the typical real asset fund delivers outcomes roughly in line with public markets: median PMEs near one and median Direct Alphas around zero to modestly positive for real estate and infrastructure, but negative on average for natural resources. At the same time, the upper quartile of Direct Alpha remains economically meaningful across most strategies, highlighting the importance of manager selection, style tilts, and cycle timing. Vintage-level stacks further underscore the role of macro conditions: crisis-era cohorts around 2007–2010 exhibit the widest dispersions and largest underperformance relative to public markets, whereas post-2012 vintages show tighter spreads and PMEs that cluster closer to or slightly above parity.

Taken together, the aggregate and cross-sectional evidence in Sections 4–5 has direct implications for how private real assets should be viewed in diversified portfolios, as discussed in Section 6. Once unsmoothed, private real assets no longer appear as low-volatility, weakly correlated "shock absorbers" in the narrow mean–variance sense. Instead, they emerge as macro-

exposed assets whose diversification value stems from differentiated cash-flow structures and distinct sensitivities to growth, inflation, and real rates. Real estate contributes income-oriented exposure with moderate inflation pass-through and equity-like downside; infrastructure provides long-duration, inflation-linked cash flows with relatively resilient drawdown behavior; natural resources offer episodic but powerful protection against commodity and supply-driven inflation shocks, at the cost of greater tail risk and cyclicality. In combination, these features suggest that the primary portfolio role of private real assets is to broaden macro risk coverage and shape the term and inflation structure of returns, rather than simply to dampen reported volatility.

Our analysis also highlights several limitations and avenues for future research. First, while the ARMA–Dimson framework offers a transparent way to connect appraisal-based returns to public factors, it remains a reduced-form representation of complex valuation and reporting practices; alternative identification strategies, such as fully specified state-space models or transaction-level analyzes, could further refine unsmoothed estimates. Second, benchmark choice and strategy mapping inevitably involve judgment: different regional or sectoral indices, or more granular matching of fund mandates to public analogues, could shift PME and beta estimates at the margin. Third, our study focuses on net-of-fee fund aggregates and does not explicitly model fees, carry structures, or intra-fund leverage, all of which matter for investor outcomes and for comparing private and public vehicles on a like-for-like basis. Finally, we treat liquidity and capital-call dynamics only indirectly; integrating unsmoothed return models with cash flow forecasts, secondary-market pricing, and stress scenarios would enrich the portfolio perspective, particularly for institutions managing liquidity constraints.

Notwithstanding these caveats, the central message of the paper is clear. Once we adjust for appraisal smoothing and align private real asset returns with appropriate public and macro benchmarks, much of the apparent stability and diversification in reported performance can be traced to accounting conventions rather than fundamental insulation from market risk. Yet real assets remain valuable precisely because of their structured exposure to inflation, real rates, and commodity dynamics, and because their cross-sectional dispersion offers scope for skill-based outperformance. For investors and researchers alike, the challenge is not to treat private real assets as a volatility mirage, but to measure and use them in a way that reflects their true risk, return, and portfolio contribution. Our ARMA–Dimson unsmoothing approach and empirical results provide

one step toward that goal, offering a unified framework for interpreting past performance and for designing more transparent, macro-aware allocations to private real assets going forward.

References

- Andonov, Aleksandar, Yael V. Hochberg, and Joshua D. Rauh (2018a) "Political Representation and Governance: Evidence from the Investment Decisions of Public Pension Funds," *Journal of Finance*, 73 (5), 2041–2086.
- Andonov, Aleksandar, Roman Kräussl, and Joshua D. Rauh (2018b) "The Subsidy to Infrastructure as an Asset Class," *NBER Working Paper Series* (w25384), 48.
- ——— (2021) "Institutional Investors and Infrastructure Investing," Stanford University Graduate School of Business Research Paper, 92.
- Bernstein, Shai, Josh Lerner, and Antoinette Schoar (2013) "The investment strategies of sovereign wealth funds," *Journal of Economic Perspectives*, 27 (2), 219–238.
- Brown, Gregory W., Andrei S. Gonçalves, and Wendy Hu (2024) "The Private Capital Alpha," *Fisher College of Business Working Paper Series*, 52.
- Bustos, Ricardo and Andrew DeMond (2025) "The MSCI Private Infrastructure Factor Model," *MSCI Inc*, 46.
- Childs, Paul D., Steven H. Ott, and Timothy J. Riddiough (2002) "Optimal Valuation of Noisy Real Assets," *Real Estate Economics*, 30, 385–414.
- Couts, Spencer J, Andrei S Gonçalves, and Andrea Rossi (2024) "Unsmoothing Returns of Illiquid Funds," *The Review of Financial Studies*, 37 (7), 2110–2155.
- Cremers, K. J. Martijn (2013) "The Performance of Direct Investments in Real Assets: Natural Resources, Infrastructure and Commercial Real Estate."
- Dichev, Ilia D. and Gwen Yu (2011) "Higher risk, lower returns: What hedge fund investors really earn," *Journal of Financial Economics*, 100 (2), 248–263.
- Dimson, Elroy (1979) "Risk measurement when shares are subject to infrequent trading," *Journal of Financial Economics*, 7 (2), 197–226.

- Fisher, Jeffrey, Dean Gatzlaff, David Geltner, and Donald Haurin (2003) "Controlling for the Impact of Variable Liquidity in Commercial Real Estate Price Indices," *Real Estate Economics*, 31 (2), 269–303.
- Fisher, Lynn M. and David J. Hartzell (2016) "Class Differences in Real Estate Private Equity Fund Performance," *Journal of Finance*, 52 (4).
- Geltner, David (1993) "Estimating market values from appraised values without assuming an efficient market," *The Journal of Real Estate Research*, 8 (3), 325–345.
- Geltner, David and William N. Goetzmann (1998) "Two Decades of Commercial Property Returns:

 A Ncreif Index Using Independent Appraisals."
- Geltner, David and David C. Ling (2006) "Considerations in the Design and Construction of Investment Real Estate Research Indices," *Journal of Real Estate Research*, 28 (4), 411–444.
- Getmansky, Mila, Andrew W. Lo, and Igor Makarov (2004) "An econometric model of serial correlation and illiquidity in hedge fund returns," *Journal of Financial Economics*, 74 (3), 529–609.
- Gredil, Oleg, Barry E. Griffiths, and Rudiger Stucke (2023) "Benchmarking private equity: The direct alpha method," *Journal of Corporate Finance*, 81, 102360.
- Haran, Martin, Daniel Lo, and Stanimira Milcheva (2019) "Performance Drivers in Private Infrastructure Funds."
- Harris, Robert S., Tim Jenkinson, and Steven N. Kaplan (2013) "Private equity performance: What do we know?" *Journal of Finance*, 43.
- Horrigan, Holly, Brad Case, David Geltner, and Henry Pollakowski (2009) "REIT-Based Property Return Indices: A New Way to Track and Trade Commercial Real Estate," *The Journal of Portfolio Management Special Real Estate*, 35 (5), 80–91.
- Kaplan, Steven N. and Berk A. Sensoy (2005) "Private Equity Performance: Returns, Persistence, and Capital Flows," *The Journal of Finance*, 60 (4), 1791–1823.
- Korteweg, Arthur and Stefan Nagel (2016) "Risk-adjusting the Returns to Venture Capital," *The Journal of Finance*, 71 (3), 1437–1470.

Lerner, Josh, Antoinette Schoar, and Wan Wongsunwai (2005) "Smart institutions, foolish choices? The limited partner performance puzzle," *NBER working paper* (w11136).

MSCI (2025) "Private Capital Classification System," MSCI Inc, 31.

Newell, Graeme, Helen W. Peng, and Anthony De Francesco (2010) "The performance of unlisted infrastructure in investment portfolios," *Journal of Property Research*, 28 (1), 59–74.

Pagliari, Joseph L. (2016) "Another Take on Real Estate's Role in Mixed-Asset Portfolio Allocations," *Real Estate Economics*, 45 (1), 75–132.

——— (2020) "Real Estate Returns by Strategy: Have Value-Added and Opportunistic Funds Pulled Their Weight?" *Real Estate Economics*, 48 (1), 89–134.

Peng, Helen W. and Graeme Newell (2007) "The significance of infrastructure in investment portfolios."

Phalippou, Ludovic and Oliver Gottschalg (2008) "The Performance of Private Equity Funds," *The Review of Financial Studies*, 22 (4), 1747–1776.

Shen, Junying and Frederic Blanc-Brude (2022) "Building Portfolios with Infrastructure: Performance, Cash Flows Portfolio Allocation," *PGIM IAS*.

Sorensen, Morten and Ravi Jagannathan (2015) "The public market equivalent and private equity performance," *Financial Analysts Journal*, 71 (4), 43–50.

A Appendix

TABLE 6: Data Dictionary

This table documents all series used in the analysis. For each variable we report the source, a brief definition, and (where relevant) the construction of the composite or factor. Private-market series are based on MSCI Private Capital Universe data TWRR (QTD) aggregates; public indices and macro variables are taken from standard vendor and academic sources. Quarterly alignment follows the conventions described in the main text: price and level series are converted to quarterly returns or growth rates, and spreads (e.g., breakevens, real rates, credit spreads) are formed as differences of the corresponding yields or indices. This table is intended to be a self-contained reference for all data inputs underlying the descriptive statistics, unsmoothing models, and factor regressions in the paper.

Series Name	Source	Definition
Private Fund ReturnsReal Assets – All	MSCI Private Capital Universe data	Capitalization-weighted aggregate of all private real asset funds (real estate, infrastructure, and natural resources). Returns are quarterly time-weighted (TWRR).
Real Estate – All	MSCI Private Capital Universe data	Combined sample of all private real estate funds.
Real Estate – Core	MSCI Private Capital Universe data	Core/generalist real estate strategies emphasizing stabilized, income-producing assets.
Real Estate – VA/Opp	MSCI Private Capital Universe data	
Infrastructure – All	MSCI Private Capital Universe data	Aggregate performance of all private infrastructure funds.
Infrastructure – Core	MSCI Private Capital Universe data	Core/generalist infrastructure strategies emphasizing contracted or regulated assets.
Infrastructure – VA/Opp	MSCI Private Capital Universe data	Opportunistic/value-added infrastructure vehicles with higher leverage and development exposure.
Natural Resources	MSCI Private Capital Universe data	Private funds investing in energy, timber, agriculture, and related resource assets.
NFI-ODCE	NCREIF Fund Index	Open-end diversified core equity index of institutional U.S. property funds (gross, unlevered).
MSCI-ACOE MSCI-AFOE	MSCI / PREA MSCI / PREA	MSCI-PREA U.S. Core Open-End Property Fund Index. MSCI-PREA U.S. All Open-End Property Fund Index (core-
	MISCI / FREA	plus and opportunistic).
Public Benchmarks Global Public Equities	Ken French	Developed markets value-weighted market return from the Fama–French data library.
S&P GSCI (Commodities)	Bloomberg (SPGSCI)	Broad commodity price index (S&P Goldman Sachs Commodity Index, total return).
Public Natural Resources	Ken French	Equal-weighted average of Agriculture, Gold, Mines, Coal, and Oil industries from Ken French 48-industry returns.
Global Public Bonds	Bloomberg (LEGATRUU)	Bloomberg Global Aggregate Bond Index, total return USD.
Global Public Infrastructure	Bloomberg (M2WO0INF)	MSCI World Infrastructure Gross Total Return USD Index (listed infrastructure equities).
Global Public Real Estate	Bloomberg (RUGL)	FTSE EPRA NAREIT Developed Total Return USD Index (listed REITs).
Global Public Real Assets	Composite (see text)	Combined real asset proxy: Global Real Estate Index through 1998, 50/50 RE–Infra blend through 2005, and S&P Global Real Assets thereafter.
Risk Factors		
5-Year Breakeven Inflation	Federal Reserve Bank of St. Louis	Market-implied five-year inflation expectation (percentage points).
5-Year Real Yield	Board of Governors of the Federal Reserve System (US)	Yield on five-year Treasury Inflation-Indexed Securities.
Gold	Ken French	Return on gold industry portfolio from Ken French 48-industry data.

TABLE 7: Lag Structure Determination

This table summarizes the information-criterion-based selection of ARMA lag orders for each private return series. For a given process, we report the autoregressive (AR) and moving-average (MA) orders that minimize the Akaike Information Criterion (AIC) and, separately, the Bayesian Information Criterion (BIC), for pure AR, pure MA, and ARMA specifications. Comparing AIC- and BIC-selected orders highlights where preferred specifications are robust (similar orders across criteria) versus where parsimony pushes toward shorter lag structures. These results provide the model-selection backbone for the unsmoothing exercises and for the ARMA specifications reported in Table 3.

AIC					BIC		
Fund	AR	MA	AIC	Fund	AR	MA	BIC
Real Assets - All	10	0	-593.51	Real Assets - All	2	0	-574.13
Real Estate - All	10	0	-581.14	Real Estate - All	6	0	-558.07
Core & Generalist	9	0	-605.58	Core & Generalist	4	0	-582.39
Value-add & Opportunistic	6	0	-546.35	Value-add & Opportunistic	6	0	-525.71
Infrastructure - All	3	0	-364.13	Infrastructure - All	1	0	-355.76
Core & Generalist	1	0	-260.42	Core & Generalist	1	0	-255.61
Value-add & Opportunistic	5	0	-367.12	Value-add & Opportunistic	2	0	-353.15
Natural Resources	3	0	-458.22	Natural Resources	2	0	-448.65
NFI-ODCE	2	0	-791.71	ODCE	2	0	-782.86
MSCI-ACOE	2	0	-354.40	ACOE	2	0	-347.70
MSCI-AFOE	2	0	-349.48	ACOE	2	0	-342.77
Real Assets - All	0	6	-603.66	Real Assets - All	0	6	-583.02
Real Estate - All	0	6	-588.75	Real Estate - All	0	6	-568.11
Core & Generalist	0	6	-606.43	Core & Generalist	0	6	-585.79
Value-add & Opportunistic	0	6	-553.66	Value-add & Opportunistic	0	6	-533.02
Infrastructure - All	0	6	-362.49	Infrastructure - All	0	1	-352.24
Core & Generalist	0	8	-261.06	Core & Generalist	0	1	-254.60
Value-add & Opportunistic	0	10	-367.15	Value-add & Opportunistic	0	5	-349.67
Natural Resources	0	6	-452.37	Natural Resources	0	2	-439.28
NFI-ODCE	0	6	-789.77	ODCE_NTR	0	6	-769.13
MSCI-ACOE	0	4	-349.55	ACOE	0	4	-338.38
MSCI-AFOE	0	6	-346.01	ACOE	0	4	-334.50
Real Assets - All	0	6	-603.66	Real Assets - All	0	6	-583.02
Real Estate - All	0	6	-588.75	Real Estate - All	0	6	-568.11
Core & Generalist	0	6	-606.43	Core & Generalist	0	6	-585.79
Value-add & Opportunistic	1	4	-552.33	Value-add & Opportunistic	1	4	-534.64
Infrastructure - All	3	0	-364.15	Infrastructure - All	1	1	-356.19
Core & Generalist	1	2	-262.78	Core & Generalist	1	0	-255.61
Value-add & Opportunistic	5	2	-372.37	Value-add & Opportunistic	1	1	-361.98
Natural Resources	3	2	-462.05	Natural Resources	1	1	-452.68
NFI-ODCE	2	0	-791.71	ODCE	2	0	-782.86
MSCI-ACOE	2	0	-354.40	ACOE	2	0	-347.70
MSCI-AFOE	2	0	-349.48	ACOE	2	0	-342.77

TABLE 8: Lag Structure and Dimson Betas

This table reports long-horizon (Dimson) beta estimates for all private real asset fund groups relative to their corresponding public benchmarks. For each series, we regress quarterly private returns on the contemporaneous and seven quarterly lags of the relevant public index (broad real assets, listed infrastructure, listed real estate, or public natural resources). The reported Lag 0–Lag 7 coefficients are the individual slope estimates, with standard errors and t-statistics, and trace out the timing of each strategy's response to public-market shocks (front-loaded versus gradual). The "Dimson" beta is the sum of the lagged coefficients and measures the total long-run benchmark sensitivity after allowing for appraisal smoothing and reporting delays. We present results for Real Assets (All), Real Estate (All/Core/Value-Added and Opportunistic), Infrastructure (All/Core/Value-Added and Opportunistic), Natural Resources, NFI-ODCE, MSCI-ACOE, and MSCI-AFOE; the bottom rows of each panel report the standard error of the Dimson estimate, adjusted R^2 , and the effective sample size in quarters. Together, these estimates benchmark the effective market exposure of private real asset vehicles across styles and structures.

Real Assets – All (Benchmark: World Real Asset)

(Deficilitia	(Deficilitark, World Real Asset)				
Variable	Coef	SE	t-stat		
Intercept	0.003	0.007	0.468		
Lag 0	0.210	0.055	3.802	***	
Lag 1	0.124	0.030	4.178	***	
Lag 2	0.072	0.028	2.565	**	
Lag 3	0.079	0.036	2.178	**	
Lag 4	0.120	0.031	3.822	***	
Lag 5	0.094	0.020	4.665	***	
Lag 6	0.059	0.019	3.102	***	
Lag 7	0.028	0.018	1.524		
Dimson	0.785	0.175	4.476	***	
SE	0.175				
Adj R-Squared	0.449				
Quarters	134				

Infrastructure – Core (Benchmark: Public infrastructure)

Variable	Coef	SE	t-stat	
Intercept	0.006	0.004	1.517	
Lag 0	0.447	0.081	5.521	***
Lag 1	0.115	0.045	2.539	**
Lag 2	0.078	0.028	2.823	***
Lag 3	0.143	0.062	2.299	**
Lag 4	0.015	0.047	0.318	
Lag 5	-0.003	0.025	-0.104	
Lag 6	0.127	0.039	3.226	***
Lag 7	-0.048	0.038	-1.265	
Dimson	0.873	0.166	5.273	***
SE	0.166			
Adj R-Squared	0.433			
Quarters	75			

Infrastructure – All (Benchmark: Public infrastructure)

Variable	Coef	SE	t-stat	
Intercept	0.010	0.003	3.052	***
Lag 0	0.302	0.051	5.913	***
Lag 1	0.098	0.033	3.013	***
Lag 2	0.053	0.042	1.259	
Lag 3	0.101	0.031	3.261	***
Lag 4	0.012	0.037	0.317	
Lag 5	0.052	0.040	1.293	
Lag 6	0.102	0.031	3.337	***
Lag 7	-0.056	0.031	-1.797	*
Dimson	0.665	0.087	7.634	***
SE	0.087			
Adj R-Squared	0.495			
Quarters	93			

Infrastructure – Value-Added/Opportunistic (Benchmark: Public infrastructure)

Variable	Coef	SE	t-stat	
Intercept	0.011	0.003	4.061	***
Lag 0	0.246	0.032	7.671	***
Lag 1	0.017	0.039	0.435	
Lag 2	0.077	0.046	1.651	*
Lag 3	0.071	0.030	2.370	**
Lag 4	0.025	0.030	0.832	
Lag 5	0.013	0.035	0.365	
Lag 6	0.121	0.058	2.077	**
Lag 7	-0.043	0.028	-1.533	
Dimson	0.526	0.149	3.534	***
SE	0.149			
Adj R-Squared	0.347			
Quarters	83			

Lag Structure and Dimson Betas (continued)

Real Estate – All (Benchmark: Public Real Estate)

Real Estate – Core (Benchmark: Public Real Estate)

Variable	Coef	SE	t-stat	
Intercept	-0.005	0.007	-0.657	
Lag 0	0.178	0.043	4.106	***
Lag 1	0.139	0.026	5.278	***
Lag 2	0.101	0.036	2.832	***
Lag 3	0.130	0.034	3.803	***
Lag 4	0.157	0.039	4.061	***
Lag 5	0.121	0.020	6.060	***
Lag 6	0.093	0.028	3.277	***
Lag 7	0.055	0.019	2.887	***
Dimson	0.972	0.180	5.401	***
SE	0.180			
Adj R-Squared	0.510			
Quarters	134			

Variable	Coef	SE	t-stat	
Intercept	-0.005	0.007	-0.695	
Lag 0	0.154	0.034	4.560	***
Lag 1	0.116	0.025	4.655	***
Lag 2	0.087	0.028	3.100	***
Lag 3	0.099	0.028	3.516	***
Lag 4	0.131	0.033	4.028	***
Lag 5	0.119	0.024	5.043	***
Lag 6	0.084	0.026	3.161	***
Lag 7	0.057	0.023	2.459	**
Dimson	0.847	0.147	5.762	***
SE	0.147			
Adj R-Squared	0.456			
Quarters	134			

Real Estate – Value-Added/Opportunistic (Benchmark: Public Real Estate)

Natural Resources (Benchmark: Public Natural Resources)

Variable	Coef	SE	t-stat	
Intercept	-0.003	0.008	-0.439	
Lag 0	0.186	0.047	3.958	***
Lag 1	0.145	0.028	5.248	***
Lag 2	0.100	0.040	2.471	**
Lag 3	0.141	0.038	3.764	***
Lag 4	0.166	0.041	4.044	***
Lag 5	0.114	0.020	5.844	***
Lag 6	0.105	0.031	3.385	***
Lag 7	0.043	0.017	2.513	**
Dimson	1.001	0.196	5.116	***
SE	0.196			
Adj R-Squared	0.499			
Quarters	134			

Variable	Coef	SE	t-stat	
Intercept	0.004	0.005	0.767	
Lag 0	0.273	0.031	8.956	***
Lag 1	0.127	0.021	6.060	***
Lag 2	0.088	0.024	3.706	***
Lag 3	0.076	0.014	5.409	***
Lag 4	0.053	0.024	2.188	**
Lag 5	0.024	0.025	0.988	
Lag 6	0.039	0.020	1.993	**
Lag 7	-0.038	0.017	-2.209	**
Dimson	0.644	0.102	6.300	***
SE	0.102			
Adj R-Squared	0.554			
Quarters	134			

NFI-ODCE
(Benchmark: Public Real Estate)

MSCI-ACOE (Benchmark: Public Real Estate)

Variable	Coef	SE	t-stat	
Intercept	0.000	0.005	0.036	
Lag 0	0.042	0.030	1.392	
Lag 1	0.084	0.027	3.101	***
Lag 2	0.112	0.026	4.383	***
Lag 3	0.118	0.026	4.613	***
Lag 4	0.108	0.026	4.078	***
Lag 5	0.094	0.020	4.809	***
Lag 6	0.064	0.019	3.381	***
Lag 7	0.042	0.020	2.170	**
Dimson	0.665	0.146	4.559	***
SE	0.146			
Adj R-Squared	0.542			
Quarters	134			

Variable	Coef	SE	t-stat	
Intercept	0.000	0.004	-0.064	
Lag 0	0.031	0.023	1.348	
Lag 1	0.120	0.029	4.097	***
Lag 2	0.140	0.034	4.124	***
Lag 3	0.165	0.025	6.709	***
Lag 4	0.143	0.022	6.363	***
Lag 5	0.124	0.022	5.510	***
Lag 6	0.074	0.029	2.533	**
Lag 7	0.047	0.023	2.040	**
Dimson	0.844	0.155	5.444	***
SE	0.155			
Adj R-Squared	0.631			
Quarters	62			

Lag Structure and Dimson Betas (continued)

MSCI-AFOE (Benchmark: Public Real Estate)

Variable	Coef	SE	t-stat	
Intercept	0.001	0.004	0.213	
Lag 0	0.031	0.025	1.247	
Lag 1	0.120	0.030	4.029	***
Lag 2	0.137	0.034	4.000	***
Lag 3	0.171	0.026	6.693	***
Lag 4	0.149	0.023	6.370	***
Lag 5	0.127	0.023	5.541	***
Lag 6	0.075	0.029	2.542	**
Lag 7	0.050	0.024	2.093	**
Dimson	0.860	0.155	5.531	***
SE	0.155			
Adj R-Squared	0.628			
Quarters	62			
, .	0.000			

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